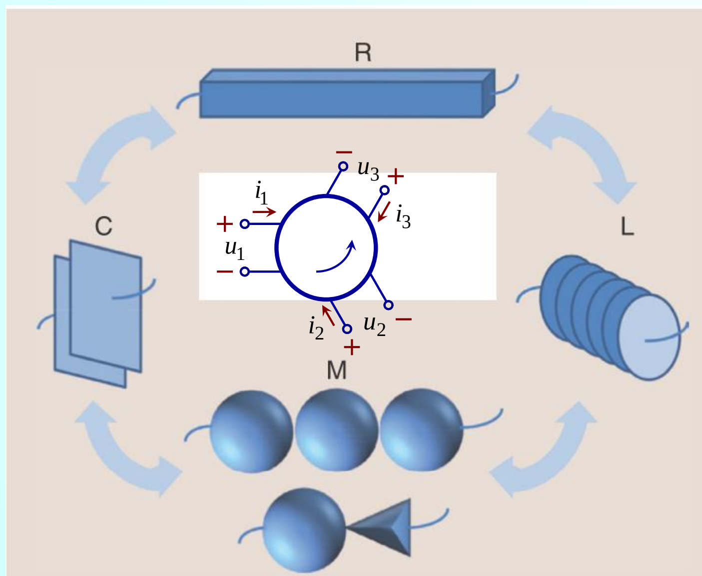
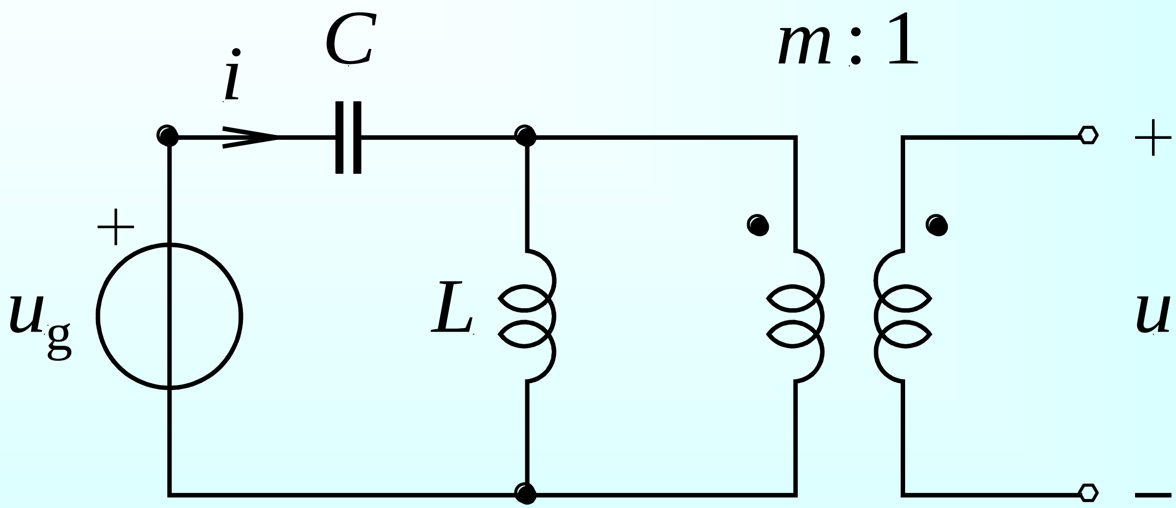
Практикум из рачунарске анализе трофазних кола
4. Временски домен. Трансформатори



