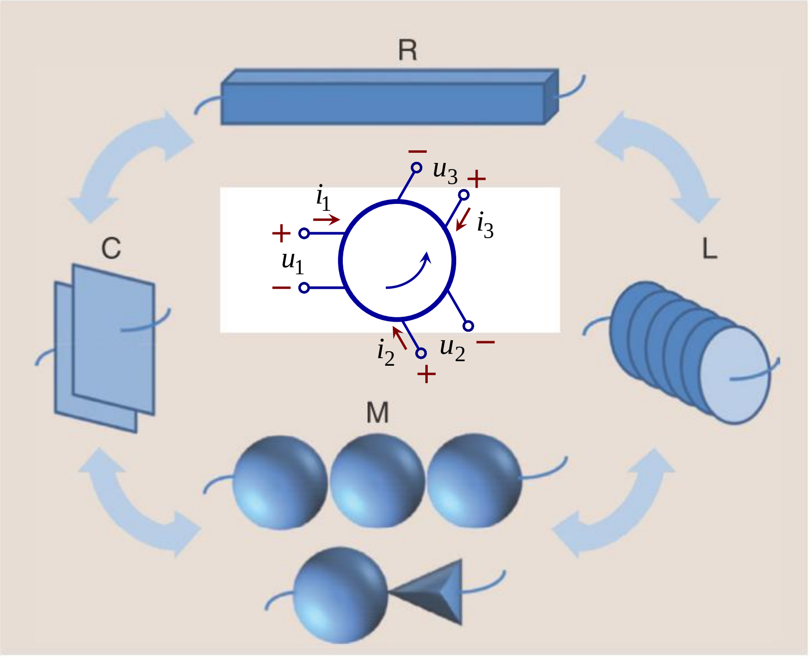
Практикум из рачунарске анализе трофазних кола
5. Фреквенцијски домен



Милка Потребић Иваниш

Никола Баста

MATLAB: Simscape
Foundation Library
Utilities
Simulink

ОДРЕЂИВАЊЕ АМПЛИТУДСКЕ & ФАЗНЕ КАРАКТЕРИСТИКЕ

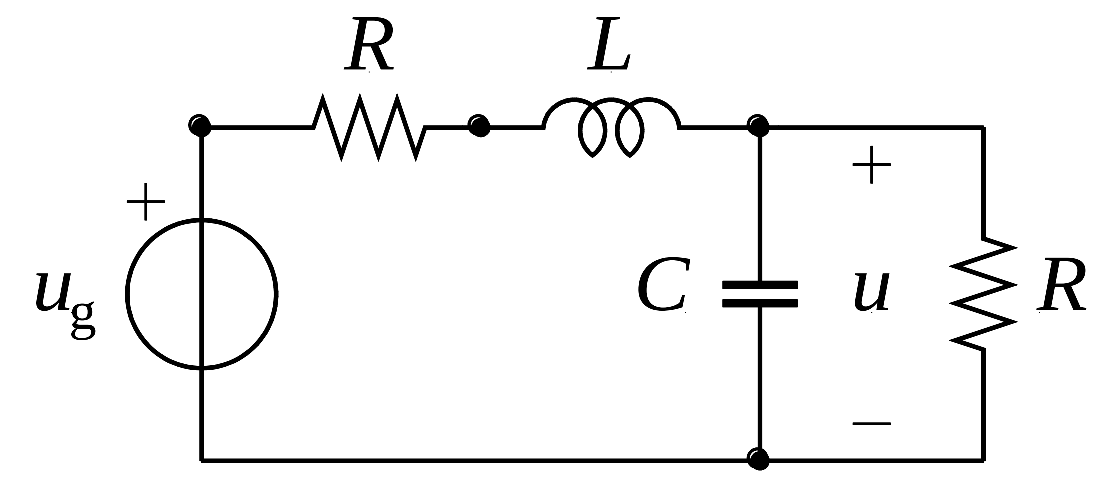
Филтар пропусник ниских учестаности

Вредности елемената електричног кола са слике су познате и постоји веза $L = R^2 C$.

(а) Одредити трансфер функцију (уопштену комплексну преносну функцију електричног кола, трансмитансу напона) $\underline{H}(\underline{s}) = \dfrac{\underline{U}(\underline{s})}{\underline{U}_{\mathrm{g}}(\underline{s})}$.

(б) Нацртати амплитудску карактеристику.

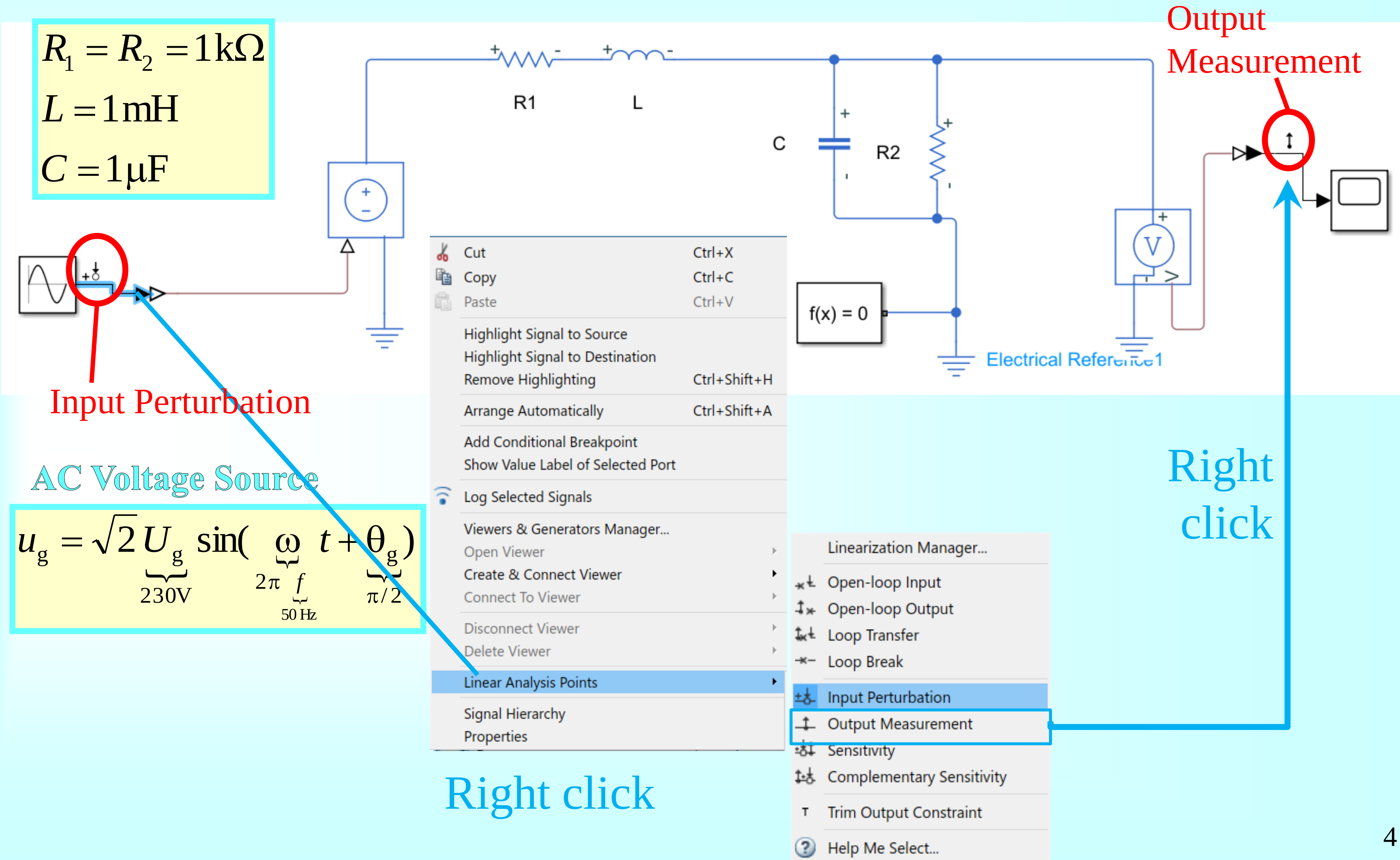
(в) Одредити пропусни опсег 3 dB.



$$u_{\mathrm{g}} = \sqrt{2} U_{\mathrm{g}} \cos(\omega t + \theta_{\mathrm{g}})$$

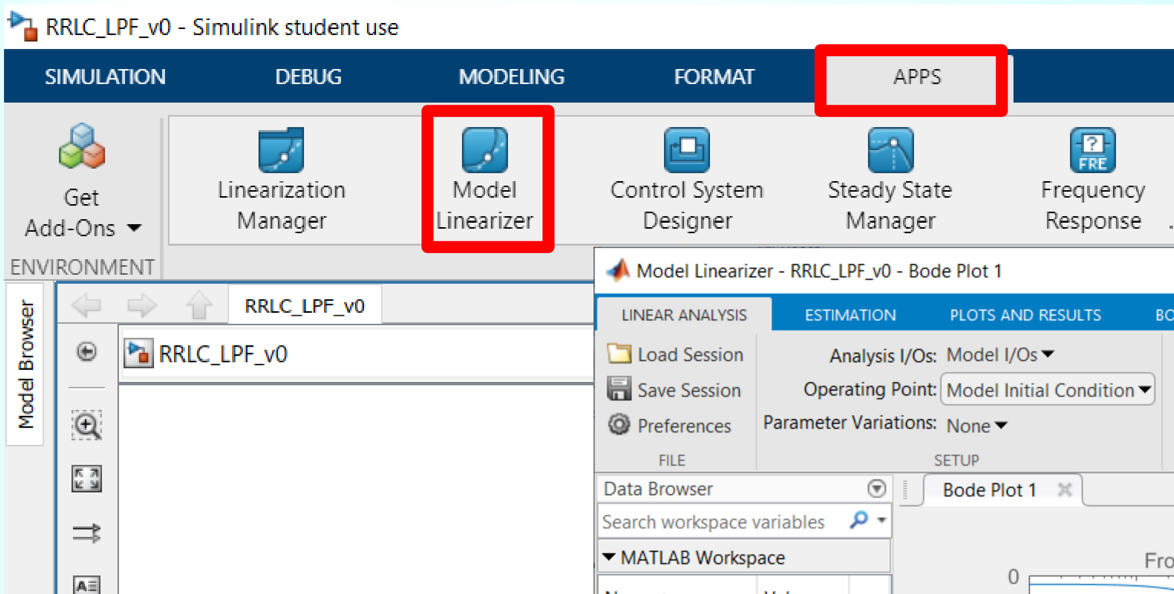
Simulink, Simscape, Foundation Library, Utilities

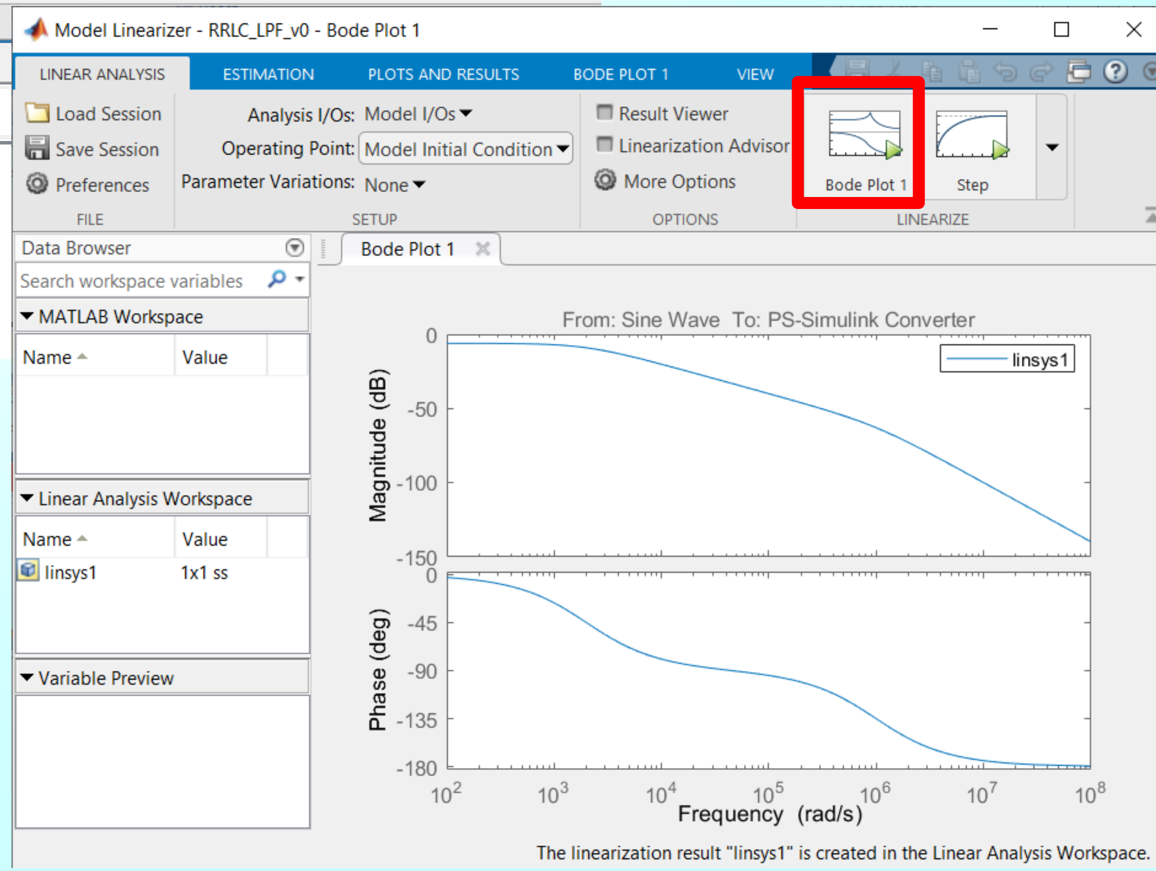
$$R_1 = R_2 = 1\,\mathrm{k\Omega}$$
$$L = 1\,\mathrm{mH}$$
$$C = 1\,\mathrm{\mu F}$$



$$u_g = \sqrt{2}\underbrace{U_g}_{230\mathrm{V}} \sin(\underbrace{\omega}_{2\pi \underbrace{f}_{50\,\mathrm{Hz}}} t + \underbrace{\theta_g}_{\pi/2})$$

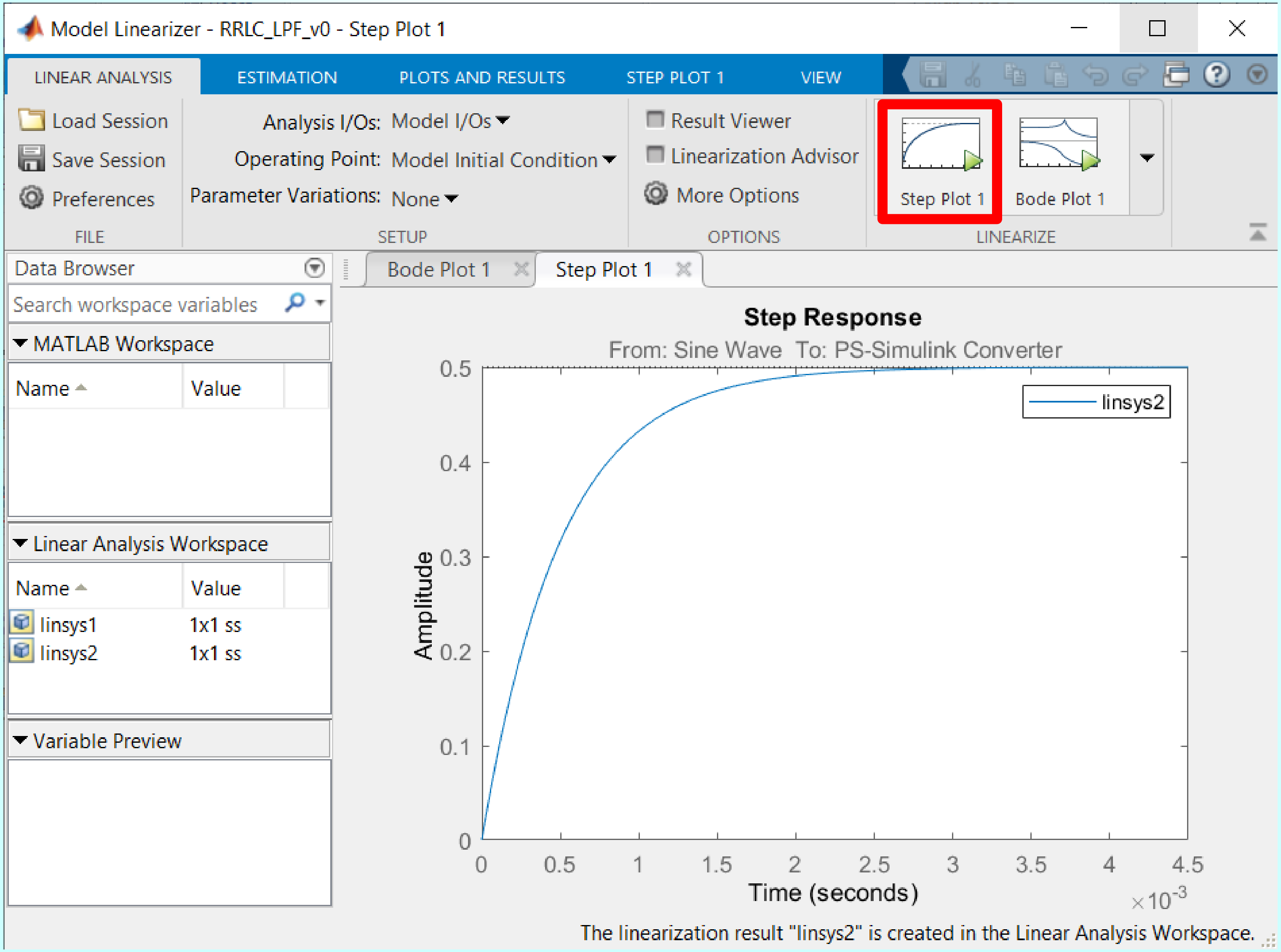
Simulink: Apps -> Model Linearizer



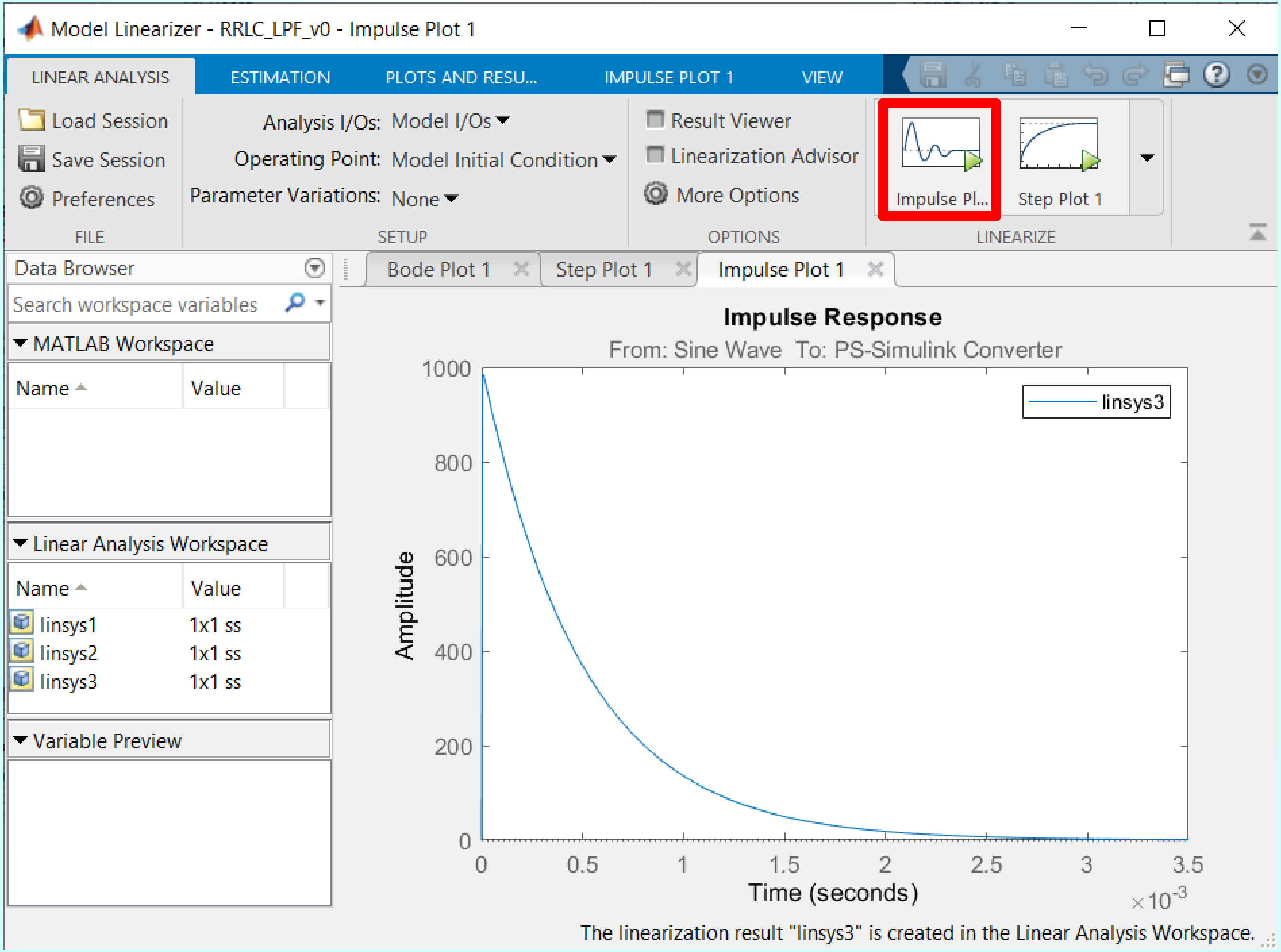


Model Linearizer:
Bode Plot –
амплитудска и фазна
карактеристика

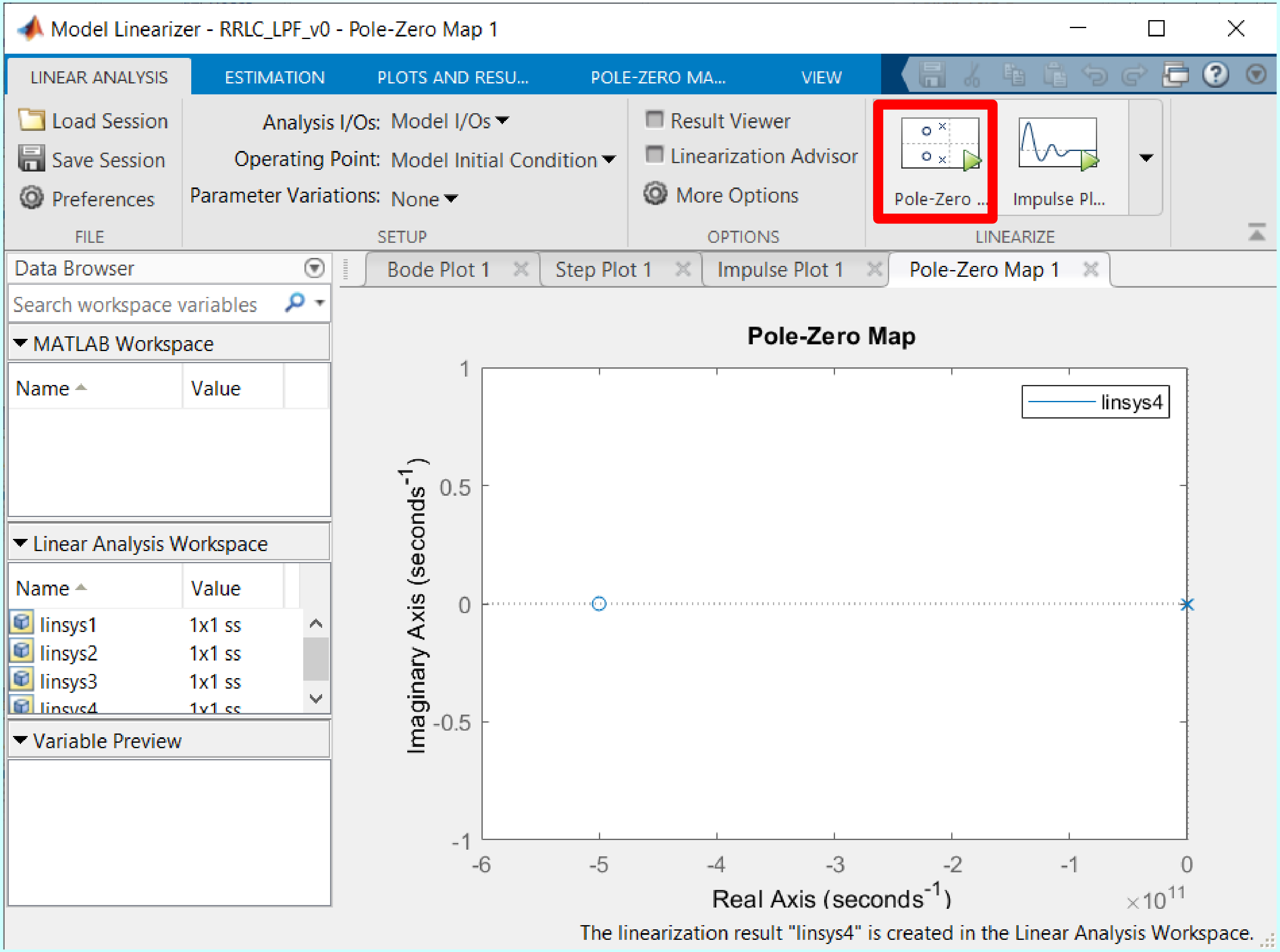
Одскочни одзив – Step Plot



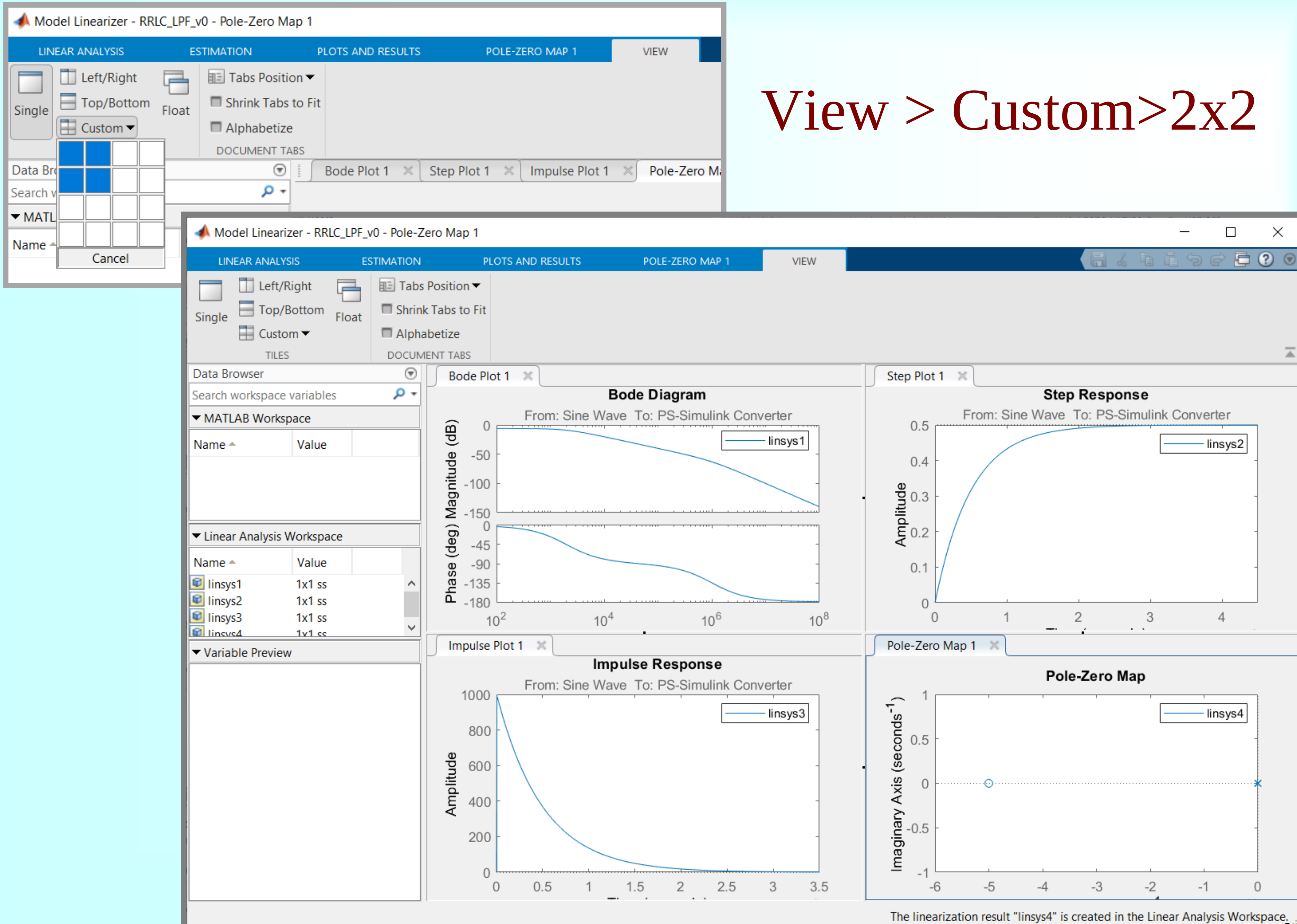
Импулсни одзив – Impulse Plot



Полови и нуле трансфер функције – Pole Zero Plot



View > Custom>2x2



ФРЕКВЕНЦИЈСКИ ОДЗИВ

Određivanje uopštene kompleksne funkcij

```
clear variables
syms w C L R
assume(0 < R & 0 < C & 0 < L )

zamena = L == C*R^2
```

zamena = $L = C\,R^2$

```
vrednosti = [C == 1, R == 1]
```

vrednosti = $(C = 1 \quad R = 1)$

```
syms s
Z1 = R + s*L
```

Z1 = $R + L\,s$

```
Z2 = 1/(s*C + 1/R)
```

Z2 =
$$\frac{1}{C\,s + \frac{1}{R}}$$

```
syms H(s)
H(s) = subs(Z2/(Z1 + Z2), lhs(zamena), rhs(zamena))
```

H(s) =
$$\frac{1}{\left(C\,s+\frac{1}{R}\right)\left(R+\frac{1}{C\,s+\frac{1}{R}}+C\,R^2\,s\right)}$$

```
simplify(H(s))
```

ТРАНСФЕР ФУНКЦИЈА

ans =
$$\frac{1}{C^2\,R^2\,s^2+2\,C\,R\,s+2}$$

АМПЛИТУДСКА И ФАЗНА КАРАКТЕРИСТИКА

Amplitudska i fazna karakteristika

```
syms Hjw(w)
Hjw(w) = simplify(subs(H(s), s, 1i*w))
```

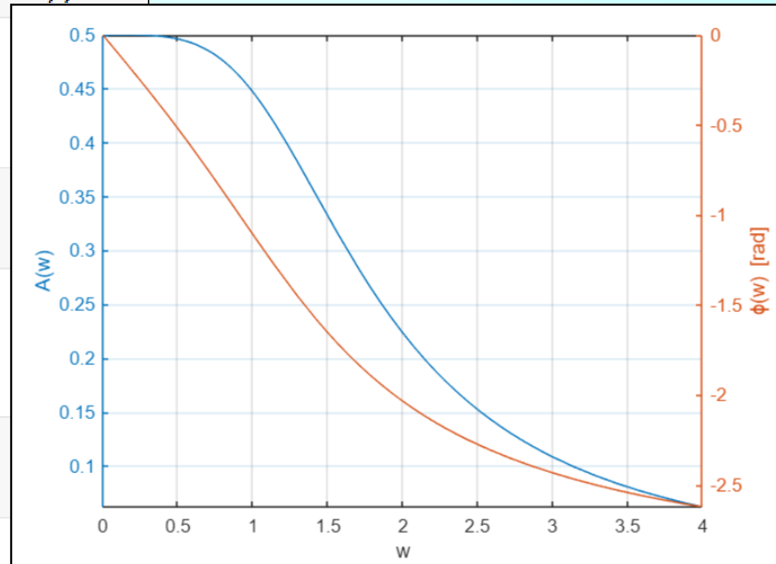
Hjw(w) =
$$-\frac{C^2\,R^2\,w^2+2\,C\,R\,w\,\mathrm{i}-2}{C^4\,R^4\,w^4+4}$$

```
syms Aw(w)
Aw(w) = simplify(abs(Hjw(w)))
```

Aw(w) =
$$\frac{\left|C^2\,R^2\,w^2+2\,C\,R\,w\,\mathrm{i}-2\right|}{\left|C^4\,R^4\,w^4+4\right|}$$

```
syms phi(w)
phi(w) = expand(angle(Hjw(w)))
```

phi(w) =
$$\mathrm{atan2}\left(-\frac{2\,C\,R\,w}{C^4\,R^4\,w^4+4}, \frac{2}{C^4\,R^4\,w^4+4}-\frac{C^2\,R^2\,w^2}{C^4\,R^4\,w^4+4}\right)$$




```
figure
yyaxis left
fplot(w, subs(Aw(w), lhs(vrednosti), rhs(vrednosti)), [0,4])
ylabel('A(w)')
yyaxis right
fplot(w, subs(phi(w), lhs(vrednosti), rhs(vrednosti)), [0,4])
ylabel('\phi(w) [rad]')


xlabel('w')
grid on
```

MATLAB: Symbolic Toolbox

Određivanje propusnog opsega

```
Aref = Aw(0)
```

Aref =
$\frac{1}{2}$

```
assume(0<w)
w3dB = solve(Aw(w) == Aref/sqrt(2), w)
```

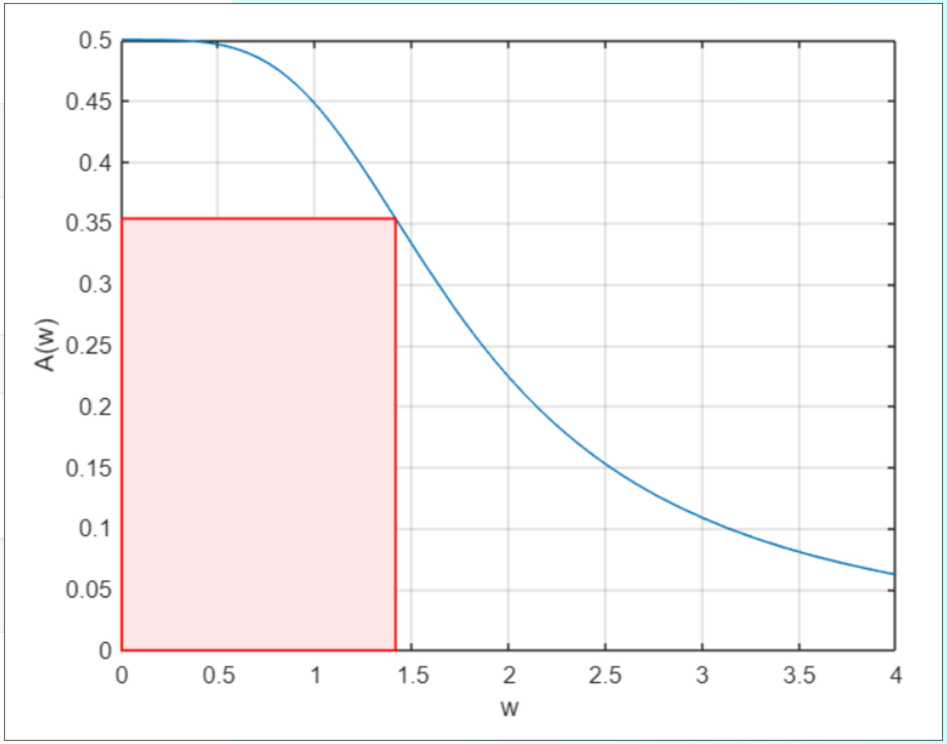
w3dB =
$\frac{\sqrt{2}}{C\,R}$

```
BandPass3dB = [0, w3dB]
```

BandPass3dB =
$\begin{pmatrix} 0 & \frac{\sqrt{2}}{C\,R} \end{pmatrix}$

```
wg1 = subs(w3dB(1), lhs(vrednosti), rhs(vrednosti))
```

wg1 = $\sqrt{2}$



Označavanje propusnog opsega u grafiku

```
figure
fplot(w, subs(Aw(w), lhs(vrednosti), rhs(vrednosti)), [0,4])
hold on
rectangle('Position',[0, 0, double(wg1), double(Aref/sqrt(2))],...
          'EdgeColor','r',...
          'FaceColor',[1 0.9 0.9],...
          'LineWidth',1)
hold off
ylabel('A(w)')
xlabel('w')
grid on
```

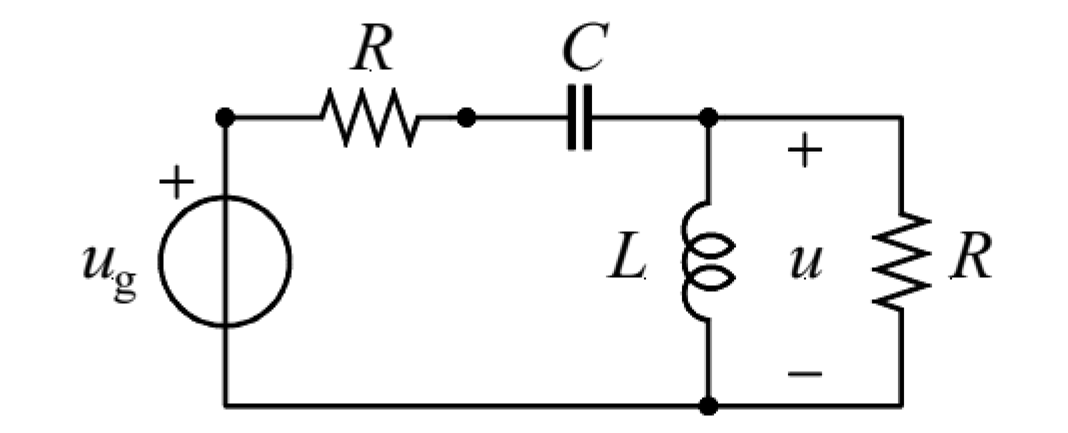
Филтар пропусник високих учестаности

Вредности елемената електричног кола са слике су познате и постоји веза $L = R^2 C$.

(а) Одредити трансфер функцију (уопштену комплексну преносну функцију електричног кола, трансмитансу напона) $\underline{H}(\underline{s}) = \frac{\underline{U}(\underline{s})}{\underline{U}_{\mathrm{g}}(\underline{s})}$.

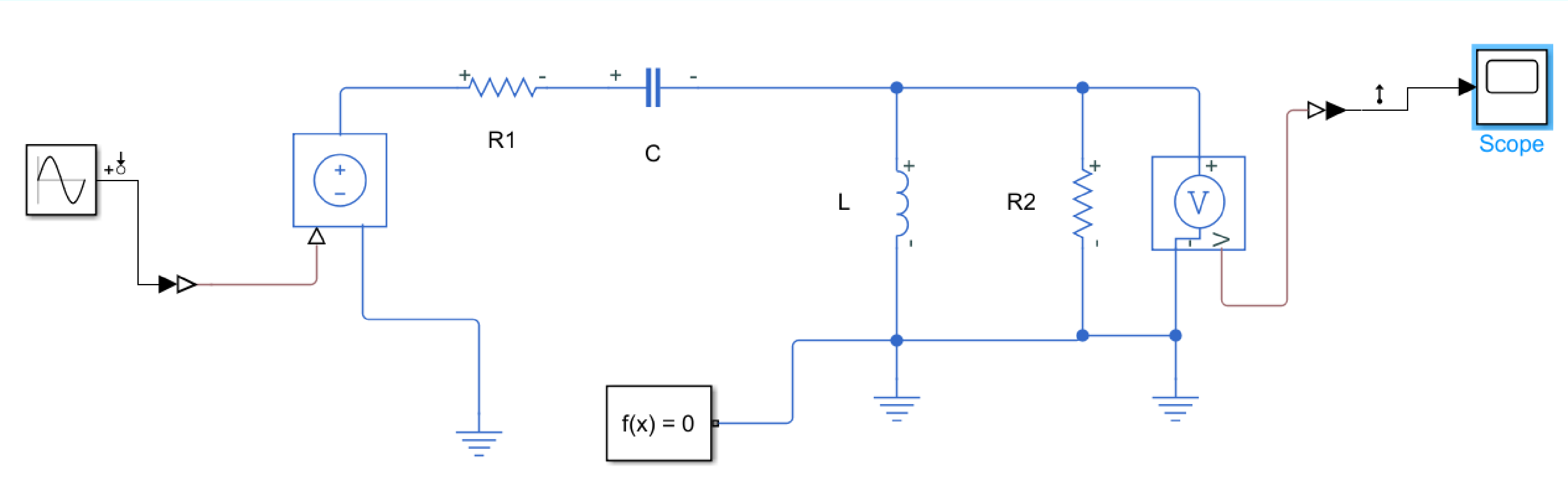
(б) Нацртати амплитудску карактеристику.