Милка Потребић Иваниш

Никола Баста

Одзив на простопериодичну побуду



$U_m = 1\text{V}$

$\omega = 10^5\ \text{rad/s}$

$L = 0.1\text{mH}$

$C = 1\mu\text{F}$

$m = 1$

$$u_g(t) = U_m \sin\left(\frac{1}{\sqrt{CL}}t\right)h(t)$$

Задатак - простопериодична побуда

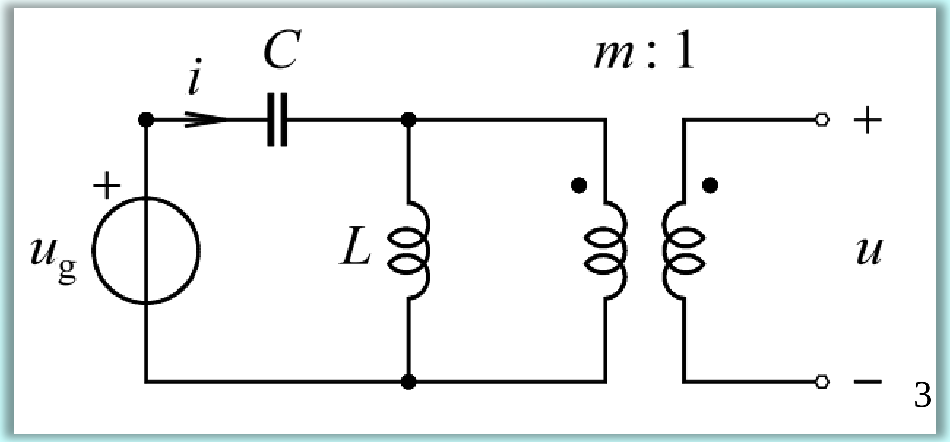
(5) Одредити једначину одзива за напон u електричног кола са слике. Вредности елемената и параметри побуде су познати,

$$u_{\mathrm{g}}(t) = U_{\mathrm{m}} \sin(\frac{1}{\sqrt{CL}} t) \mathrm{h}(t) .$$

Нема сакупљене енергије.

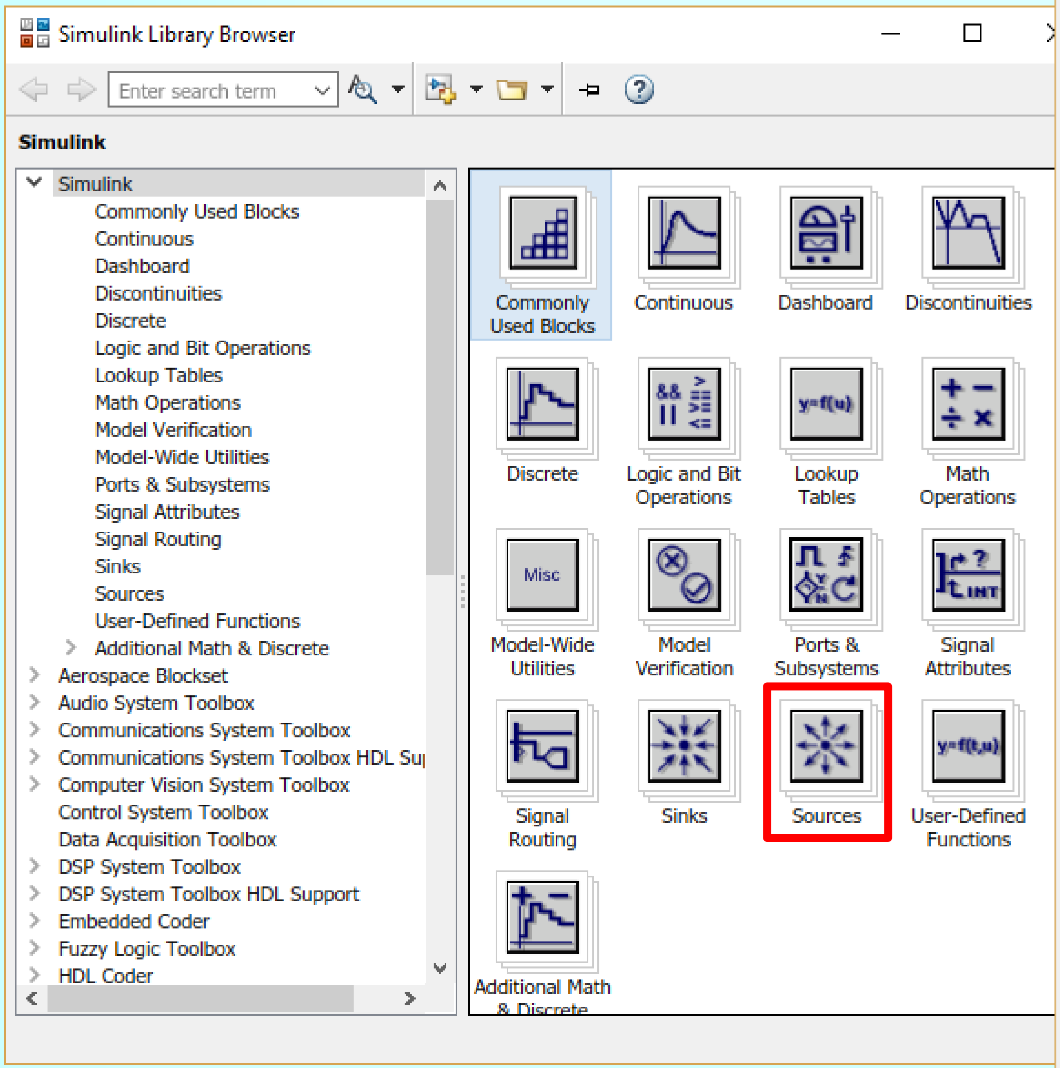
(5) Одредити струју извора, i, и нацртати график струје извора.