(в) Одредити пропусни опсег 3 dB.



$$u_{\mathrm{g}} = \sqrt{2}U_{\mathrm{g}}\cos(\omega t + \theta_{\mathrm{g}})$$

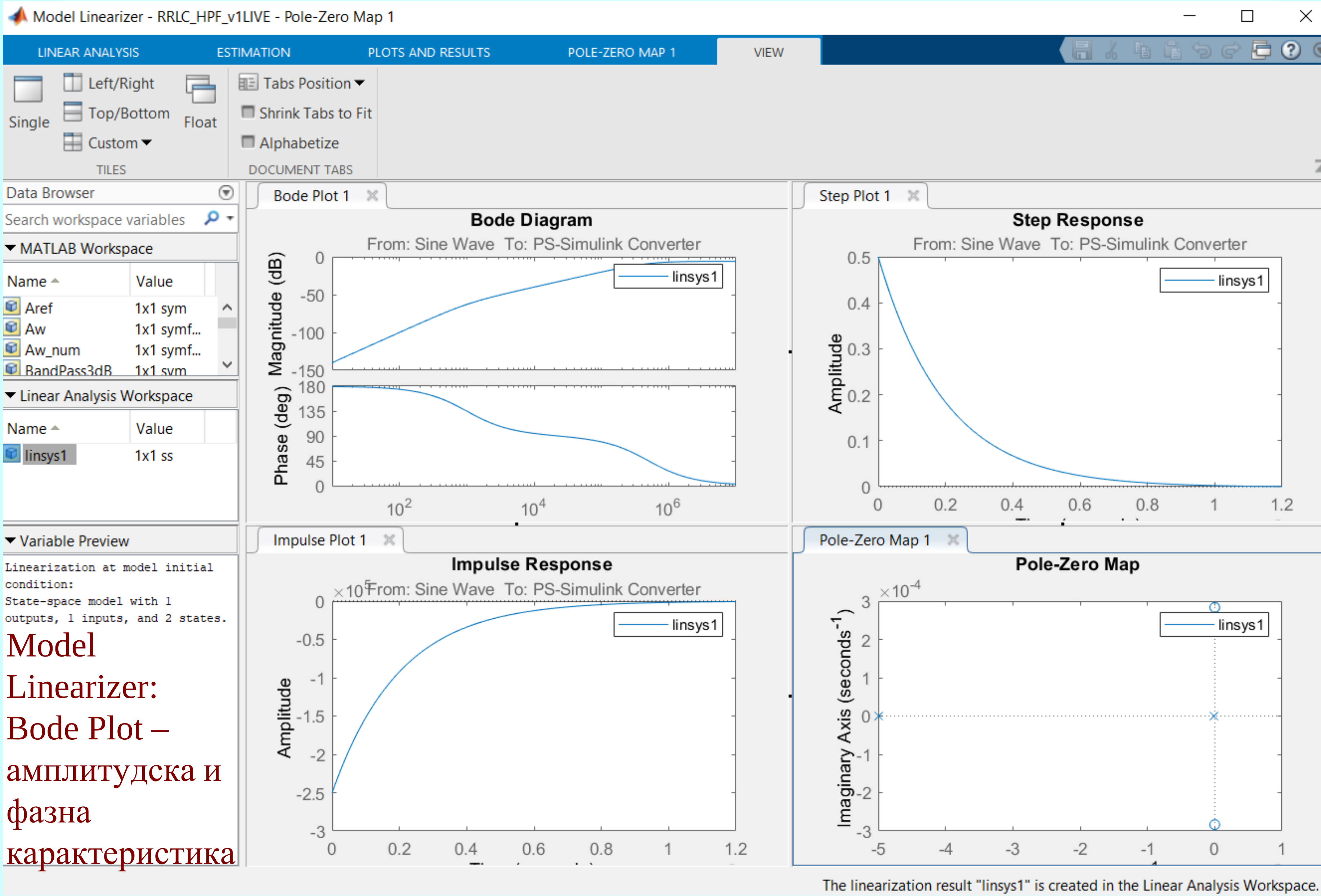
Simulink, Simscape, Foundation Library, Utilities



AC Voltage Source

$$u_g = \sqrt{2}\underbrace{U_g}_{230\text{V}}\sin(\underbrace{\omega}_{2\pi\underbrace{f}_{50\text{ Hz}}}t+\underbrace{\theta_g}_{\pi/2})$$

$$R_1 = R_2 = 1\text{k}\Omega$$
$$L = 1\text{mH}$$
$$C = 1\mu\text{F}$$



Model Linearizer: Bode Plot – амплитудска и фазна карактеристика

MATLAB: Symbolic Toolbox

Određivanje uopštene kompleksne funkcije

```
clear variables
syms w C L R
assume(0 < R & 0 < C & 0 < L )

zamena = L == C*R^2
```

zamena = $L = C\,R^2$

```
vrednosti = [C == 1, R == 1]
```

vrednosti = $\left(C = 1 \quad R = 1\right)$

```
syms s
Z1 = R + 1/(s*C)
```

Z1 =

$$R + \frac{1}{C\,s}$$

```
Z2 = 1/(1/(s*L) + 1/R)
```

Z2 =

$$\frac{1}{\frac{1}{R} + \frac{1}{L\,s}}$$

```
syms H(s)
H(s) = subs(Z2/(Z1 + Z2), lhs(zamena), rhs(zamena))
```

H(s) =

$$\frac{1}{\left(\frac{1}{R} + \frac{1}{C\,R^2\,s}\right)\left(R + \frac{1}{C\,s} + \frac{1}{\frac{1}{R} + \frac{1}{C\,R^2\,s}}\right)}$$

```
simplify(H(s))
```

ans =

$$\frac{C^2\,R^2\,s^2}{2\,C^2\,R^2\,s^2 + 2\,C\,R\,s + 1}$$

Amplitudska i fazna karakteristika

```
syms Hjw(w)
Hjw(w) = subs(simplify(H(s)), s, 1i*w)
```

Hjw(w) =

$$-\frac{C^2\,R^2\,w^2}{-2\,C^2\,R^2\,w^2 + 2\,C\,R\,w\,\mathrm{i} + 1}$$

```
syms Aw(w)
Aw(w) = simplify(abs(Hjw(w)))
```

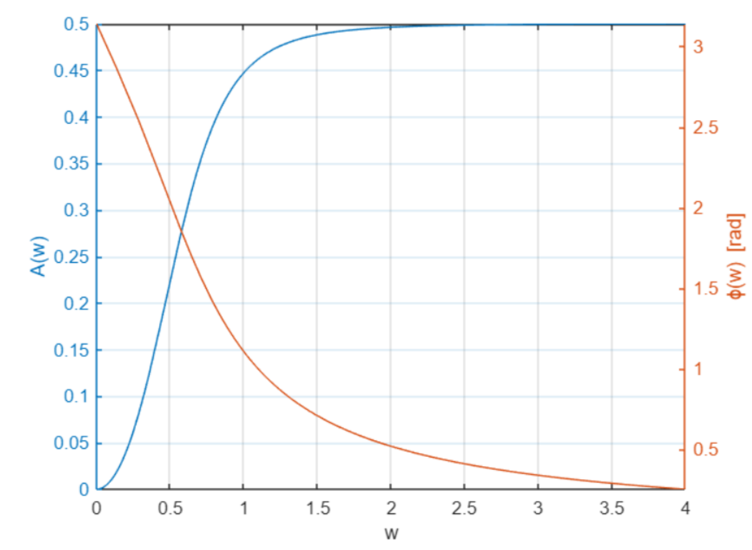
Aw(w) =

$$\frac{C^2\,R^2\,|w|^2}{\left|-2\,C^2\,R^2\,w^2 + 2\,C\,R\,w\,\mathrm{i} + 1\right|}$$

```
syms phi(w)
phi(w) = expand(angle(Hjw(w)))
```

phi(w) =

$$\mathrm{angle}\left(-\frac{w^2}{-2\,C^2\,R^2\,w^2 + 2\,C\,R\,w\,\mathrm{i} + 1}\right)$$



Crtanje amplitudske i fazne karakteristike

```
figure
yyaxis left
fplot(w, subs(Aw(w), lhs(vrednosti), rhs(vrednosti)), [0,4])
ylabel('A(w)')
yyaxis right
fplot(w, subs(phi(w), lhs(vrednosti), rhs(vrednosti)), [0,4])
ylabel('\phi(w) [rad]')
xlabel('w')
grid on
```

Određivanje propusnog opsega

```
Aref = limit(Aw(w), w, Inf)
```

Aref =

$$\frac{1}{2}$$

```
assume(0<w)
w3dB = solve(Aw(w) == Aref/sqrt(2), w)
```

w3dB =

$$\frac{\sqrt{2}}{2\,C\,R}$$

```
BandPass3dB = [ w3dB, Inf]
```

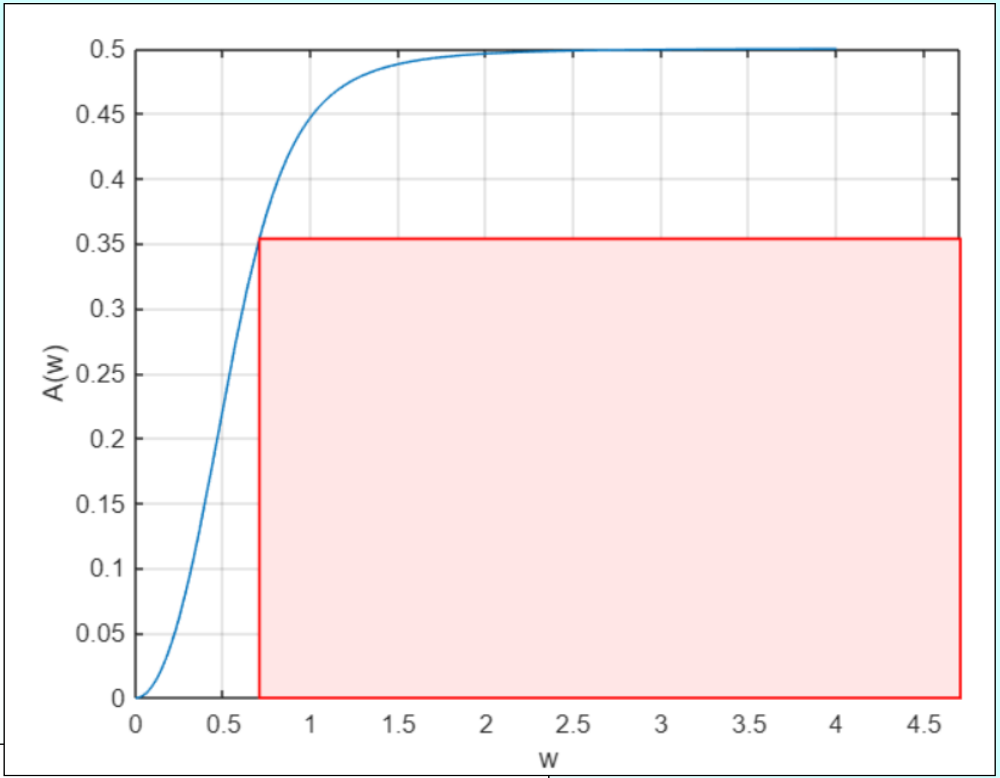
BandPass3dB =

$$\left(\frac{\sqrt{2}}{2\,C\,R}\quad \infty\right)$$

```
wg1 = subs(w3dB(1), lhs(vrednosti), rhs(vrednosti))
```

wg1 =

$$\frac{\sqrt{2}}{2}$$



Označavanje propusnog opsega u grafiku

```
figure
fplot(w, subs(Aw(w), lhs(vrednosti), rhs(vrednosti)), [0,4])
hold on
rectangle('Position',[double(wg1), 0, 4, double(Aref/sqrt(2)) ], ...
          'EdgeColor','r',...
          'FaceColor',[1 0.9 0.9],...
          'LineWidth',1)
hold off
ylabel('A(w)')
xlabel('w')
grid on
```

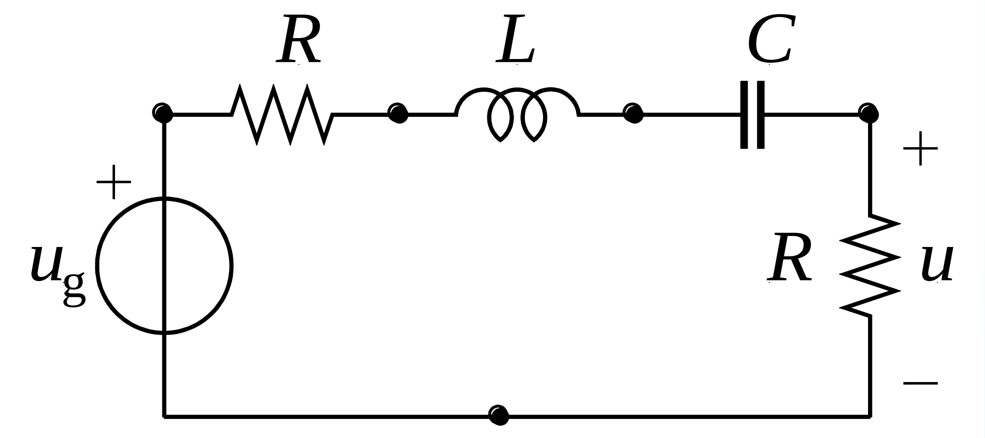
Филтар пропусник опсега учестаности

Вредности елемената електричног кола са слике су познате и постоји веза $L = R^2 C$.

(а) Одредити трансфер функцију (уопштену комплексну преносну функцију електричног кола, трансмитансу напона) $\underline{H}(\underline{s}) = \dfrac{\underline{U}(\underline{s})}{\underline{U}_{\mathrm{g}}(\underline{s})}$.

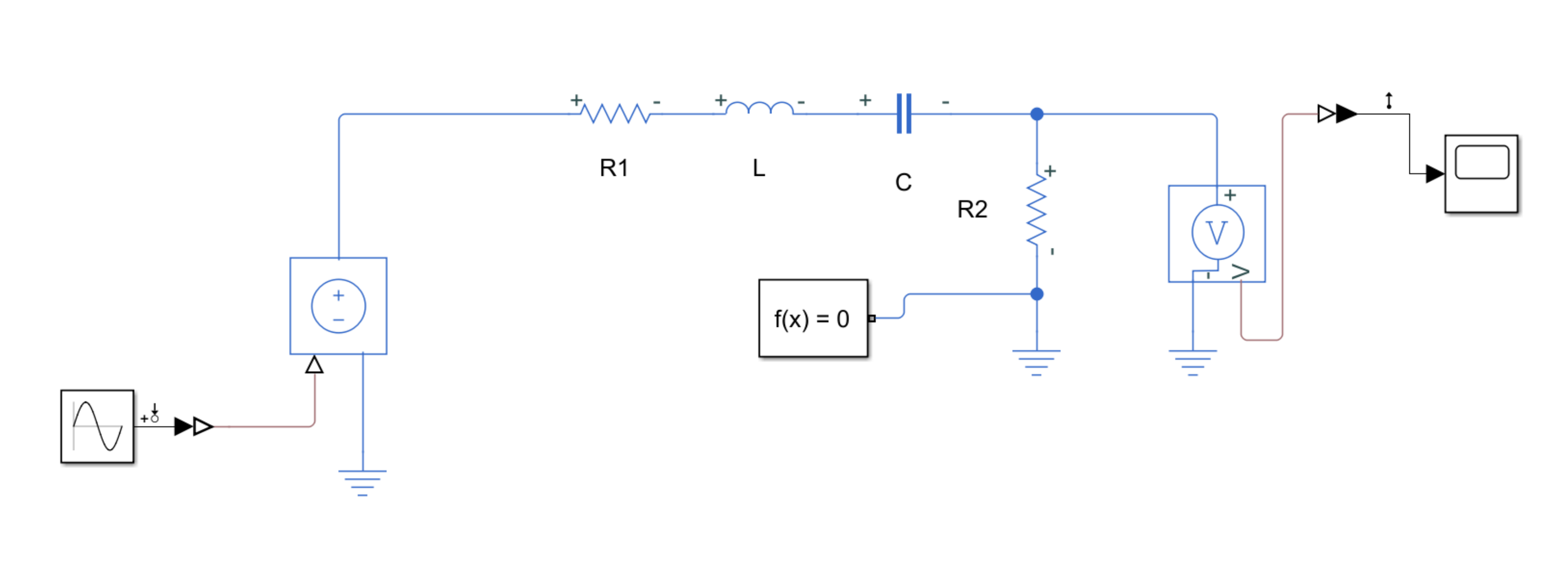
(б) Нацртати амплитудску карактеристику.

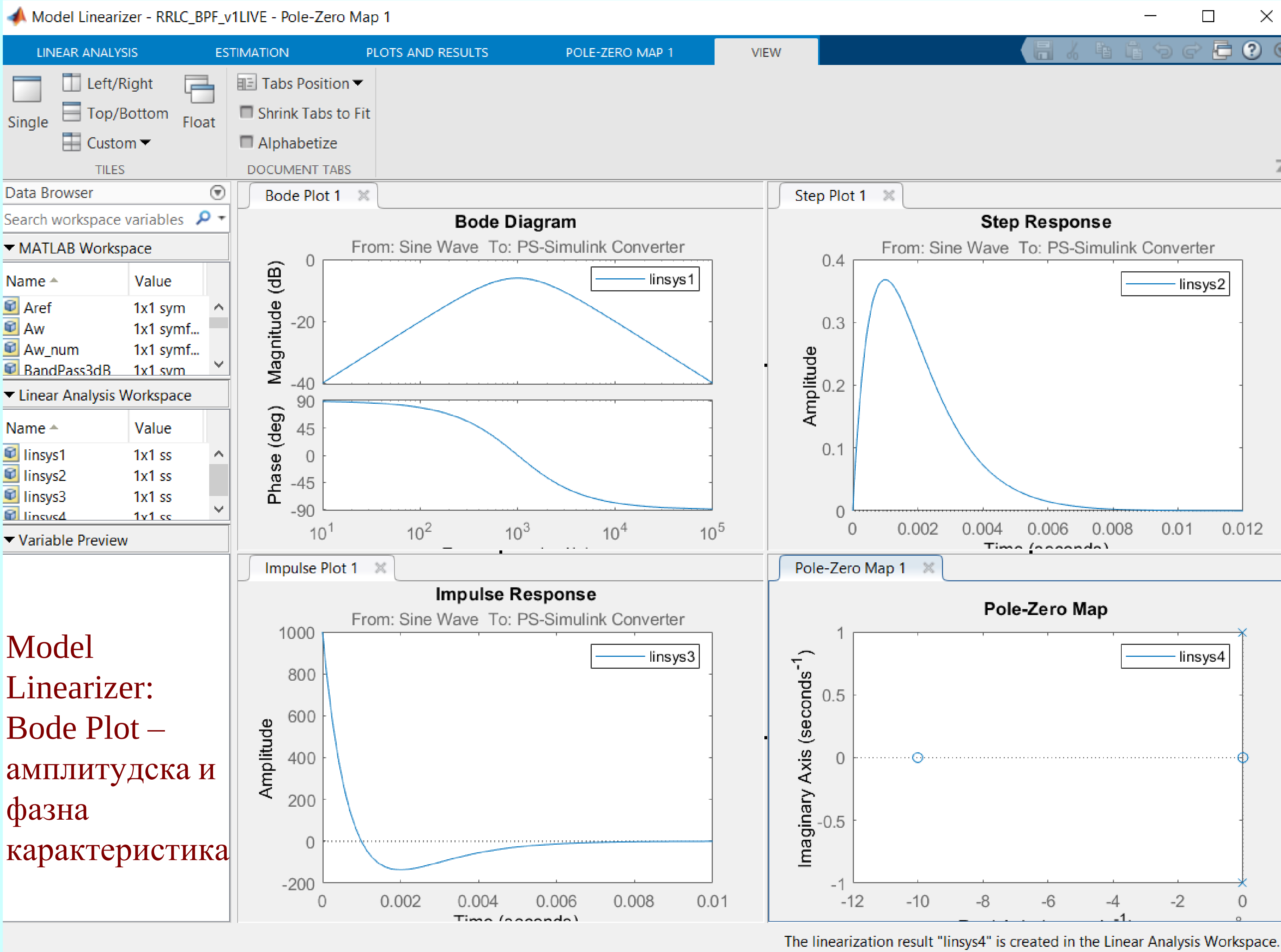
(в) Одредити пропусни опсег 3 dB.



$$u_{\mathrm{g}} = \sqrt{2} U_{\mathrm{g}} \cos(\omega t + \theta_{\mathrm{g}})$$

Simulink Simscape Foundation Library Utilities





Model Linearizer: Bode Plot – амплитудска и фазна карактеристика

Određivanje uopštene kompleksne funkcije

MATLAB: Symbolic Toolbox

```
clear variables
syms w C L R
assume(0 < R & 0 < C & 0 < L )

zamena = L == C*R^2
```

zamena = $L = C\,R^2$

```
vrednosti = [C == 1, R == 1]
```

vrednosti = $(C = 1 \quad R = 1)$

```
syms s H(s)
H(s) = simplify(subs(R/(R + s*L + 1/(s*C) + R), lhs(zamena), rhs(zamena)))
```

H(s) =

$$\frac{C\,R\,s}{(C\,R\,s+1)^2}$$

Amplitudska i fazna karakteristika

```
syms Hjw(w)
Hjw(w) = simplify(subs(H(s), s, 1i*w))
```

Hjw(w) =

$$\frac{C\,R\,w\,\mathrm{i}}{(1+C\,R\,w\,\mathrm{i})^2}$$

```
syms Aw(w)
assume(0 < w)
Aw(w) = simplify(abs(Hjw(w)))
```

Aw(w) =

$$\frac{C\,R\,w}{|1+C\,R\,w\,\mathrm{i}|^2}$$

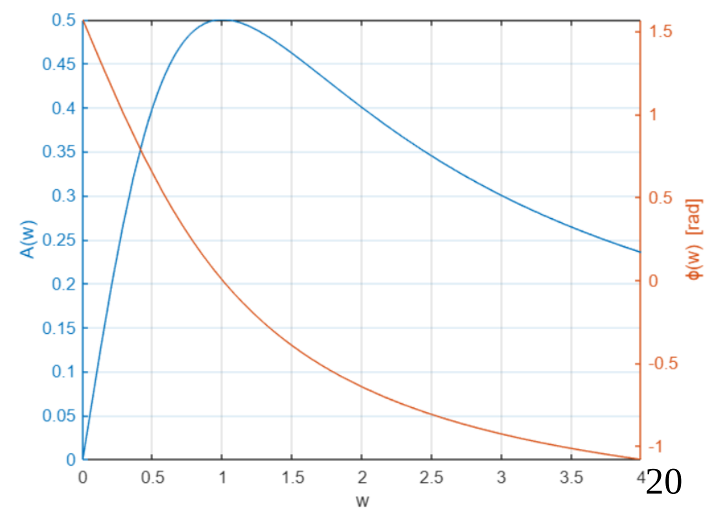
```
syms phi(w)
phi(w) = angle(Hjw(w))
```

phi(w) =

$$\mathrm{angle}\left(\frac{w\,\mathrm{i}}{(1+C\,R\,w\,\mathrm{i})^2}\right)$$

Crtanje amplitudske i fazne karakteristike

```
figure

yyaxis left
fplot(w, subs(Aw(w), lhs(vrednosti), rhs(vrednosti)), [0,4])
ylabel('A(w)')

yyaxis right
fplot(w, subs(phi(w), lhs(vrednosti), rhs(vrednosti)), [0,4])
ylabel('\phi(w) [rad]')
xlabel('w')
grid on
```



Određivanje propusnog opsega

```
syms Aw_num(w)
assume(0 < w)
Aw_num(w) = subs(Aw, lhs(vrednosti), rhs(vrednosti))
```