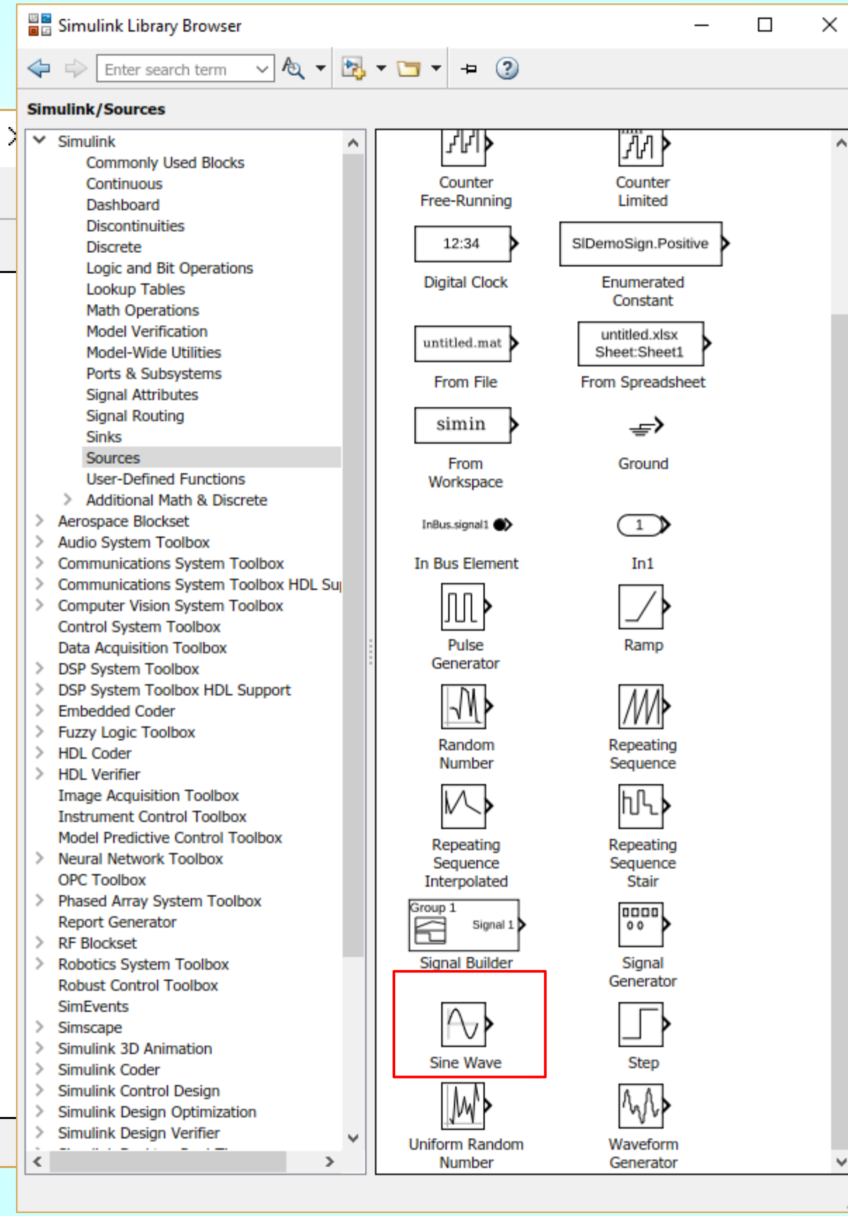
(5) Како гласе једначине стања кола? Написати их у матричном облику.