Aw_num(w) =

$$\frac{w}{w^2+1}$$

```
wg = solve(diff(Aw_num,w) == sym(0), w, 'Real', true)
```

wg = 1

```
Aref = subs(Aw_num(wg))
```

Aref =

$$\frac{1}{2}$$

```
w3dB = simplify(solve(Aw(w) == Aref/sqrt(2)))
```

w3dB =

$$\begin{pmatrix} \frac{\sqrt{2}-1}{C\,R} \\ \frac{\sqrt{2}+1}{C\,R} \end{pmatrix}$$

```
BandPass3dB = w3dB.'
```

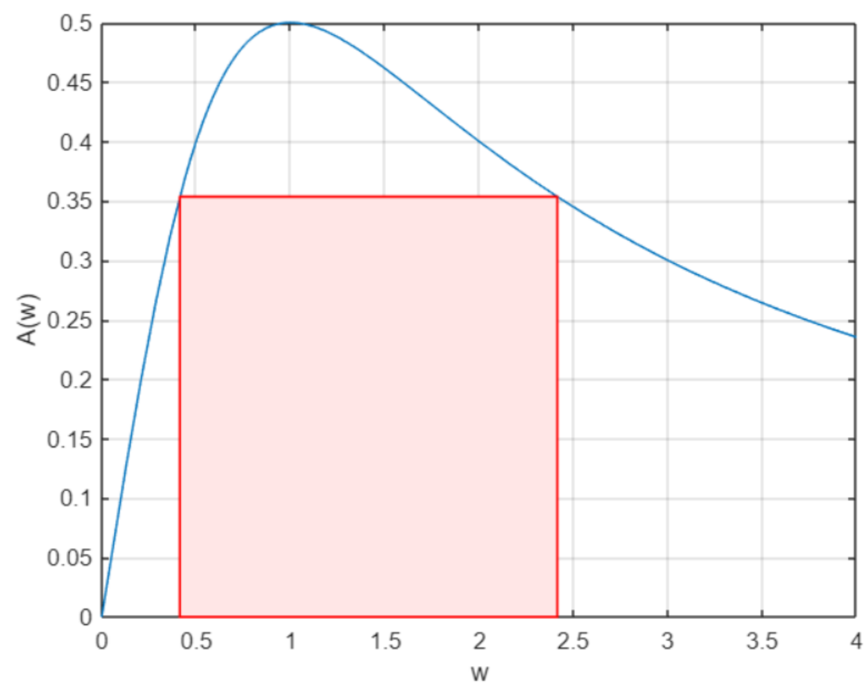
BandPass3dB =

$$\begin{pmatrix} \frac{\sqrt{2}-1}{C\,R} & \frac{\sqrt{2}+1}{C\,R} \end{pmatrix}$$

```
wg1 = subs(w3dB(1), lhs(vrednosti), rhs(vrednosti))
```

wg1 = $\sqrt{2}-1$

```
wg2 = subs(w3dB(2), lhs(vrednosti), rhs(vrednosti))
```



Označavanje propusnog opsega u grafiku

```
figure
fplot(w, subs(Aw(w), lhs(vrednosti), rhs(vrednosti)), [0,4])
hold on
rectangle('Position',[double(wg1), 0, double(wg2-wg1), double(Aref/sqrt(2))],...
          'EdgeColor','r',...
          'FaceColor',[1 0.9 0.9],...
          'LineWidth',1)
hold off
ylabel('A(w)')
xlabel('w')
grid on
```

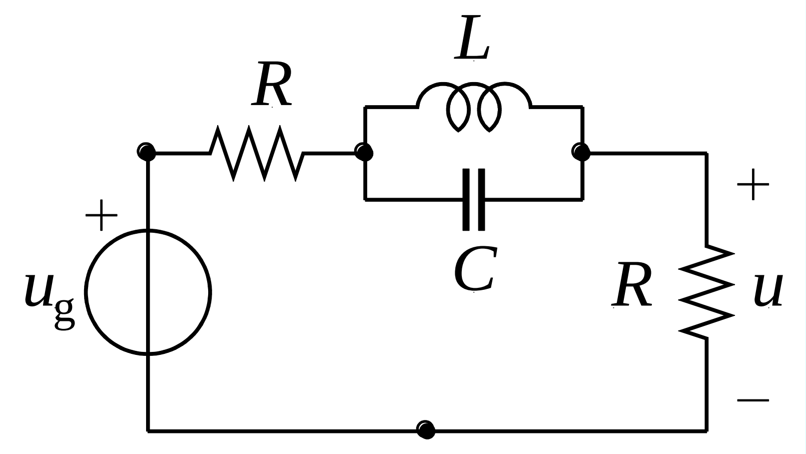
Филтар непропусник опсега учестаности

Вредности елемената електричног кола са слике су познате и постоји веза $L = R^2 C$.

(а) Одредити трансфер функцију (уопштену комплексну преносну функцију електричног кола, трансмитансу напона) $\underline{H}(\underline{s}) = \dfrac{\underline{U}(\underline{s})}{\underline{U}_\mathrm{g}(\underline{s})}$.

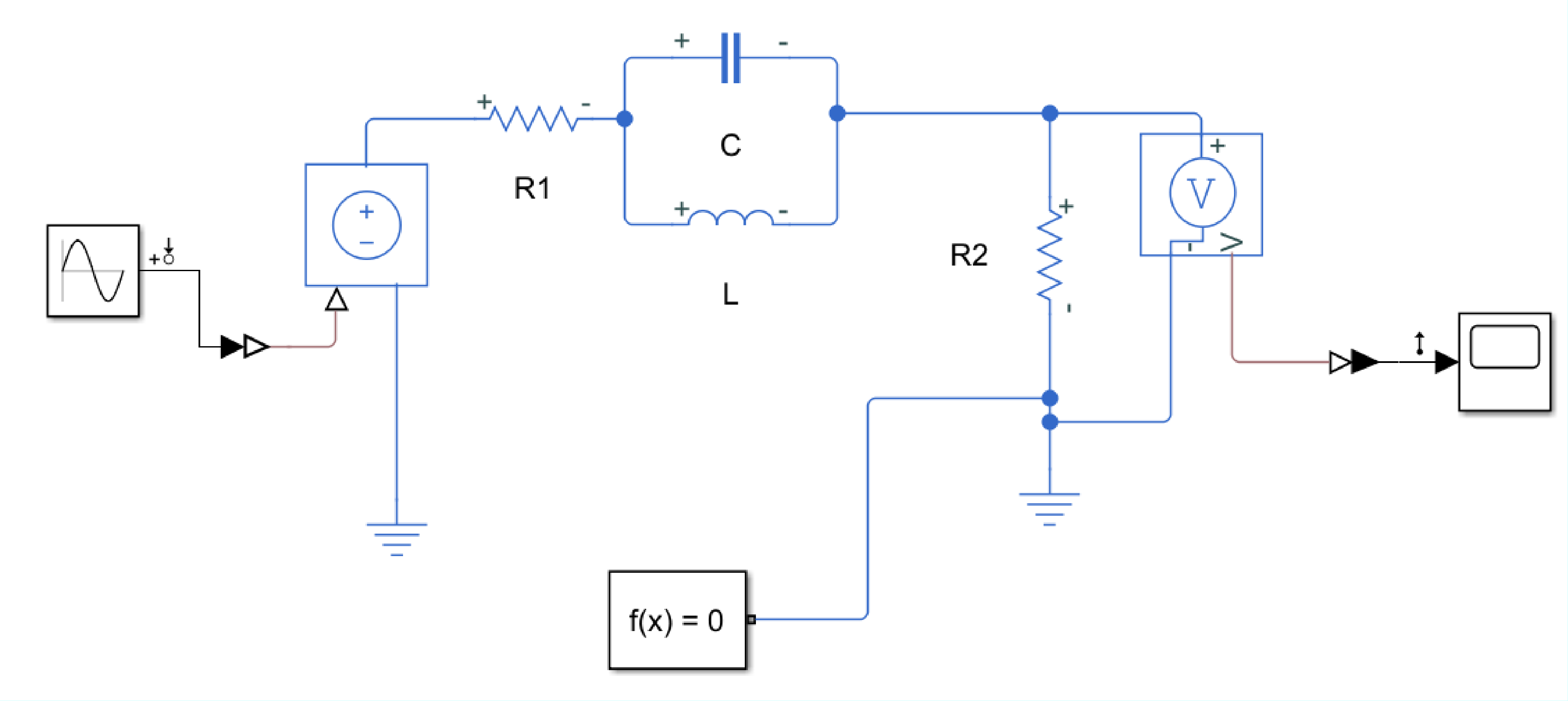
(б) Нацртати амплитудску карактеристику.

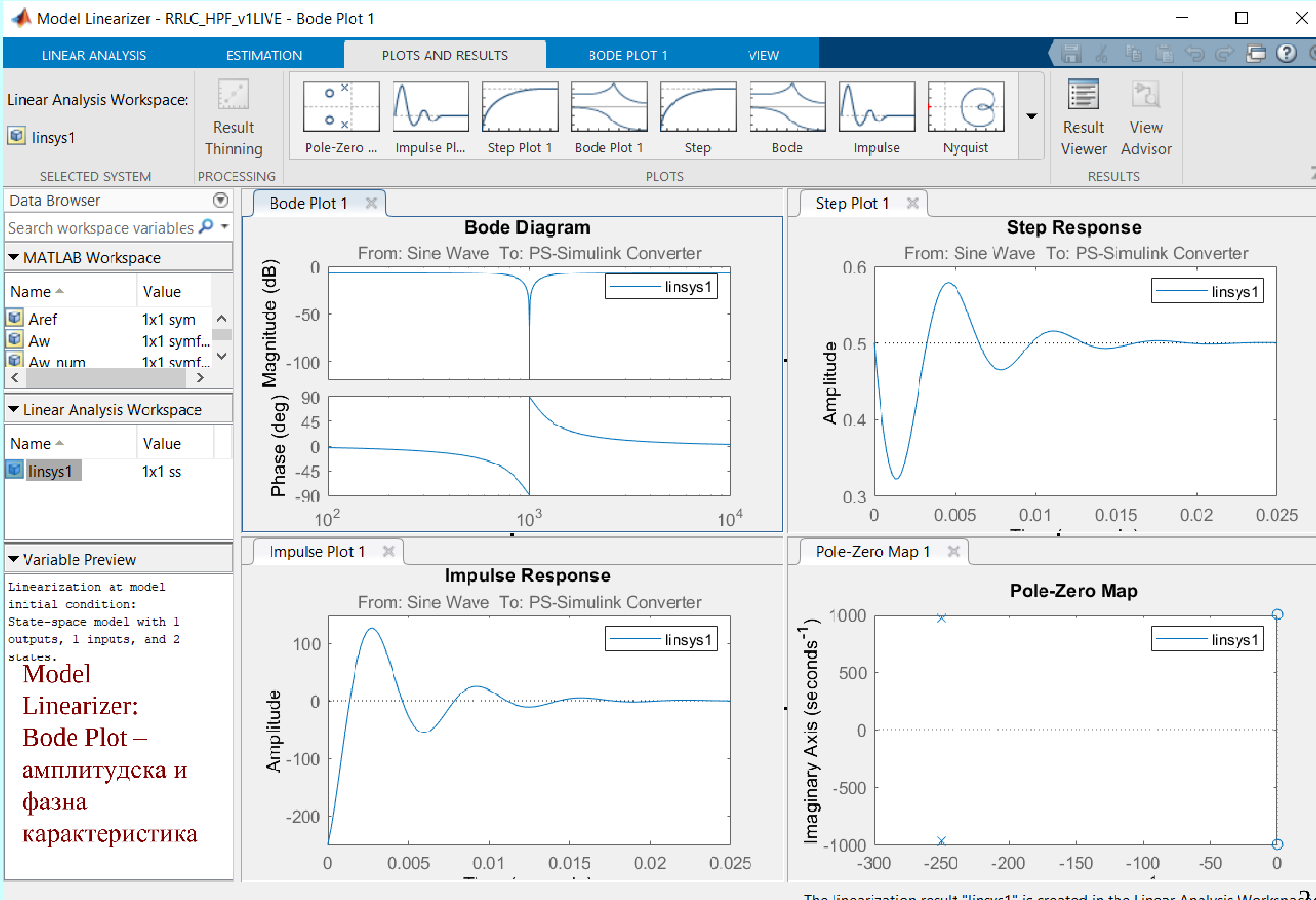
(в) Одредити пропусни опсег 3 dB.



$$u_\mathrm{g} = \sqrt{2}U_\mathrm{g}\cos(\omega t + \theta_\mathrm{g})$$

Simulink Simscape Foundation Library Utilities





Model Linearizer: Bode Plot – амплитудска и фазна карактеристика

MATLAB: Symbolic Toolbox

Određivanje uopštene kompleksne funkcije

```
clear variables
syms w C L R
assume(0 < R & 0 < C & 0 < L )

zamena = L == C*R^2
```

zamena = $L = C\,R^2$

```
vrednosti = [C == 1, R == 1]
```

vrednosti = $\left(C = 1 \quad R = 1\right)$

```
syms s H(s)
H(s) = simplify(subs(R/(R + 1/(1/(s*L) + s*C) + R), lhs(zamena), rhs(zamen
```

H(s) =

$$\frac{C^2\,R^2\,s^2 + 1}{2\,C^2\,R^2\,s^2 + C\,R\,s + 2}$$

Amplitudska i fazna karakteristika

```
syms Hjw(w)
Hjw(w) = simplify(subs(H(s), s, 1i*w))
```

Hjw(w) =

$$-\frac{C^2\,R^2\,w^2 - 1}{-2\,C^2\,R^2\,w^2 + C\,R\,w\,\mathrm{i} + 2}$$

```
syms Aw(w)
assume(0 < w)
Aw(w) = simplify(abs(Hjw(w)))
```

Aw(w) =

$$\frac{\left|C^2\,R^2\,w^2 - 1\right|}{\left|-2\,C^2\,R^2\,w^2 + C\,R\,w\,\mathrm{i} + 2\right|}$$

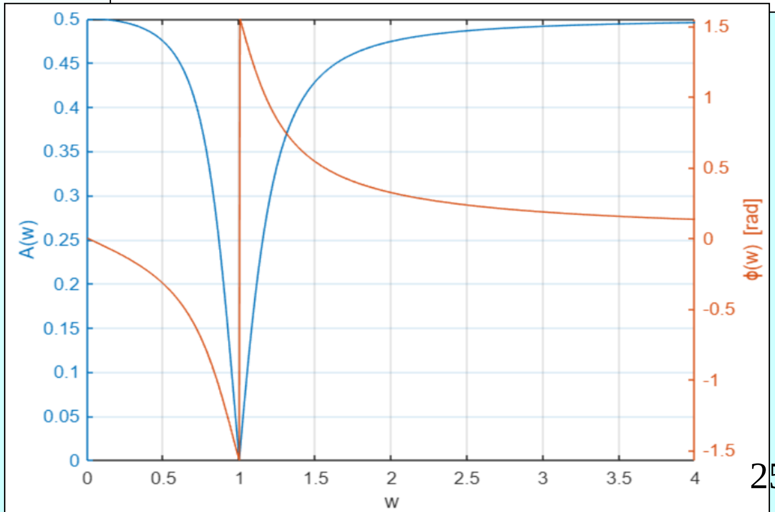
```
syms phi(w)
phi(w) = angle(Hjw(w))
```

phi(w) =

$$\mathrm{angle}\left(-\frac{C^2\,R^2\,w^2 - 1}{-2\,C^2\,R^2\,w^2 + C\,R\,w\,\mathrm{i} + 2}\right)$$

Crtanje amplitudske i fazne karakteristike

```
figure

yyaxis left
fplot(w, subs(Aw(w), lhs(vrednosti), rhs(vrednosti)), [0,4])
ylabel('A(w)')

yyaxis right
fplot(w, subs(phi(w), lhs(vrednosti), rhs(vrednosti)), [0,4])
ylabel('\phi(w) [rad]')
xlabel('w')
grid on
```



Određivanje propusnog opsega

```
syms Aw_num(w)
Aref = Aw(0)
```

Aref =

$$\frac{1}{2}$$

```
assume(w>0)
w3dB = expand(solve(Aw(w) == Aref/sqrt(2), w))
```

w3dB =

$$\begin{pmatrix} \frac{\sqrt{\frac{9}{2}-\frac{\sqrt{17}}{2}}}{2\,C\,R} \\ \frac{\sqrt{\frac{\sqrt{17}}{2}+\frac{9}{2}}}{2\,C\,R} \end{pmatrix}$$

```
Aw_num(w) = subs(Aw, lhs(vrednosti), rhs(vrednosti))
```

Aw_num(w) =

$$\frac{|w^2-1|}{\sqrt{w^2+(2\,w^2-2)^2}}$$

```
wg1 = subs(w3dB(1), lhs(vrednosti), rhs(vrednosti))
```

wg1 =

$$\frac{\sqrt{\frac{9}{2}-\frac{\sqrt{17}}{2}}}{2}$$

```
wg2 = subs(w3dB(2), lhs(vrednosti), rhs(vrednosti))
```

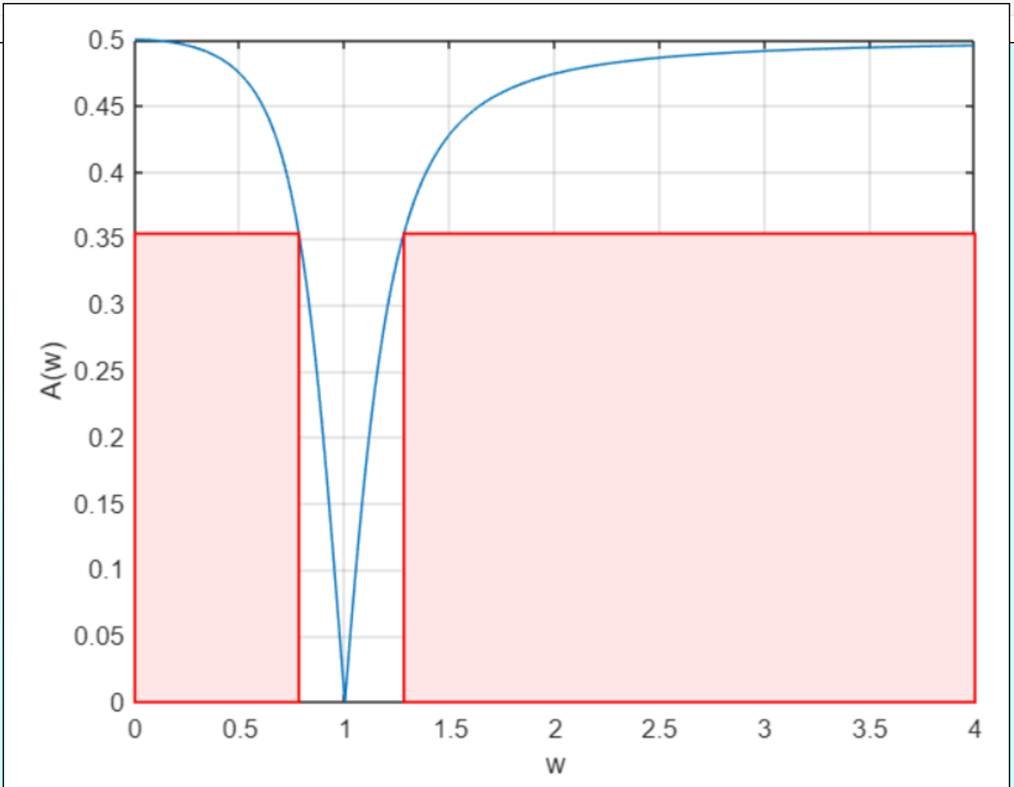
wg2 =

$$\frac{\sqrt{\frac{\sqrt{17}}{2}+\frac{9}{2}}}{2}$$

Označavanje propusnog opsega u grafiku

```
figure
fplot(w, subs(Aw(w), lhs(vrednosti), rhs(vrednosti)), [0,4])
hold on
rectangle('Position',[0, 0, double(wg1),  double(Aref/sqrt(2))],...
          'EdgeColor','r',...
          'FaceColor',[1 0.9 0.9],...
          'LineWidth',1)
rectangle('Position',[double(wg2), 0, double(4-wg2), double(Aref/sqrt(2))],...
          'EdgeColor','r',...
          'FaceColor',[1 0.9 0.9],...
          'LineWidth',1)
hold off
ylabel('A(w)')
xlabel('w')
grid on
```

MATLAB: Symbolic Toolbox



Устаљен сложенопериодичан одзив

Електрично коло са слике има познате вредности елемената:

$L_1 = L$, $L_2 = 2L$,

$C_1 = C$, $C_2 = 2C$,

$R_1 = R_2 = R$.

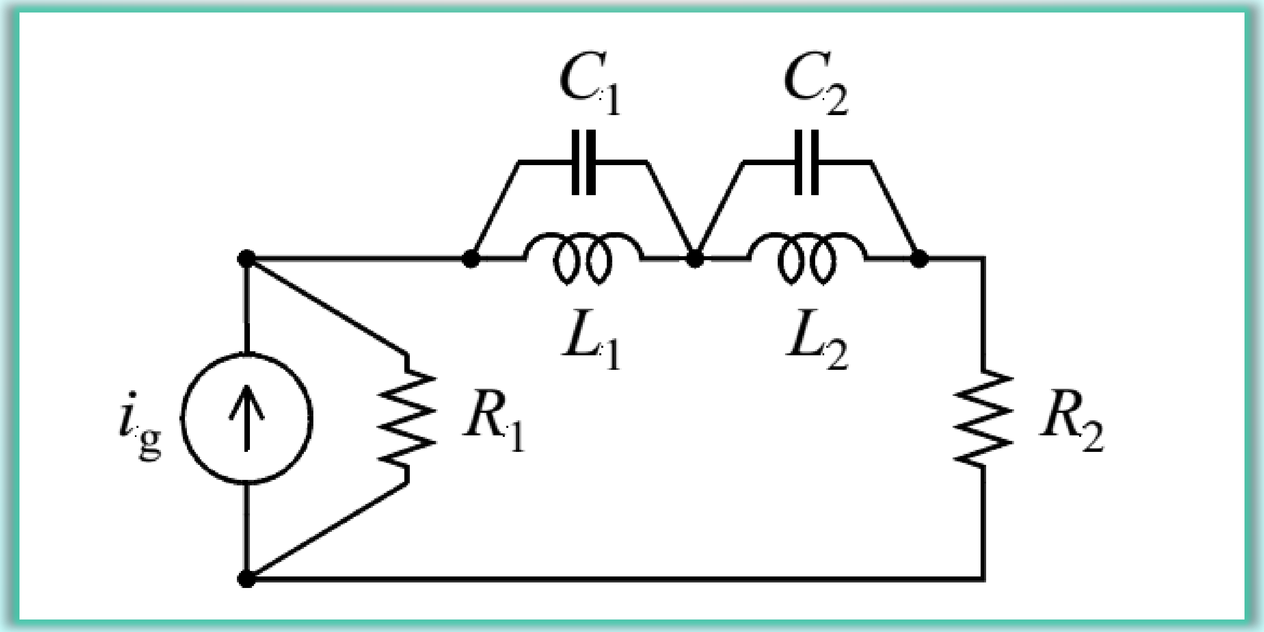
Побуда (екситација) је $i_g(t) = I_m + I_m \cos(\frac{1}{2\sqrt{CL}} t) + I_m \sin(\frac{1}{\sqrt{CL}} t)$

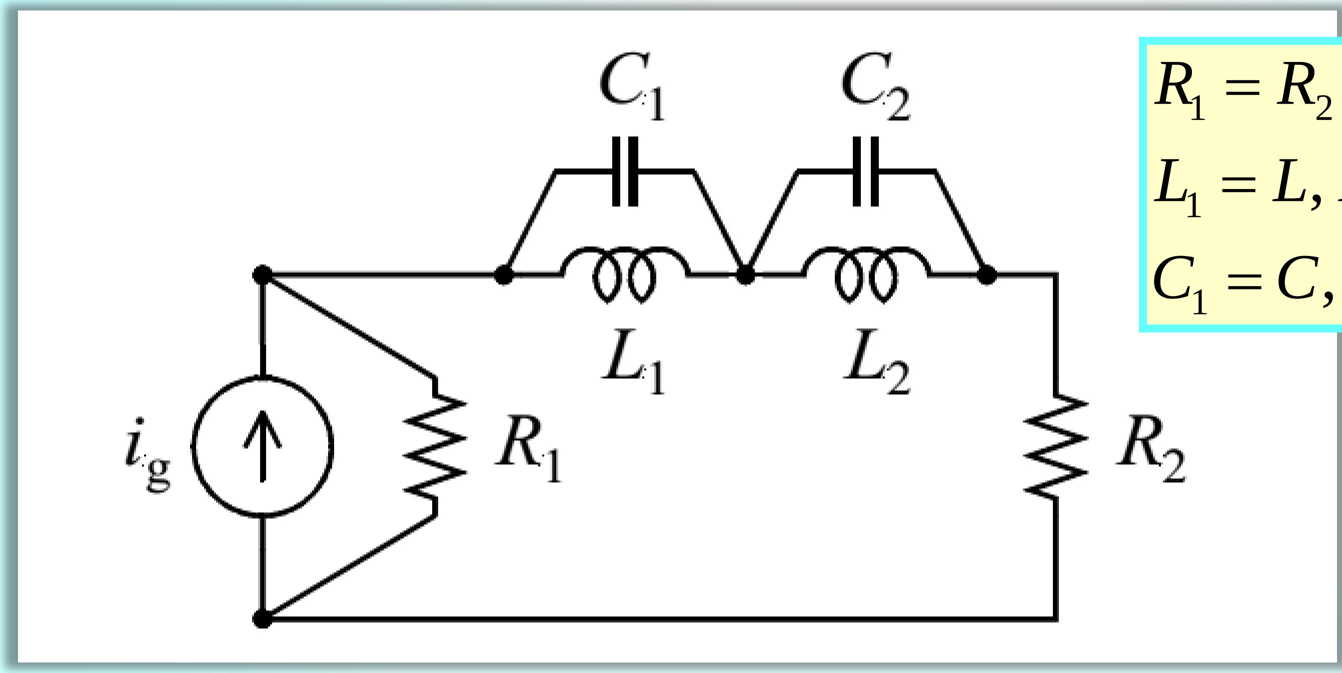
Одзив је устаљен.

(а) Одредити напон отпорника R_2.

(б) Одредити напон струјног извора (струјног генератора).

(б) Ког реда је ово коло?





$$R_1 = R_2 = 1\,\mathrm{k\Omega}$$

$$L_1 = L,\; L_2 = 2L,\; L = 0.5\,\mathrm{mH}$$

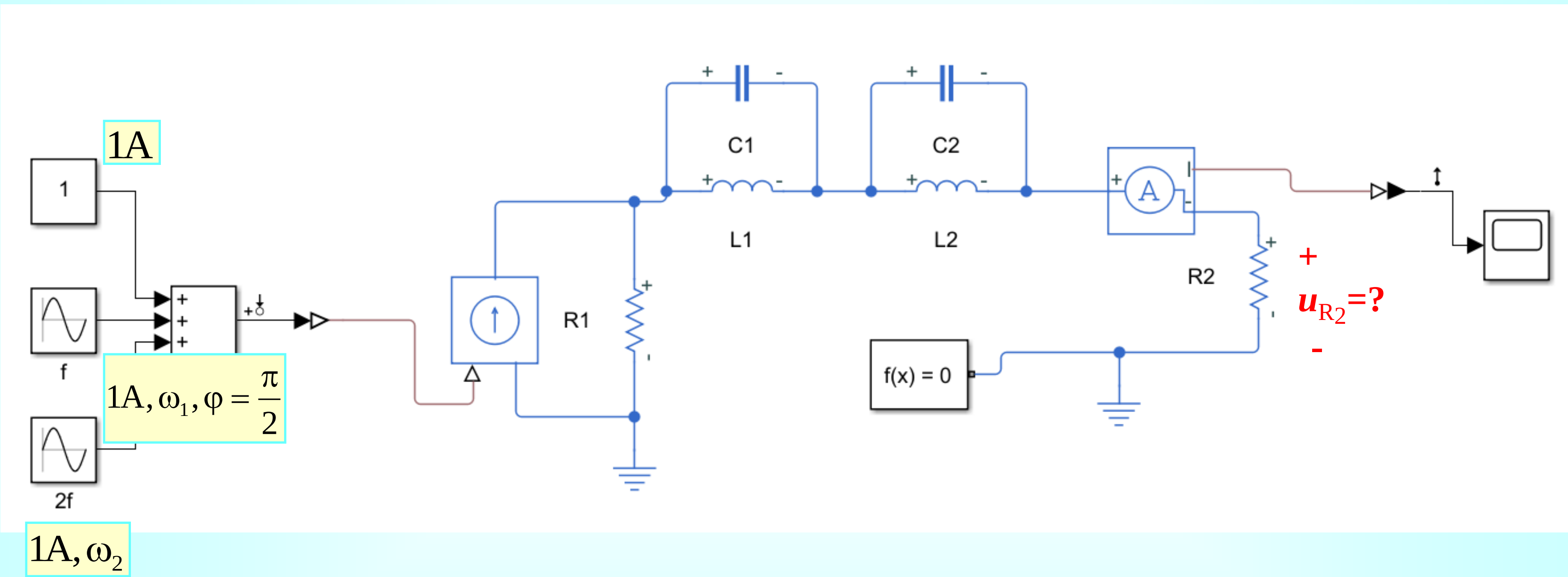
$$C_1 = C,\; C_2 = 2C,\; C = 5\,\mu\mathrm{F}$$

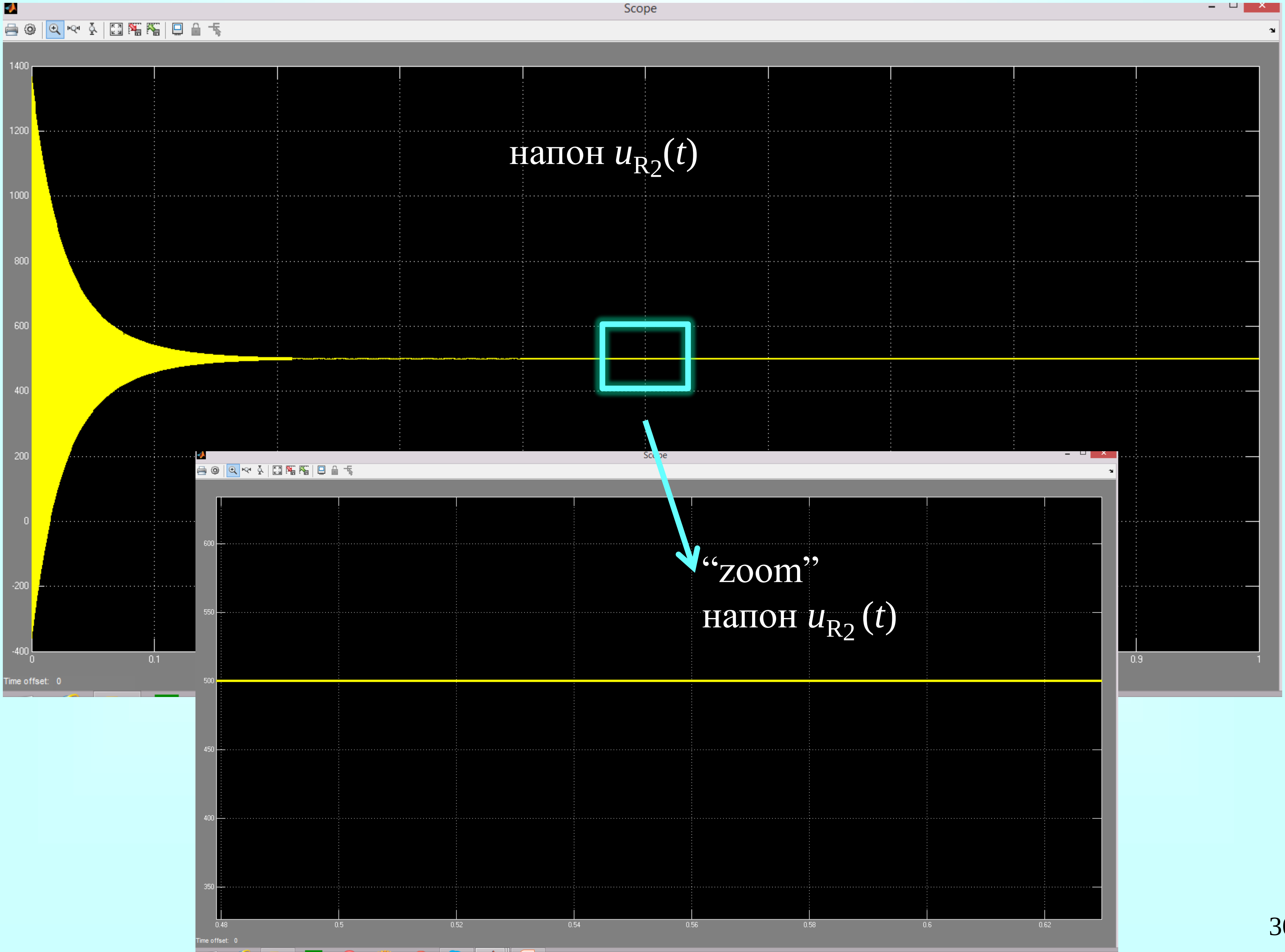
$$i_g(t) = \underbrace{I_\mathrm{m}}_{1\mathrm{A}} + I_\mathrm{m}\cos(\omega_1 t) + I_\mathrm{m}\sin(\omega_2 t)$$

$$\omega_1 = \frac{1}{\sqrt{L_2 C_2}} = \frac{1}{2\sqrt{LC}} = 10^4\ \mathrm{rad/s}$$

$$\omega_2 = \frac{1}{\sqrt{L_1 C_1}} = \frac{1}{\sqrt{LC}} = 2\cdot 10^4\ \mathrm{rad/s}$$

Simulink Simscape > Foundation Library, Utilities




Scope
напон $u_{R2}(t)$
"zoom"
напон $u_{R2}(t)$
Time offset: 0

```
clear variables
close all

syms C C1 C2 I_m L L1 L2 R R1 R2
assume(0 < L1 & 0 < L2 & 0 < C1 & 0 < C2 & 0 < R1 & 0 < R2 & 0 < L & 0 < C & 0 < R & 0 < I_m)
```

Frekvencije, pobude, zamene

```
zamene = [L1 == L, C1 == C, L2 == 2 * L, C2 == 2*C, R1 == R, R2 == R]
```

zamene = $\left(L_1 = L \quad C_1 = C \quad L_2 = 2\,L \quad C_2 = 2\,C \quad R_1 = R \quad R_2 = R\right)$

```
syms w0 w1 w2 I_g0 I_g1 I_g2

pobuda = [I_g0 == I_m,  I_g1 == I_m/sqrt(2),  I_g2 == I_m/sqrt(2) * exp(-1i*pi/sym(2))]
```

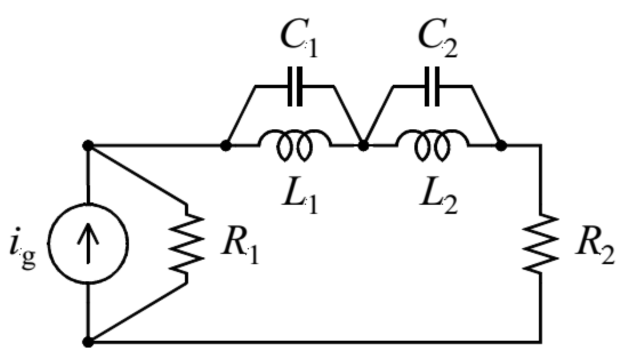
pobuda =

$$\left(I_{g0} = I_m \quad I_{g1} = \frac{\sqrt{2}\,I_m}{2} \quad I_{g2} = -\frac{\sqrt{2}\,I_m\,\mathrm{i}}{2}\right)$$

```
frekvencije = [w0 == 0, w1 == 1/(2*sqrt(L*C)), w2 == 1/sqrt(L*C)]
```

frekvencije =

$$\left(w_0 = 0 \quad w_1 = \frac{1}{2\,\sqrt{C}\,\sqrt{L}} \quad w_2 = \frac{1}{\sqrt{C}\,\sqrt{L}}\right)$$



Određivanje kompleksne funkcije UR2

```
syms UR2(s) I_g
UR2(s) = (R1*I_g*R2)/(R1 + 1/(s*C1 + 1/(s*L1)) + 1/(s*C2 + 1/(s*L2)) + R2)
```

UR2(s) =

$$\frac{I_g\,R_1\,R_2}{R_1 + R_2 + \dfrac{1}{C_1\,s + \dfrac{1}{L_1\,s}} + \dfrac{1}{C_2\,s + \dfrac{1}{L_2\,s}}}$$

```
syms UR2_jw(w)
UR2_jw(w) = simplify(subs(subs(UR2(s), lhs(zamene), rhs(zamene)), s, 1i*w))
```

UR2_jw(w) =

$$\frac{I_g\,R^2\,\left(4\,C^2\,L^2\,w^4 - 5\,C\,L\,w^2 + 1\right)}{8\,R\,C^2\,L^2\,w^4 - 6\,C\,L^2\,w^3\,\mathrm{i} - 10\,R\,C\,L\,w^2 + 3\,L\,w\,\mathrm{i} + 2\,R}$$

Хармоници, Временски домен

```
UR2_0 = simplify(subs(subs(subs(UR2_jw(w), [w I_g], [w0 I_g0]), lhs(frekvencije), rhs(frekvencije)), lhs(pobuda), rhs(pobuda)))
```

UR2_0 =

$$\frac{I_m R}{2}$$

```
UR2_wg1 = simplify(subs(subs(subs(UR2_jw(w), [w I_g], [w1 I_g1]), lhs(frekvencije), rhs(frekvencije)), lhs(pobuda), rhs(pobuda)))
```

UR2_wg1 = 0

```
UR2_wg2 = simplify(subs(subs(subs(UR2_jw(w), [w I_g], [w2 I_g2]), lhs(frekvencije), rhs(frekvencije)), lhs(pobuda), rhs(pobuda)))
```

UR2_wg2 = 0

Vremenski domen za uR2

```
syms U0 Uwg1 Uwg2
syms uR2(t)

uR2(t) = subs(UR2_0 + sqrt(2)*abs(UR2_wg1)*cos(w1*t + angle(UR2_wg1)) + sqrt(2)*abs(UR2_wg2)*cos(w2*t + angle(UR2_wg2)), lhs(frekvencije), rhs(frekvencije))
```

uR2(t) =

$$\frac{I_m R}{2}$$

Određivanje kompleksne funkcije UR1

```
syms Z(s)
Z(s) = simplify(1/(s * C1 + 1/(s*L1)) + 1/(s*C2 + 1/(s*L2)) + R2)
```

Z(s) =

$$R_2 + \frac{L_1\,s}{C_1\,L_1\,s^2+1} + \frac{L_2\,s}{C_2\,L_2\,s^2+1}$$

```
syms UR1(s)
UR1(s) = expand(subs( I_g*R1*Z(s)/(R1 + Z(s)), lhs(zamene), rhs(zamene) ) )
```

UR1(s) =

$$\frac{I_g\,R^2}{2\,R+\frac{L\,s}{\sigma_1}+\sigma_2} + \frac{I_g\,L\,R\,s}{2\,R+\frac{L\,s}{\sigma_1}+\sigma_2+2\,C\,L\,R\,s^2+\frac{C\,L^2\,s^3}{\sigma_1}+\frac{2\,C\,L^2\,s^3}{\sigma_3}} + \frac{2\,I_g\,L\,R\,s}{2\,R+\frac{L\,s}{\sigma_1}+\sigma_2+8\,C\,L\,R\,s^2+\frac{4\,C\,L^2\,s^3}{\sigma_1}+\frac{8\,C\,L^2\,s^3}{\sigma_3}}$$

where

$$\sigma_1 = C\,L\,s^2+1$$

$$\sigma_2 = \frac{2\,L\,s}{\sigma_3}$$

$$\sigma_3 = 4\,C\,L\,s^2+1$$

```
syms UR1_jw(w)
UR1_jw(w) = simplify(subs(UR1(s), s, 1i*w))
```

UR1_jw(w) =

$$-\frac{(3\,L\,w-6\,C\,L^2\,w^3)\,\sigma_1\,\mathrm{i}-(8\,R\,C^2\,L^2\,w^4-10\,R\,C\,L\,w^2+2\,R)\,\sigma_1}{64\,C^4\,L^4\,R^2\,w^8-160\,C^3\,L^3\,R^2\,w^6+36\,C^2\,L^4\,w^6+132\,C^2\,L^2\,R^2\,w^4-36\,C\,L^3\,w^4-40\,C\,L\,R^2\,w^2+9\,L^2\,w^2+4\,R^2}$$

where

$$\sigma_1 = 4\,I_g\,C^2\,L^2\,R^2\,w^4-6\,\mathrm{i}\,I_g\,C\,L^2\,R\,w^3-5\,I_g\,C\,L\,R^2\,w^2+3\,\mathrm{i}\,I_g\,L\,R\,w+I_g\,R^2$$

Хармоници, Временски домен

```
UR1_0 = simplify(subs(subs(subs(UR1_jw(w), [w I_g], [w0 I_g0]), lhs(frekvencije), rhs(frekvencije)), lhs(pobuda), rhs(pobuda)))
```

UR1_0 =

$$\frac{I_m R}{2}$$

```
UR1_wg1 = simplify(subs(subs(subs(UR1_jw(w), [w I_g], [w1 I_g1]), lhs(pobuda), rhs(pobuda)), lhs(frekvencije), rhs(frekvencije)))
```

UR1_wg1 =

$$\frac{\sqrt{2}\, I_m R}{2}$$

```
UR1_wg2 = simplify(subs(subs(subs(UR1_jw(w), [w I_g], [w2 I_g2]), lhs(pobuda), rhs(pobuda)), lhs(frekvencije), rhs(frekvencije)))
```

UR1_wg2 =

$$-\frac{\sqrt{2}\, I_m R\, \mathrm{i}}{2}$$

Vremenski domen za uR1

```
syms uR1(t)
uR1(t) = subs(UR1_0 + sqrt(sym(2)) * abs(UR1_wg1) * cos(w1 * t + angle(UR1_wg1)) + sqrt(sym(2)) * abs(UR1_wg2) * cos(w2 * t + angle(UR1_wg2)), lhs(frekvencije), rhs(frekvencije))
```

uR1(t) =

$$\frac{I_m R}{2} + I_m R \cos\left(\frac{t}{2\,\sqrt{C}\,\sqrt{L}}\right) + I_m R \cos\left(\frac{\pi}{2} - \frac{t}{\sqrt{C}\,\sqrt{L}}\right)$$

```
simplify(uR1(t))
```