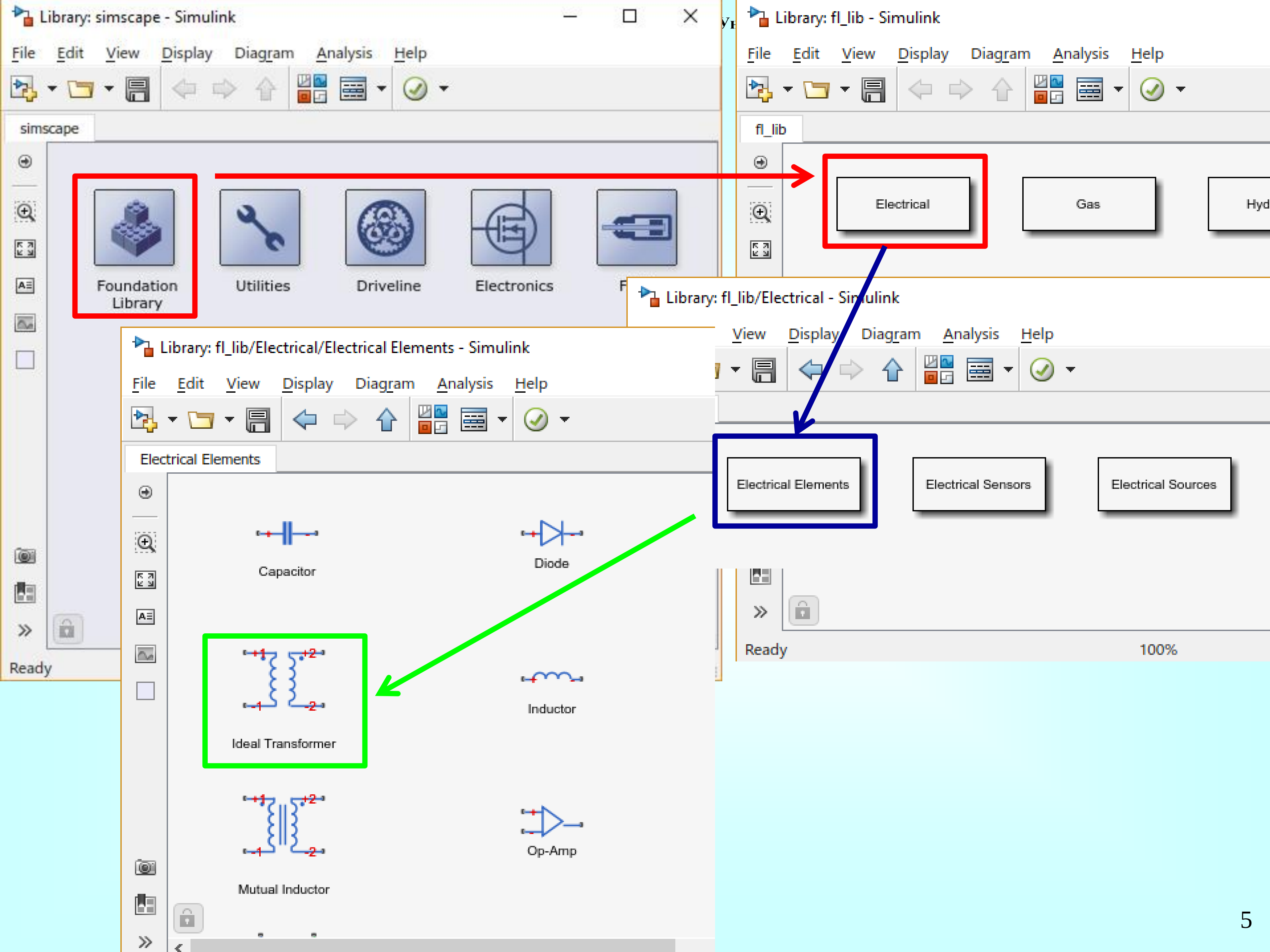


MATLAB: Simulink

Sources



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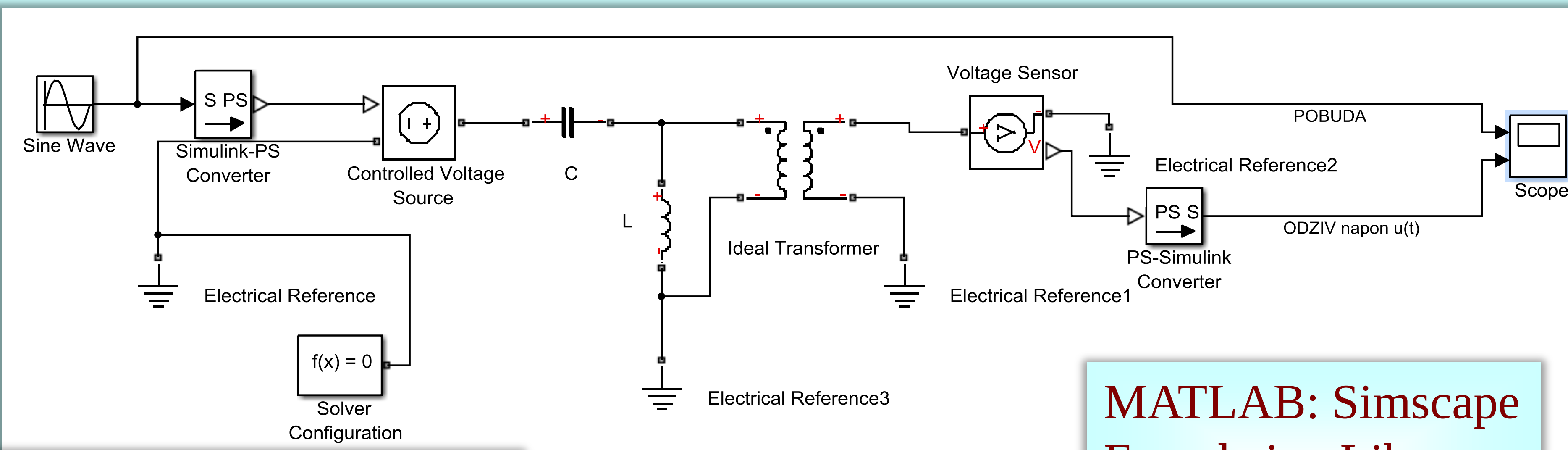
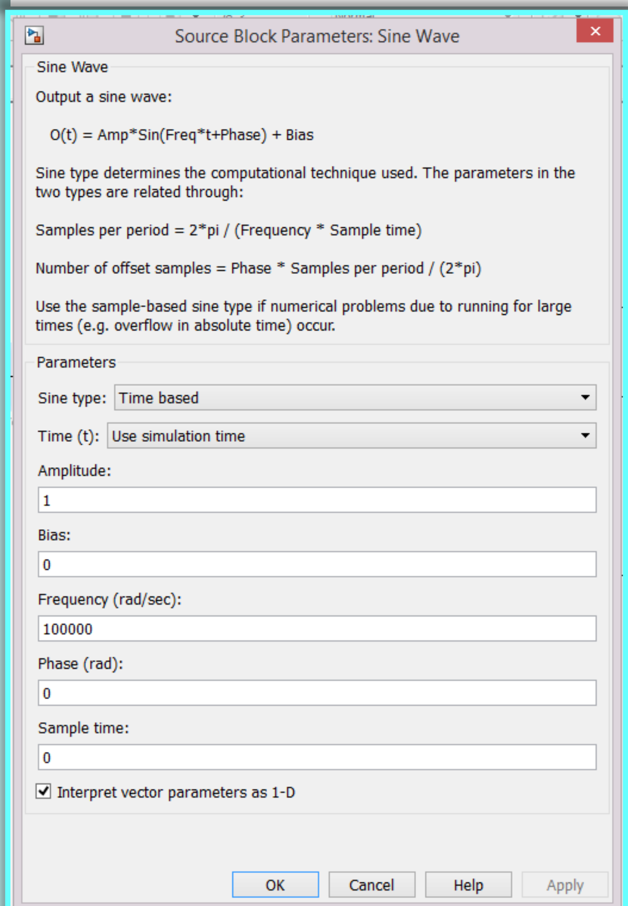
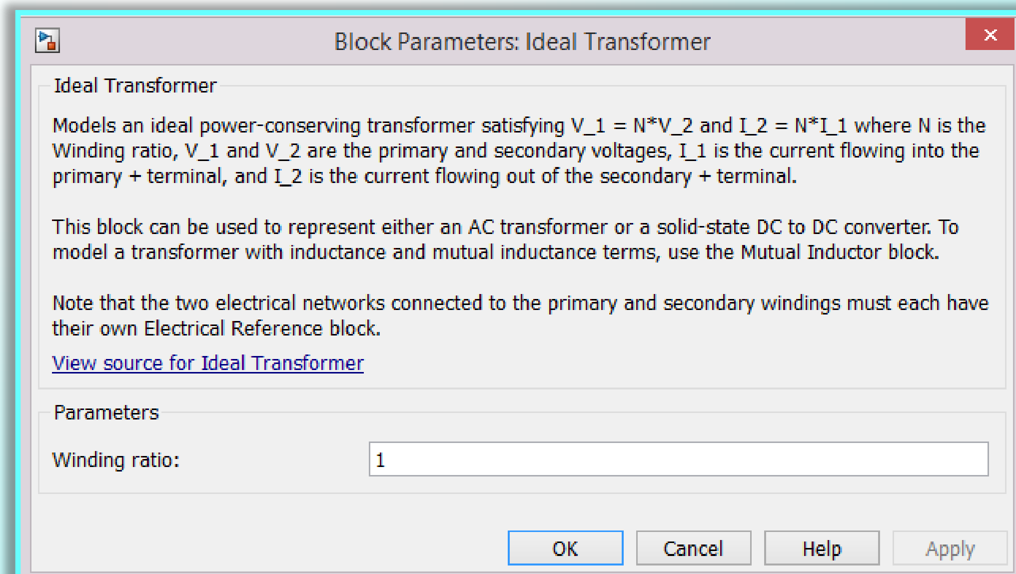
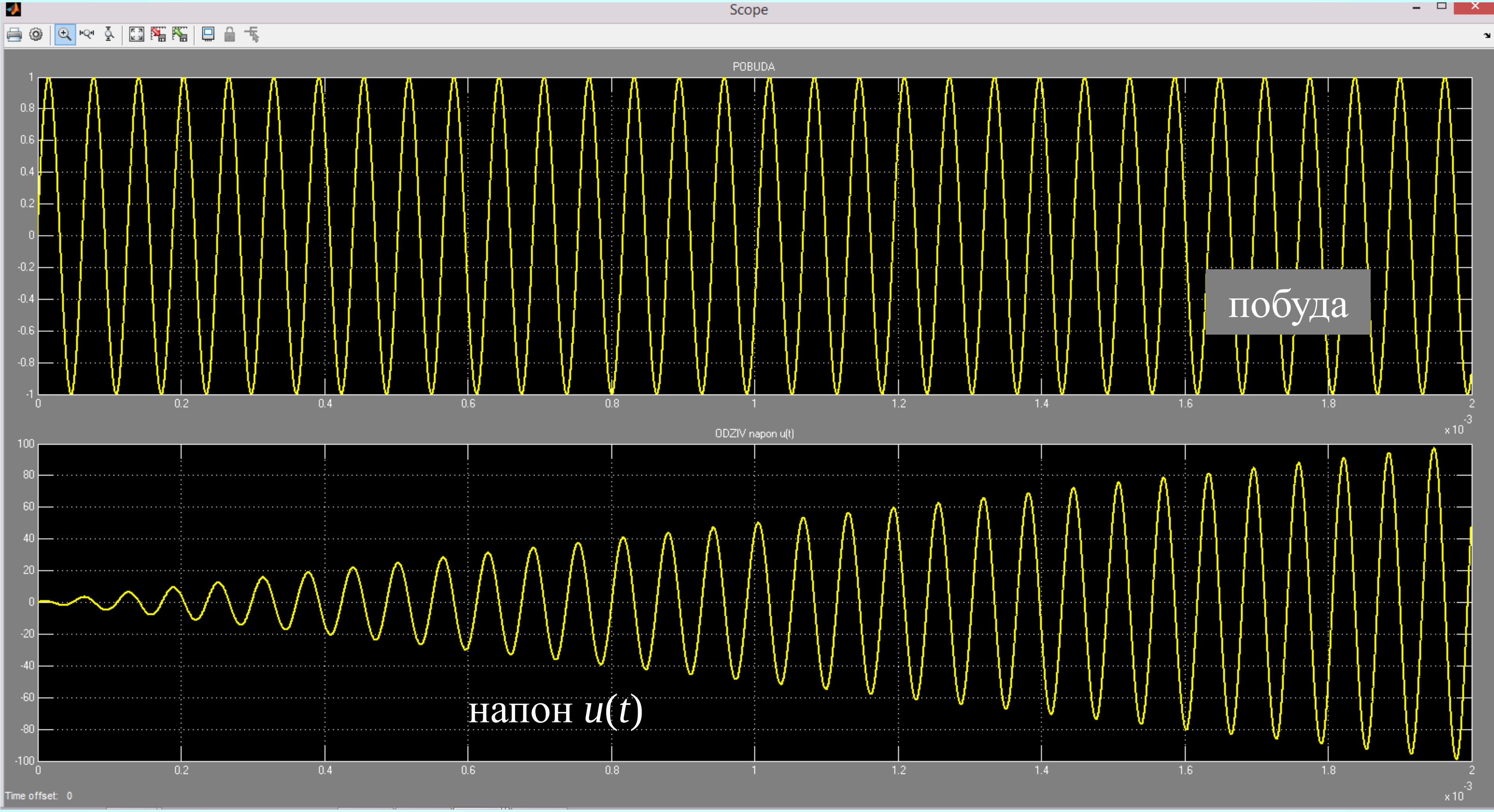
simscape
Foundation Library
Utilities
Driveline
Electronics
Library: fl_lib - Simulink
fl_lib
Electrical
Gas
Library: fl_lib/Electrical - Simulink
Electrical Elements
Electrical Sensors
Electrical Sources
Library: fl_lib/Electrical/Electrical Elements - Simulink
Electrical Elements
Capacitor
Diode
Ideal Transformer
Inductor
Mutual Inductor
Op-Amp