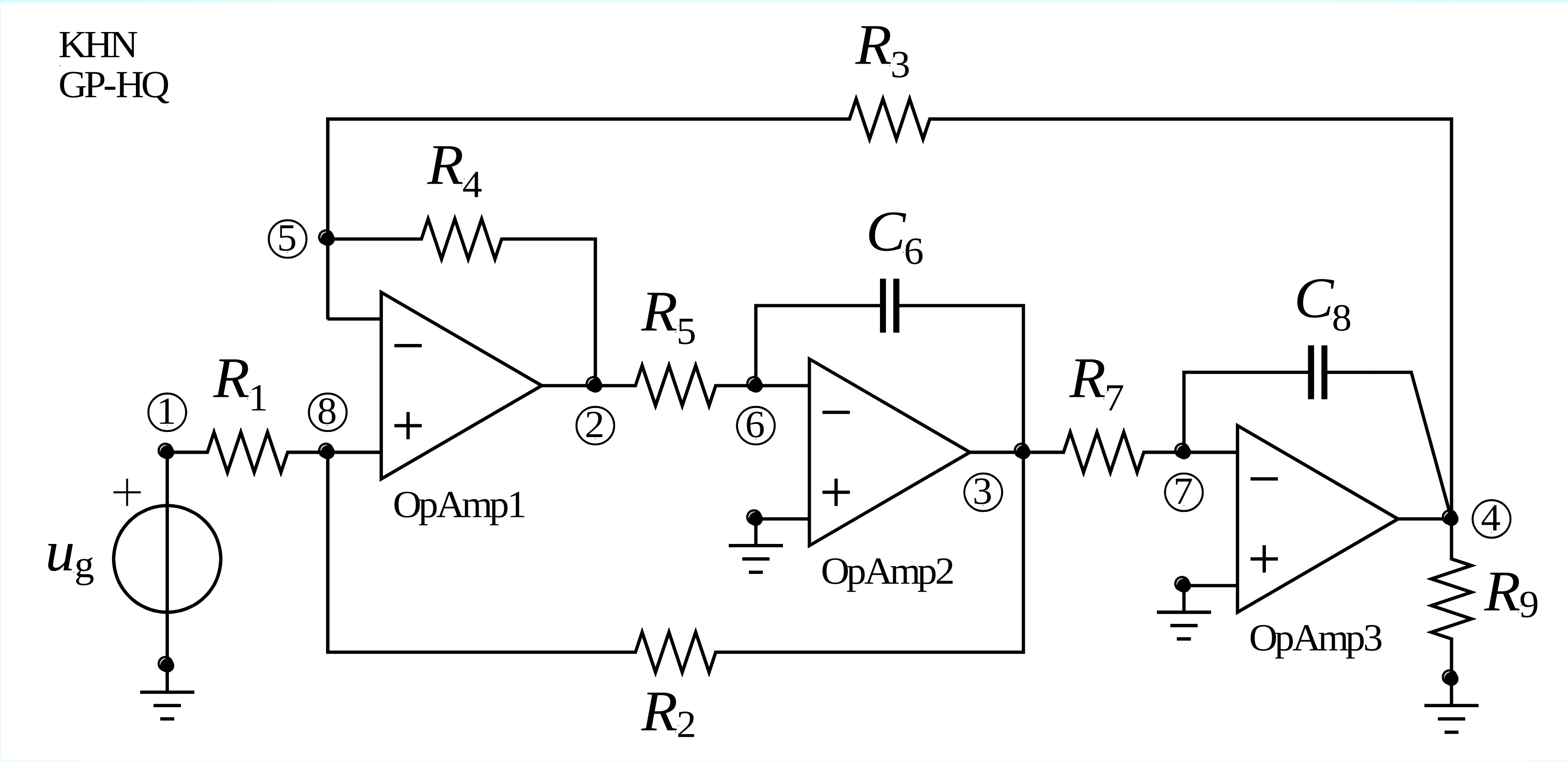
ans =

$$\frac{I_m R \left(2\cos\left(\frac{t}{2\,\sqrt{C}\,\sqrt{L}}\right) + 2\sin\left(\frac{t}{\sqrt{C}\,\sqrt{L}}\right) + 1\right)}{2}$$

MATLAB: Symbolic Toolbox

KHN active filter

Kerwin-Huelsman-Newcomb, state-variable biquad, UAF42

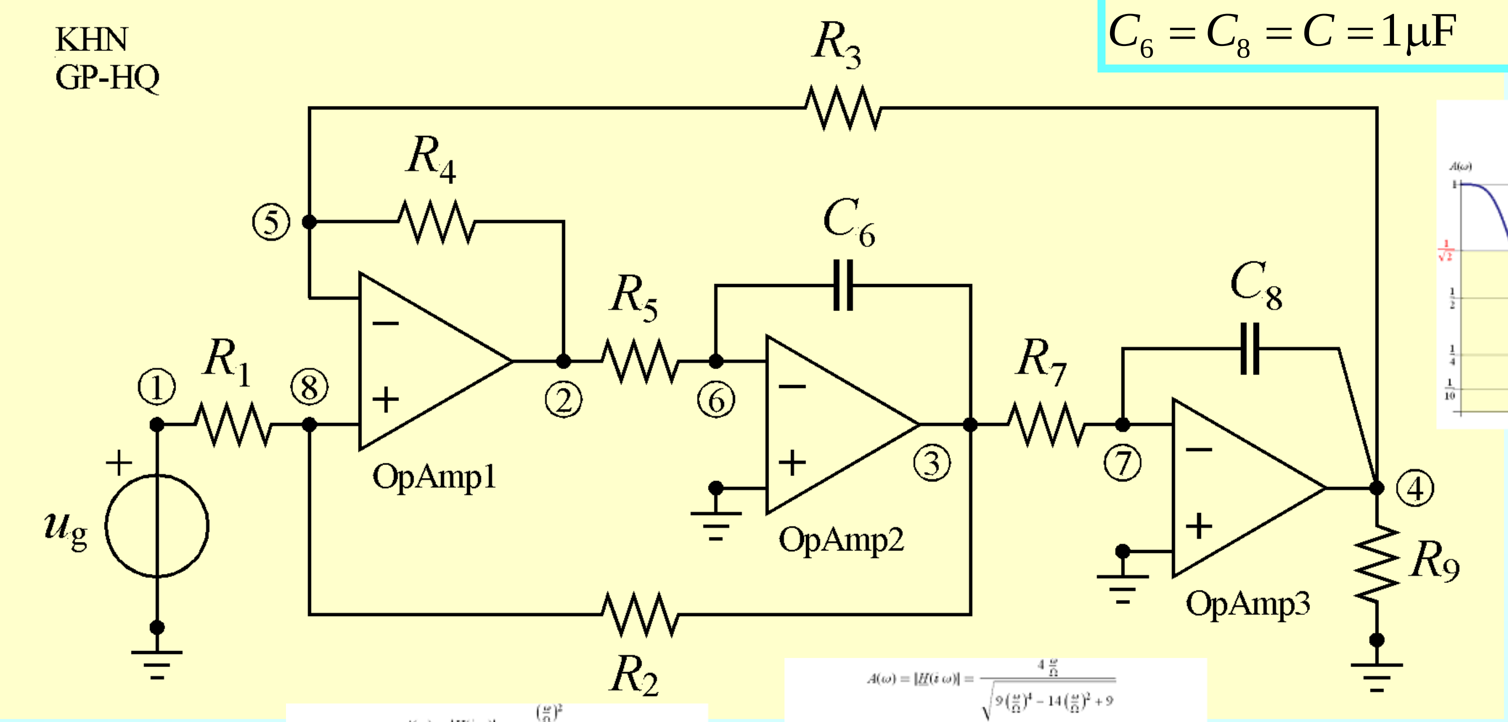


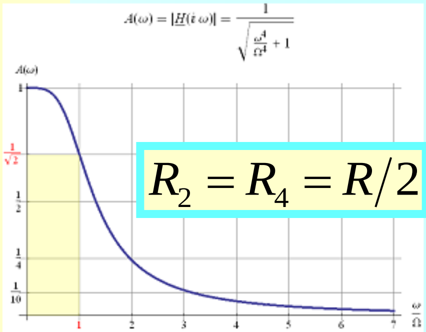
Kerwin-Huelsman-Newcomb, state-variable biquad, UAF42

KHN active filter

$R_1 = R_3 = R_5 = R_7 = R_9 = R = 1\text{k}\Omega$

$C_6 = C_8 = C = 1\mu\text{F}$





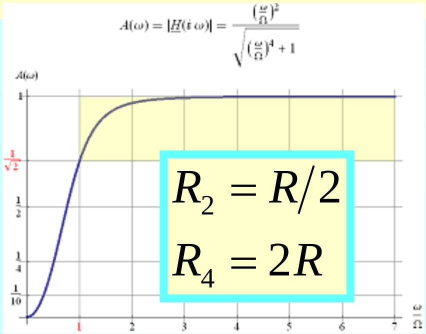
$R_2 = R_4 = R/2$

$\underline{H}(s) = \frac{\underline{V}_4(s)}{\underline{U}_g(s)}$

LowPass, LP

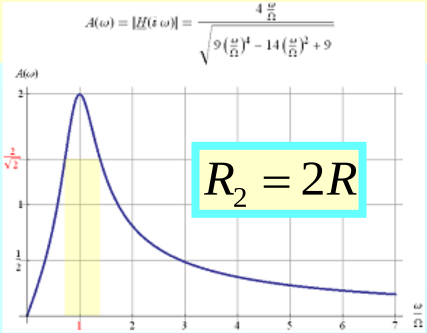
HighPass, HP

$\underline{H}(s) = \frac{\underline{V}_2(s)}{\underline{U}_g(s)}$



$R_2 = R/2$

$R_4 = 2R$



$R_2 = 2R$

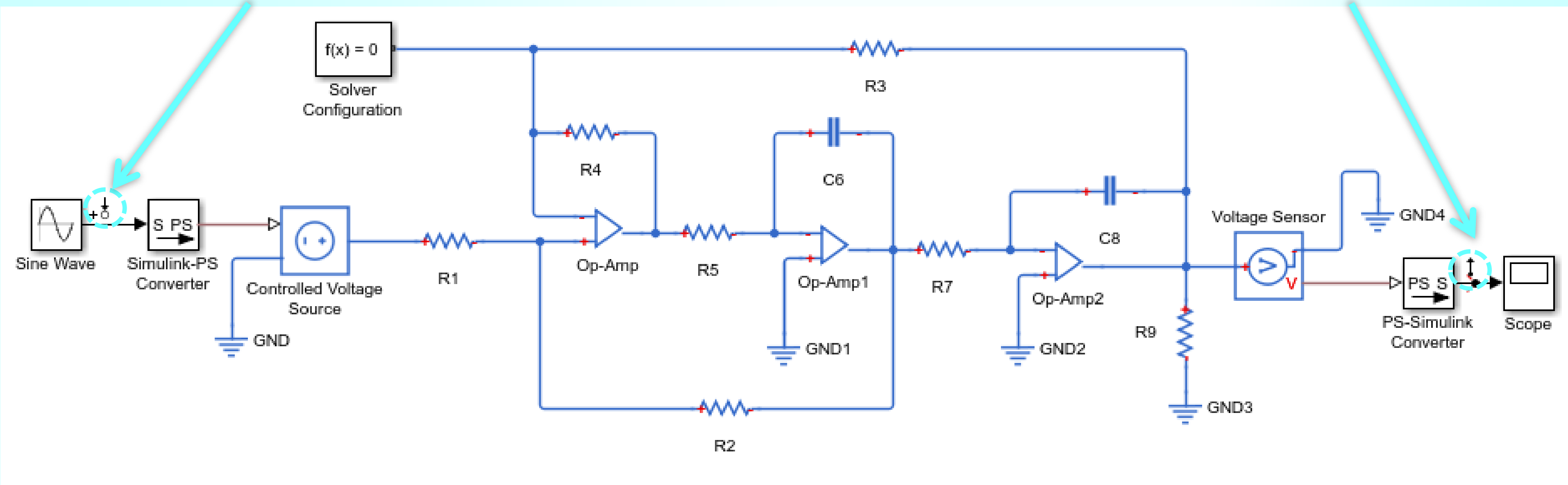
$\underline{H}(s) = \frac{\underline{V}_3(s)}{\underline{U}_g(s)}$