MATLAB: Simscape Foundation Library Utilities Simulink








Scope
POBUDA
побуда
ODZIV napon u(t)
напон u(t)
Time offset: 0
x 10^-3

Једначине

Postavljanje jednacina

```
clear variables
syms ug uC uL L C DiL DuC m u iC iL Um t
jednacine = [ug == uC + uL,...
    uL == L*DiL,...
    iC == C*DuC,...
    uL==m*u,...
    iC==iL]
```

jednacine = $\left(ug = uC + uL \quad uL = DiL\, L \quad iC = C\, DuC \quad uL = m\, u \quad iC = iL\right)$

Eliminacija i redukcija sistema

```
sistem = eliminate(jednacine, [uL,iC,u])
```

sistem = $\left[uC - ug + DiL\, L, C\, DuC - iL\right]$

Resavanje po izvodima stanja

```
resenje = solve(sistem, [DiL,DuC])
```

```
resenje = struct with fields:
    DiL: -(uC - ug)/L
    DuC: iL/C
```

Решавање система

Resavanje sistema diferencijalnih jednacina

```
syms iL(t) uC(t)

jednacineStanjaFun = subs([diff(uC)==resenje.DuC; diff(iL)==resenje.DiL], {iL,uC}, {iL(t),uC(t)})
```

jednacineStanjaFun(t) =

$$\begin{pmatrix} \frac{\partial}{\partial t}\,\mathrm{uC}(t) = \frac{\mathrm{iL}(t)}{C} \\ \frac{\partial}{\partial t}\,\mathrm{iL}(t) = \frac{\mathrm{ug} - \mathrm{uC}(t)}{L} \end{pmatrix}$$

```
zamene = ug==Um*sin(t/sqrt(C*L))*heaviside(t)
```