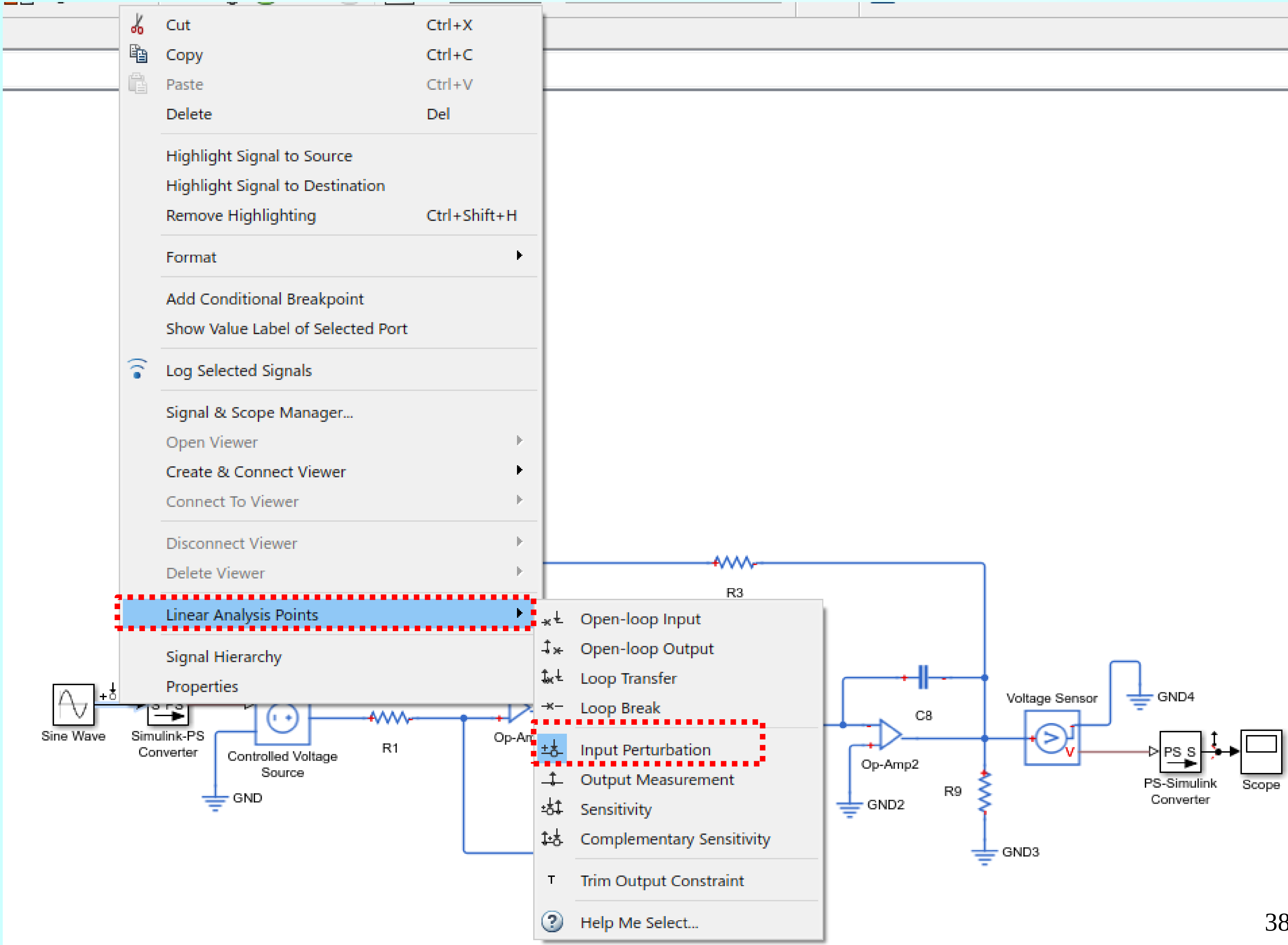
BandPass, BP

УЛАЗ – ИЗЛАЗ КОЛА (СИСТЕМА)
Филтар пропусник ниских учестаности

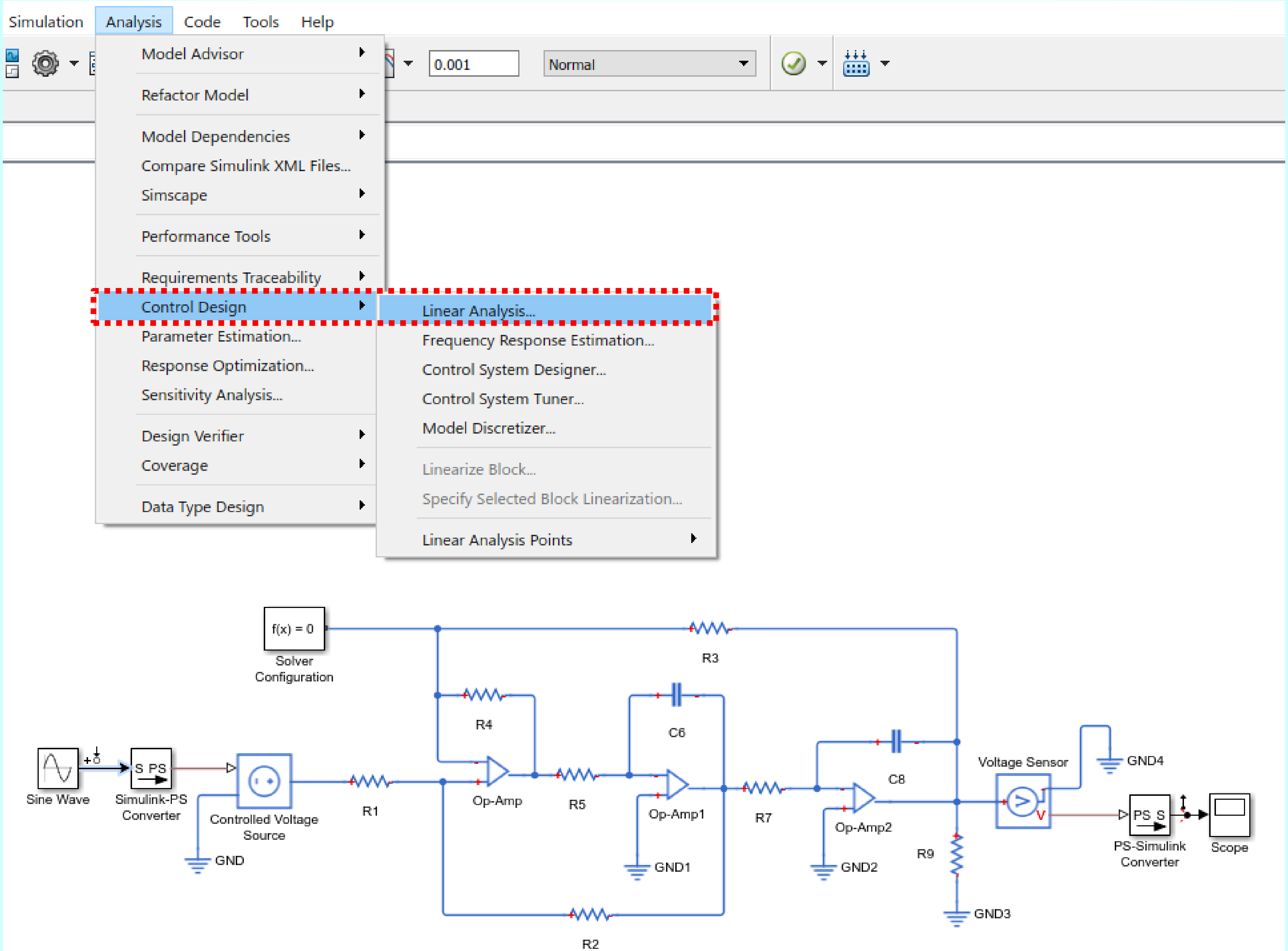
Input Perturbation

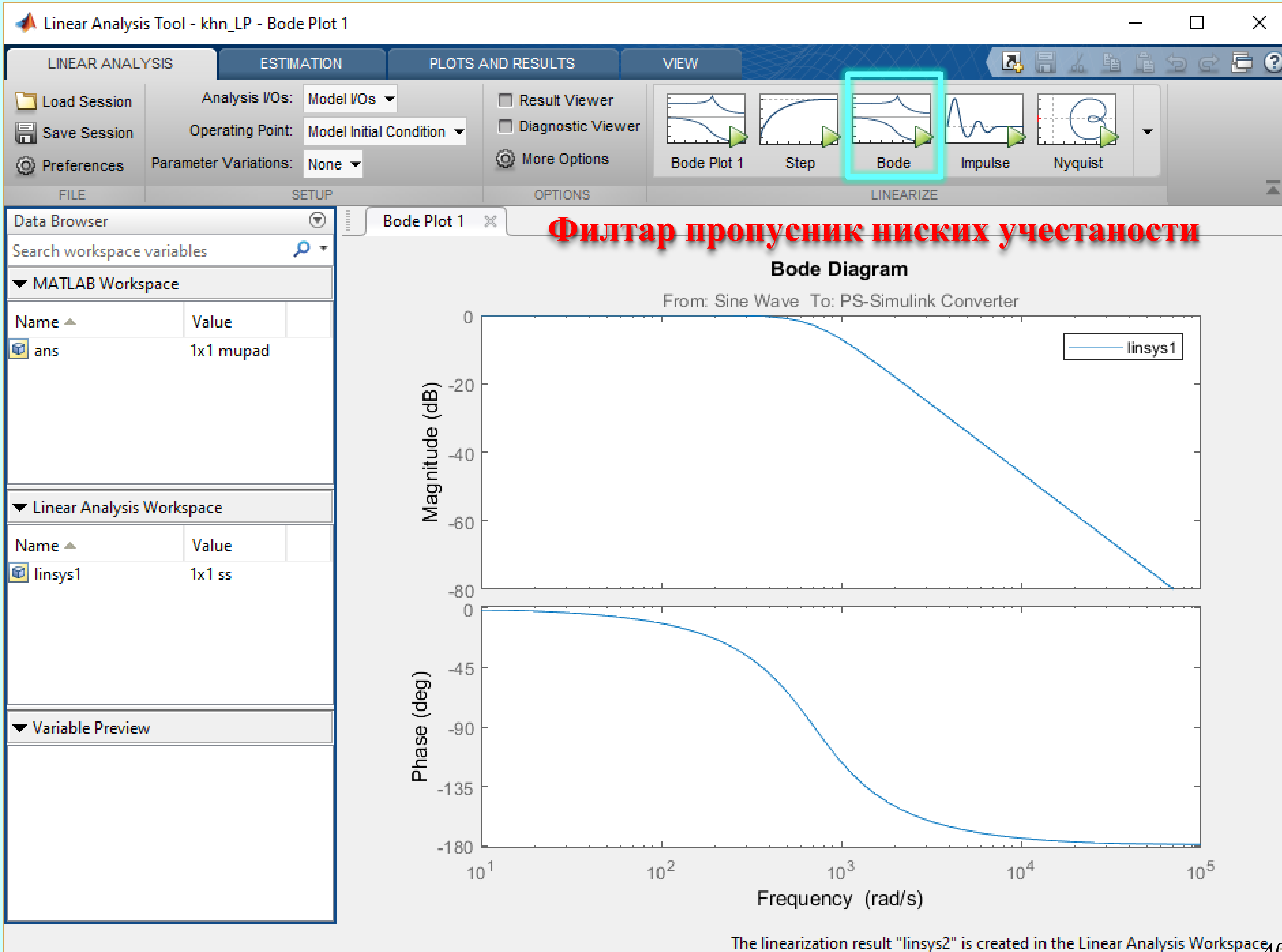
Output Measurement



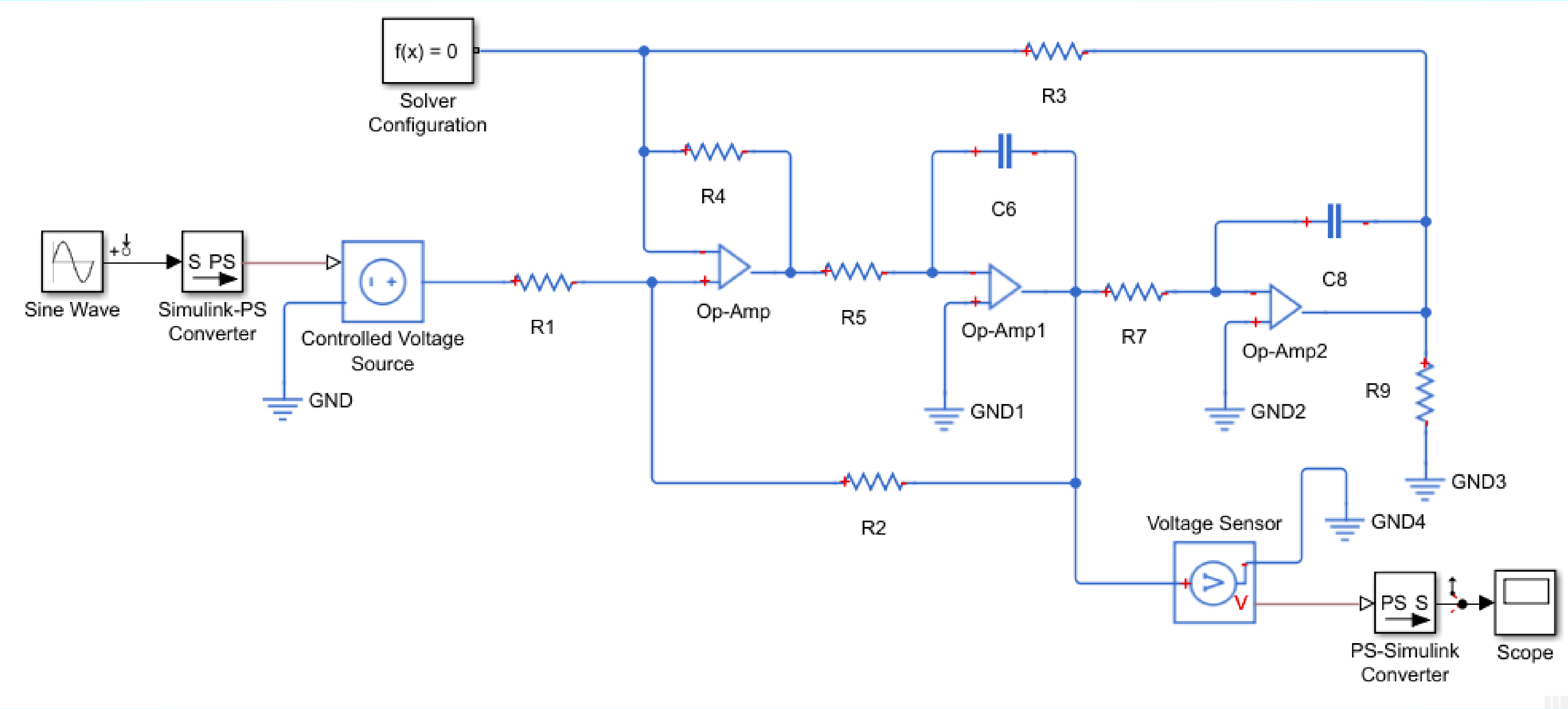


Cut Ctrl+X
Copy Ctrl+C
Paste Ctrl+V
Delete Del
Highlight Signal to Source
Highlight Signal to Destination
Remove Highlighting Ctrl+Shift+H
Format
Add Conditional Breakpoint
Show Value Label of Selected Port
Log Selected Signals
Signal & Scope Manager...
Open Viewer
Create & Connect Viewer
Connect To Viewer
Disconnect Viewer
Delete Viewer
Linear Analysis Points
Signal Hierarchy
Properties
Open-loop Input
Open-loop Output
Loop Transfer
Loop Break
Input Perturbation
Output Measurement
Sensitivity
Complementary Sensitivity
Trim Output Constraint
Help Me Select...
R3
C8
Voltage Sensor
GND4
Sine Wave
Simulink-PS Converter
Controlled Voltage Source
R1
GND
Op-Amp2
R9
GND2
GND3
PS-Simulink Converter
Scope

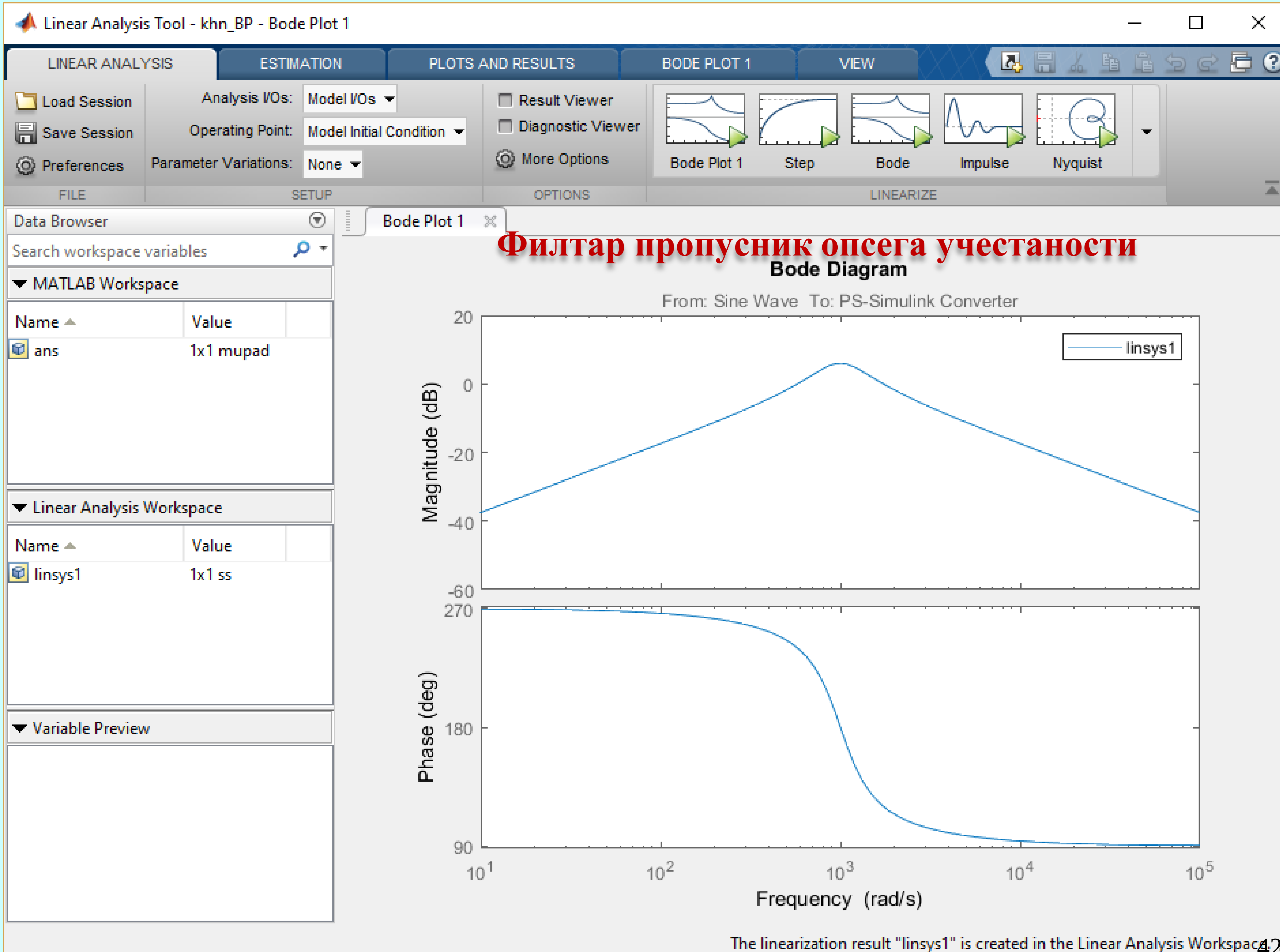

Simulation
Analysis
Code
Tools
Help
0.001
Normal
Model Advisor
Refactor Model
Model Dependencies
Compare Simulink XML Files...
Simscape
Performance Tools
Requirements Traceability
Control Design
Parameter Estimation...
Response Optimization...
Sensitivity Analysis...
Design Verifier
Coverage
Data Type Design
Linear Analysis...
Frequency Response Estimation...
Control System Designer...
Control System Tuner...
Model Discretizer...
Linearize Block...
Specify Selected Block Linearization...
Linear Analysis Points
f(x) = 0
Solver Configuration
R3
R4
C6
C8
Sine Wave
Simulink-PS Converter
Controlled Voltage Source
GND
R1
Op-Amp
R5
Op-Amp1
GND1
R7
Op-Amp2
GND2
R9
GND3
R2
Voltage Sensor
GND4
PS-Simulink Converter
Scope


Linear Analysis Tool - khn_LP - Bode Plot 1
LINEAR ANALYSIS
ESTIMATION
PLOTS AND RESULTS
VIEW
Load Session
Save Session
Preferences
FILE
Analysis I/Os:
Model I/Os
Operating Point:
Model Initial Condition
Parameter Variations:
None
SETUP
Result Viewer
Diagnostic Viewer
More Options
OPTIONS
Bode Plot 1
Step
Bode
Impulse
Nyquist
LINEARIZE
Data Browser
Search workspace variables
MATLAB Workspace
Name
Value
ans
1x1 mupad
Linear Analysis Workspace
Name
Value
linsys1
1x1 ss
Variable Preview
Bode Plot 1
Филтар пропусник ниских учестаности
Bode Diagram
From: Sine Wave To: PS-Simulink Converter
Magnitude (dB)
0
-20
-40
-60
-80
linsys1
Phase (deg)
0
-45
-90
-135
-180
10^1
10^2
10^3
10^4
10^5
Frequency (rad/s)
The linearization result "linsys2" is created in the Linear Analysis Workspace.

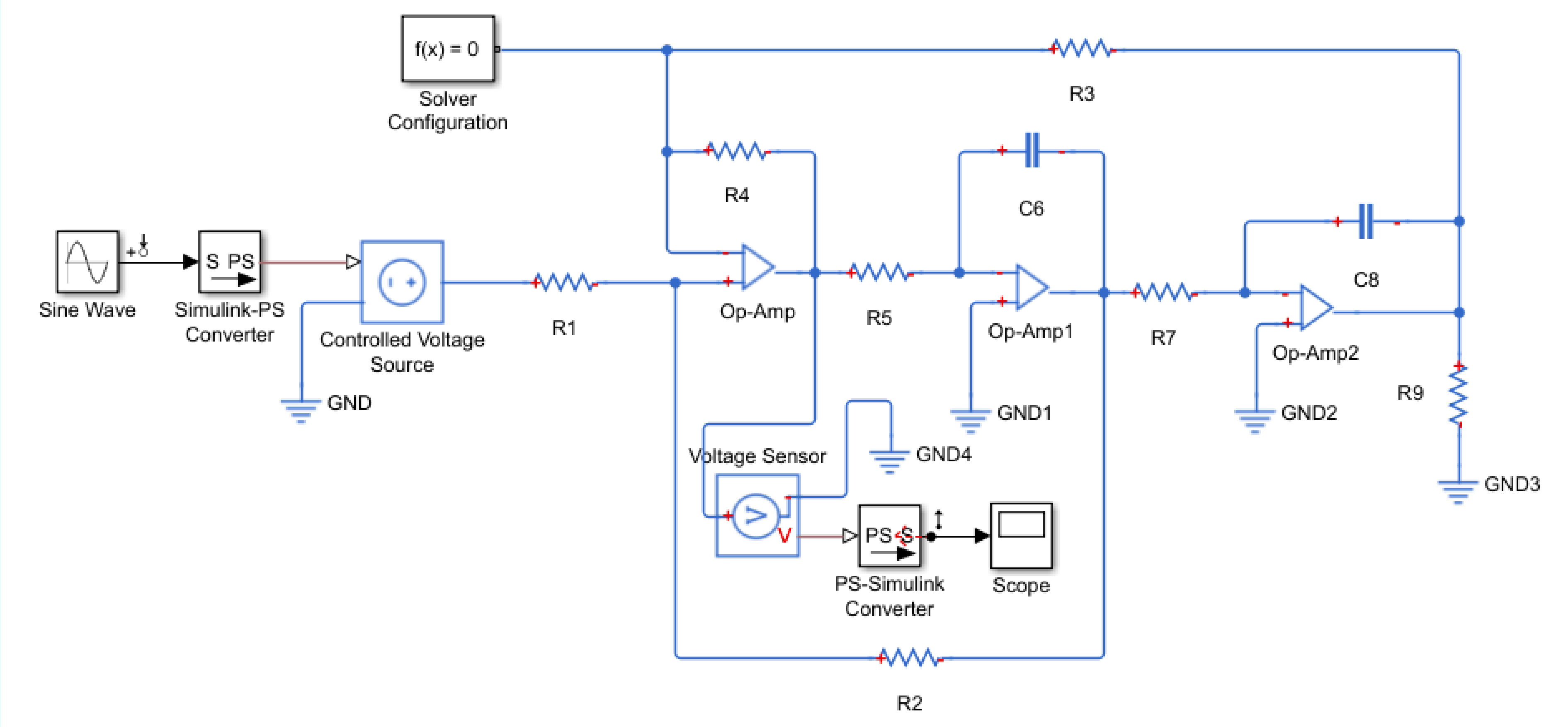
Филтар пропусник опсега учестаности



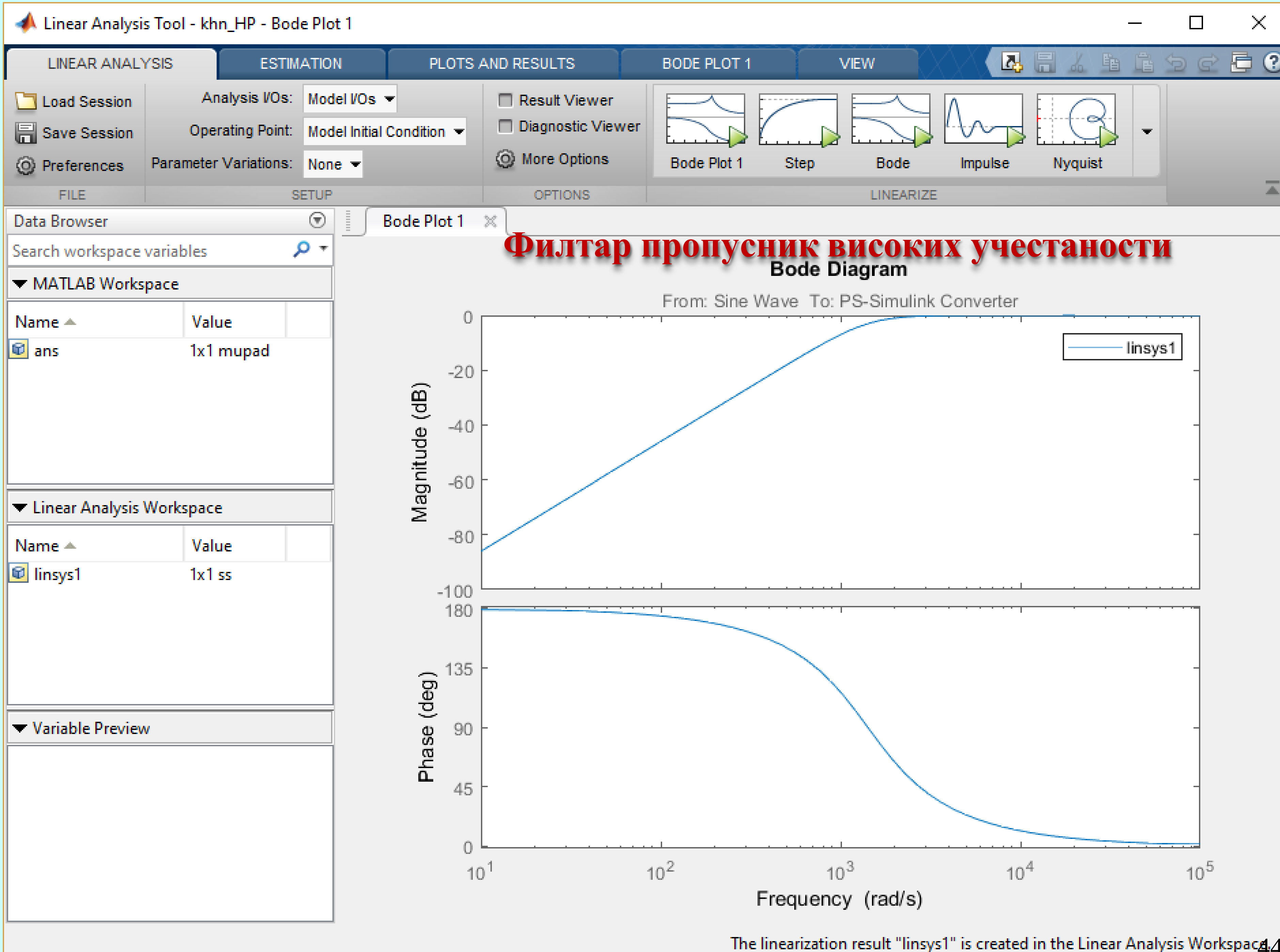
Филтар пропусник опсега учестаности



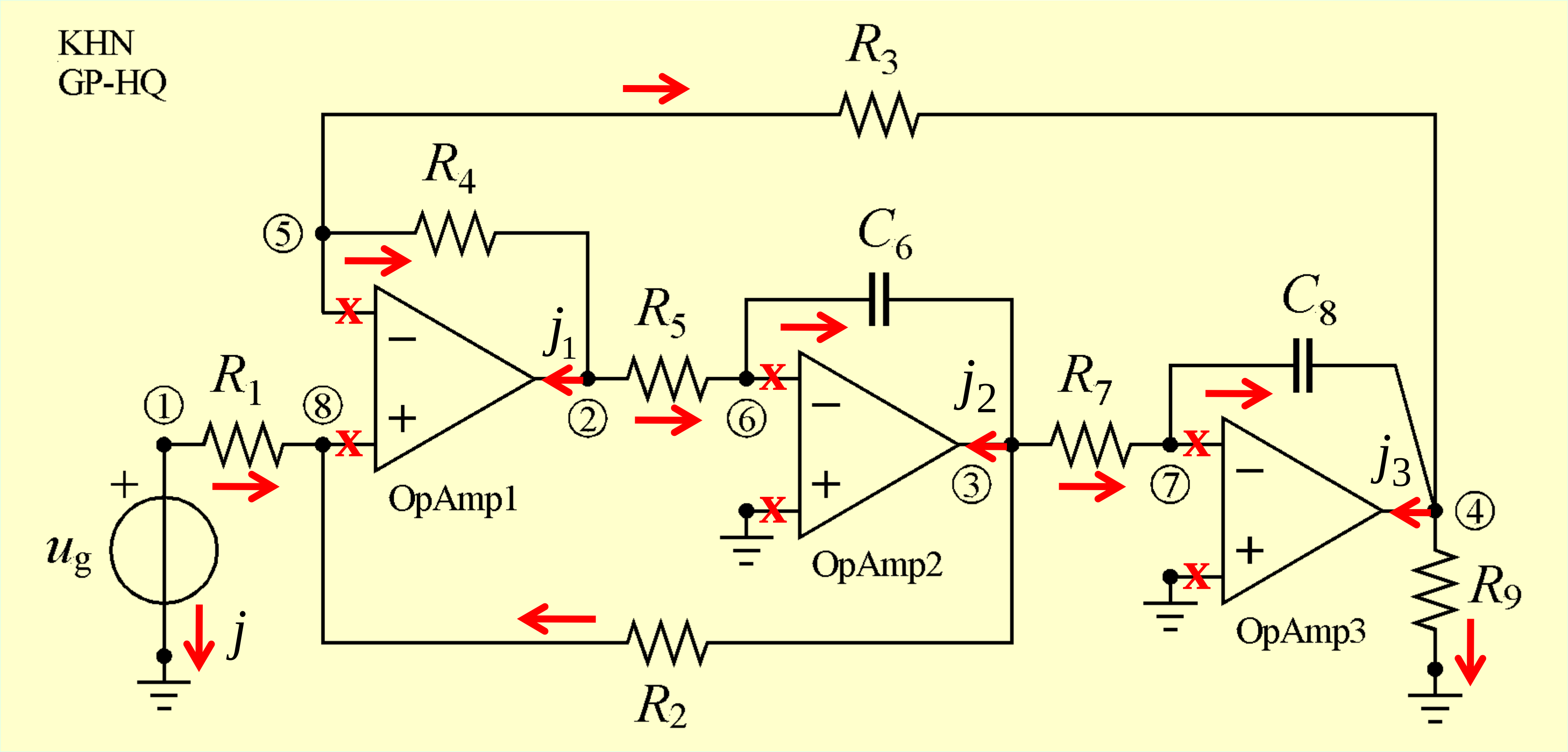
Филтар пропусник високих учестаности



Филтар пропусник високих учестаности



KHN active filter

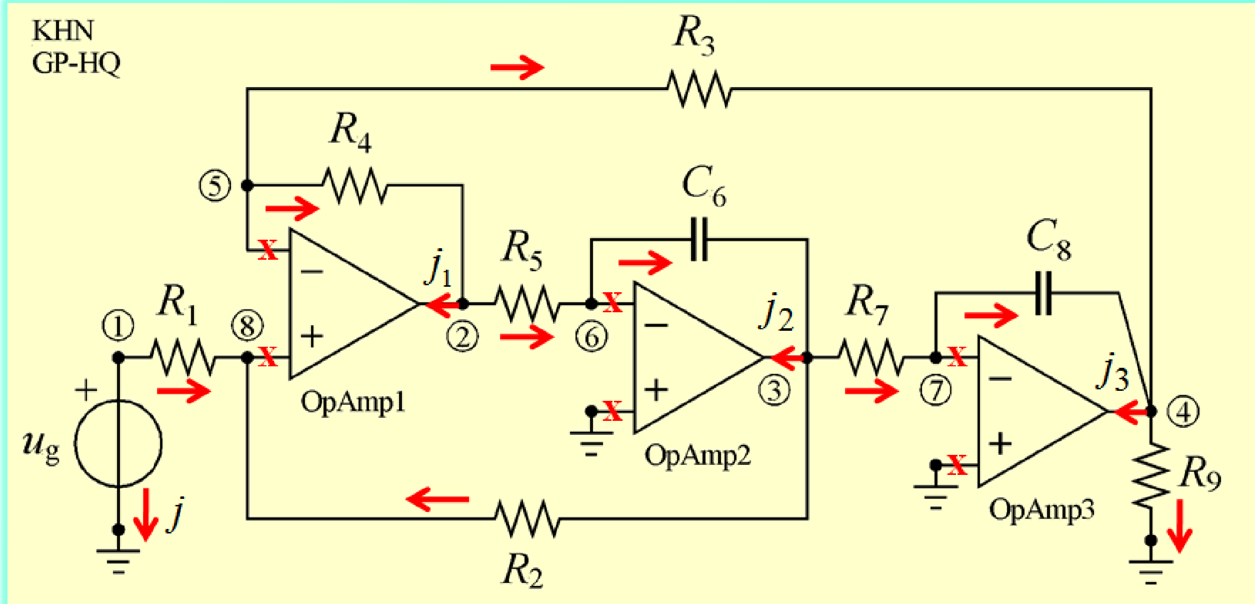


$$\underline{H}_{\text{LowPass}}(\underline{s}) = \frac{\underline{V}_4(\underline{s})}{\underline{V}_1(\underline{s})} = ?$$

$$\underline{H}_{\text{BandPass}}(\underline{s}) = \frac{\underline{V}_3(\underline{s})}{\underline{V}_1(\underline{s})} = ?$$

$$\underline{H}_{\text{HighPass}}(\underline{s}) = \frac{\underline{V}_2(\underline{s})}{\underline{V}_1(\underline{s})} = ?$$

KHN active filter



$$(1)\ \underline{J}+\left(\underline{V}_1-\underline{V}_8\right)/R_1=0$$

$$(2)\ \underline{J}_1+\left(\underline{V}_2-\underline{V}_6\right)/R_5-\left(\underline{V}_5-\underline{V}_2\right)/R_4=0$$

$$(3)\ \underline{J}_2+\left(\underline{V}_3-\underline{V}_8\right)/R_2+\left(\underline{V}_3-\underline{V}_7\right)/R_7-sC_6\left(\underline{V}_6-\underline{V}_3\right)=0$$

$$(4)\ \underline{J}_3-\left(\underline{V}_5-\underline{V}_4\right)/R_3+\underline{V}_4/R_9-sC_8\left(\underline{V}_7-\underline{V}_4\right)=0$$

$$(5)\ \left(\underline{V}_5-\underline{V}_2\right)/R_4+\left(\underline{V}_5-\underline{V}_4\right)/R_3=0$$

$$(6)\ -\left(\underline{V}_2-\underline{V}_6\right)/R_5+sC_6\left(\underline{V}_6-\underline{V}_3\right)=0$$

$$(7)\ -\left(\underline{V}_3-\underline{V}_7\right)/R_7+sC_8\left(\underline{V}_7-\underline{V}_4\right)=0$$

$$(8)\ -\left(\underline{V}_1-\underline{V}_8\right)/R_1-\left(\underline{V}_3-\underline{V}_8\right)/R_2=0$$

$$\underline{V}_5=\underline{V}_8$$

$$\underline{V}_6=0$$

$$\underline{V}_7=0$$

$$\underline{V}_1=\underline{U}_g$$

```
clear variables
close all

syms C C6 C8 R R1 R2 R3 R4 R5 R7 Ug s w
assume(0<R1 & 0<R2 & 0<R3 & 0<R4 & 0<R5 & 0<C6 & 0<R7 & 0<C8 & 0<R & 0<C & 0<Ug & s~= 0 & 0<w)
```

Zamene

```
zamena_LowPass = [R1 == R, R2 == R/2, R3 == R, R4 == R/2, R5 == R, C6 == C, R7 == R, C8 == C]
```

zamena_LowPass =

$$\left(R_1 = R \quad R_2 = \frac{R}{2} \quad R_3 = R \quad R_4 = \frac{R}{2} \quad R_5 = R \quad C_6 = C \quad R_7 = R \quad C_8 = C\right)$$

```
zamena_BandPass = [R1 == R, R2 == 2 * R, R3 == R, R4 == R, R5 == R, C6 == C, R7 == R, C8 == C]
```

zamena_BandPass = $\left(R_1 = R \quad R_2 = 2\,R \quad R_3 = R \quad R_4 = R \quad R_5 = R \quad C_6 = C \quad R_7 = R \quad C_8 = C\right)$

```
zamena_HighPass = [R1 == R, R2 == R/2, R3 == R, R4 == 2 * R, R5 == R, C6 == C, R7 == R, C8 == C]
```

zamena_HighPass =

$$\left(R_1 = R \quad R_2 = \frac{R}{2} \quad R_3 = R \quad R_4 = 2\,R \quad R_5 = R \quad C_6 = C \quad R_7 = R \quad C_8 = C\right)$$

Vrednosti

```
vrednosti = [R == 1, C == 1]
```

vrednosti = $\left(R = 1 \quad C = 1\right)$

```
w0 = 1/(R*C)
```

w0 =

$$\frac{1}{C\,R}$$

MATLAB: Symbolic Toolbox

Jednacine

```
syms J J1 J2 J3 R9 V1 V2 V3 V4 V5 V6 V7 V8 v4
jednacine = [J + (V1 - V8)/R1 == 0,...
            J1 + (V2 - V6)/R5 - (V5 - V2)/R4 == 0,...
            J2 + (V3 - V8)/R2 + (V3 - V7)/R7 - s*C6*(V6 - V3) == 0,...
            J3 - (V5 - V4)/R3 + v4/R9 - s * C8 * (V7 - V4) == 0,...
            (V5 - V2)/R4 + (V5 - V4)/R3 == 0,...
            -((V2 - V6)/R5) + s * C6 * (V6 - V3) == 0,...
            -((V3 - V7)/R7) + s * C8 * (V7 - V4) == 0,...
            -((V1 - V8)/R1) - (V3 - V8)/R2 == 0,...
            V5 == V8, V6 == 0, V7 == 0, V1 == Ug]
```

jednacine =

$$\left(J + \frac{V_1 - V_8}{R_1} = 0 \quad J_1 + \frac{V_2 - V_5}{R_4} + \frac{V_2 - V_6}{R_5} = 0 \quad J_2 + \frac{V_3 - V_8}{R_2} + \frac{V_3 - V_7}{R_7} + \sigma_2 = 0 \quad J_3 + \frac{V_4 -}{R} \right.$$

where

$$\sigma_1 = C_8\, s\, (V_4 - V_7)$$

$$\sigma_2 = C_6\, s\, (V_3 - V_6)$$

Promenljive

```
promenljive = [V1, V2, V3, V4, V5, V6, V7, V8, J, J1, J2, J3]
```

promenljive = $\left(V_1 \quad V_2 \quad V_3 \quad V_4 \quad V_5 \quad V_6 \quad V_7 \quad V_8 \quad J \quad J_1 \quad J_2 \quad J_3 \right)$

```
odziv = solve(jednacine, promenljive)
```

Low Pass odziv

```
syms V4_1(s) H_LowPass(s) Hjw_LowPass(w)

V4_1(s) = subs(odziv.V4, lhs(zamena_LowPass), rhs(zamena_LowPass))
```

V4_1(s) =

$$\frac{3 R^2 \mathrm{Ug}}{4 \left(\frac{3 C^2 R^4 s^2}{2} + \frac{3 C R^3 s}{2} + \frac{3 R^2}{4}\right)}$$

```
H_LowPass(s) = simplify(V4_1(s)/Ug)
```

H_LowPass(s) =

$$\frac{1}{2 C^2 R^2 s^2 + 2 C R s + 1}$$

```
Hjw_LowPass(w) = subs(H_LowPass(s), s, 1i*w)
```

Hjw_LowPass(w) =

$$\frac{1}{-2 C^2 R^2 w^2 + 2 C R w \mathrm{i} + 1}$$

```
Aw_LowPass(w) = simplify(abs(Hjw_LowPass(w)))
```

Aw_LowPass(w) =

$$\frac{1}{\left|-2 C^2 R^2 w^2 + 2 C R w \mathrm{i} + 1\right|}$$

```
phi_LowPass(w) = simplify(angle(Hjw_LowPass(w)))
```

phi_LowPass(w) =

$$\mathrm{angle}\left(\frac{1}{-2 C^2 R^2 w^2 + 2 C R w \mathrm{i} + 1}\right)$$

Band Pass odziv

```
syms V3_1(s) H_BandPass(s) Hjw_BandPass(w)
V3_1(s) = subs(odziv.V3, lhs(zamena_BandPass), rhs(zamena_BandPass))
```

V3_1(s) =

$$-\frac{4 C R^3 \mathrm{Ug}\, s}{3 C^2 R^4 s^2 + 2 C R^3 s + 3 R^2}$$

```
H_BandPass(s) = simplify(V3_1(s)/Ug)
```

H_BandPass(s) =

$$-\frac{4 C R s}{3 C^2 R^2 s^2 + 2 C R s + 3}$$

```
Hjw_BandPass(w) = subs(H_BandPass(s), s, 1i*w)
```

Hjw_BandPass(w) =

$$-\frac{4 C R w \mathrm{i}}{-3 C^2 R^2 w^2 + 2 C R w \mathrm{i} + 3}$$

```
Aw_BandPass(w) = simplify(abs(Hjw_BandPass(w)))
```

Aw_BandPass(w) =

$$\frac{4 C R |w|}{\left|-3 C^2 R^2 w^2 + 2 C R w \mathrm{i} + 3\right|}$$

```
phi_BandPass(w) = simplify(angle(Hjw_BandPass(w)))
```

phi_BandPass(w) =

$$\mathrm{angle}\left(-\frac{w \mathrm{i}}{-3 C^2 R^2 w^2 + 2 C R w \mathrm{i} + 3}\right)$$

MATLAB: Symbolic Toolbox

High Pass odziv

```
syms V2_1(s) H_HighPass(s)
V2_1(s) = subs(odziv.V2, lhs(zamena_HighPass), rhs(zamena_HighPass))
```

V2_1(s) =

$$\frac{3\,C^2\,R^4\,\mathrm{Ug}\,s^2}{2\left(\frac{3\,C^2\,R^4\,s^2}{2}+3\,C\,R^3\,s+3\,R^2\right)}$$

```
H_HighPass(s) = simplify(V2_1(s)/Ug)
```

H_HighPass(s) =

$$\frac{C^2\,R^2\,s^2}{C^2\,R^2\,s^2+2\,C\,R\,s+2}$$

```
Hjw_HighPass(w) = subs(H_HighPass(s), s, 1i*w)
```

Hjw_HighPass(w) =

$$-\frac{C^2\,R^2\,w^2}{-C^2\,R^2\,w^2+2\,C\,R\,w\,\mathrm{i}+2}$$

```
Aw_HighPass(w) = simplify(abs(Hjw_HighPass(w)))
```

Aw_HighPass(w) =

$$\frac{C^2\,R^2\,|w|^2}{|-C^2\,R^2\,w^2+2\,C\,R\,w\,\mathrm{i}+2|}$$

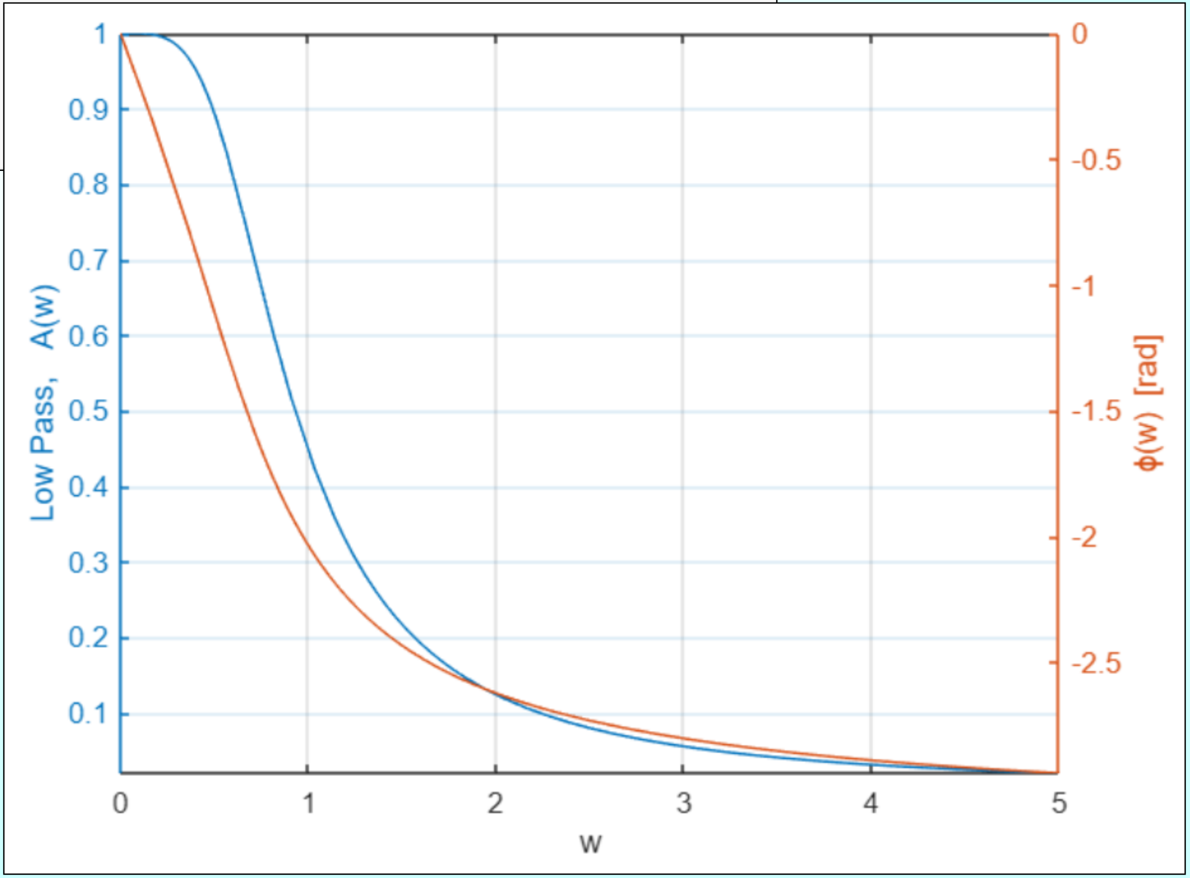
```
phi_HighPass(w) = simplify(angle(Hjw_HighPass(w)))
```

phi_HighPass(w) =

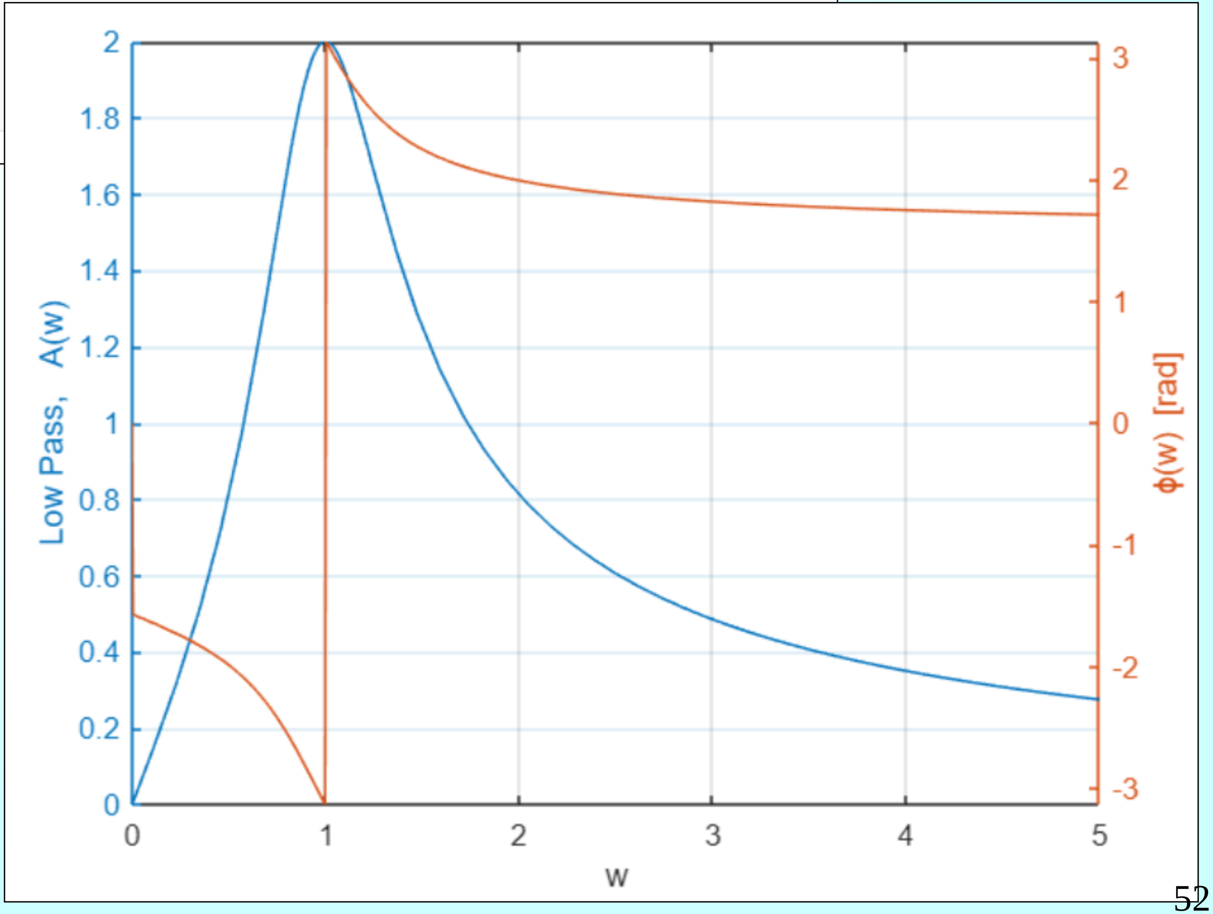
$$\mathrm{angle}\left(-\frac{w^2}{-C^2\,R^2\,w^2+2\,C\,R\,w\,\mathrm{i}+2}\right)$$

Crtanje grafika frekvencijskih karakteristika

```
figure
yyaxis left
fplot(subs(Aw_LowPass(w), lhs(vrednosti), rhs(vrednosti)), [0, 5])
ylabel('Low Pass,    A(w)')

yyaxis right
fplot(w, subs(phi_LowPass(w), lhs(vrednosti), rhs(vrednosti)), [0, 5])
ylabel('\phi(w) [rad]')

xlabel('w')
grid on
```



```
figure
yyaxis left
fplot(subs(Aw_BandPass(w), lhs(vrednosti), rhs(vrednosti)), [0, 5])
ylabel('Low Pass,   A(w)')

yyaxis right
fplot(w, subs(phi_BandPass(w), lhs(vrednosti), rhs(vrednosti)), [0, 5])
ylabel('\phi(w) [rad]')

xlabel('w')
grid on
```



```
figure
yyaxis left
fplot(subs(Aw_HighPass(w), lhs(vrednosti), rhs(vrednosti)), [0, 5])
ylabel('Low Pass,   A(w)')

yyaxis right
fplot(w, subs(phi_HighPass(w), lhs(vrednosti), rhs(vrednosti)), [0,5])
ylabel('\phi(w) [rad]')

xlabel('w')
grid on
```