zamene =

$$\mathrm{ug} = \mathrm{Um}\sin\left(\frac{t}{\sqrt{C\,L}}\right)\mathrm{heaviside}(t)$$

```
jednacineStanjaFunR=subs(jednacineStanjaFun,lhs(zamene),rhs(zamene))
```

jednacineStanjaFunR(t) =

$$\begin{pmatrix} \frac{\partial}{\partial t}\,\mathrm{uC}(t) = \frac{\mathrm{iL}(t)}{C} \\ \frac{\partial}{\partial t}\,\mathrm{iL}(t) = -\frac{\mathrm{uC}(t) - \mathrm{Um}\sin\left(\frac{t}{\sqrt{C\,L}}\right)\mathrm{heaviside}(t)}{L} \end{pmatrix}$$

```
simplify(resenjeDiff.iL)
```

ans =

$$\frac{\mathrm{Um}\,t\sin\left(\frac{t}{\sqrt{C}\,\sqrt{L}}\right)}{2\,L}$$

```
assume(t>0 & C>0 & L>0)
resenjeDiff=dsolve(jednacineStanjaFunR,[uC(0)==0,iL(0)==0],'IgnoreAnalyticConstraints',false)
```

```
resenjeDiff = struct with fields:
    iL: cos(t/(C^(1/2)*L^(1/2)))*((C^(1/2)*Um)/(2*L^(1/2)) - (C^(1/2)*Um*cos(t/(C^(1/2)*L^(1/2)))^2)/(2*L^(1/2))) +
    uC: (L^(1/2)*sin(t/(C^(1/2)*L^(1/2)))*((C^(1/2)*Um)/(2*L^(1/2)) - (C^(1/2)*Um*cos(t/(C^(1/2)*L^(1/2)))^2)/(2*L^(
```

График функције одзива

```
simplify(resenjeDiff.iL)
```

ans =

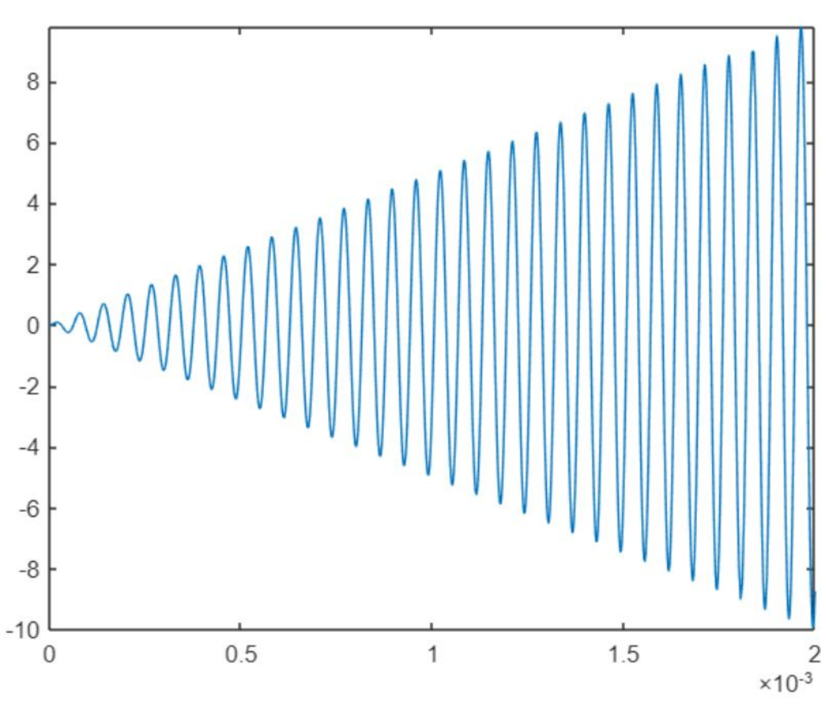
$$\frac{\mathrm{Um}\, t \sin\left(\frac{t}{\sqrt{C}\ \sqrt{L}}\right)}{2\,L}$$

```
numzamene=[L==1e-4, C==1e-6, Um==1, m==1]
```

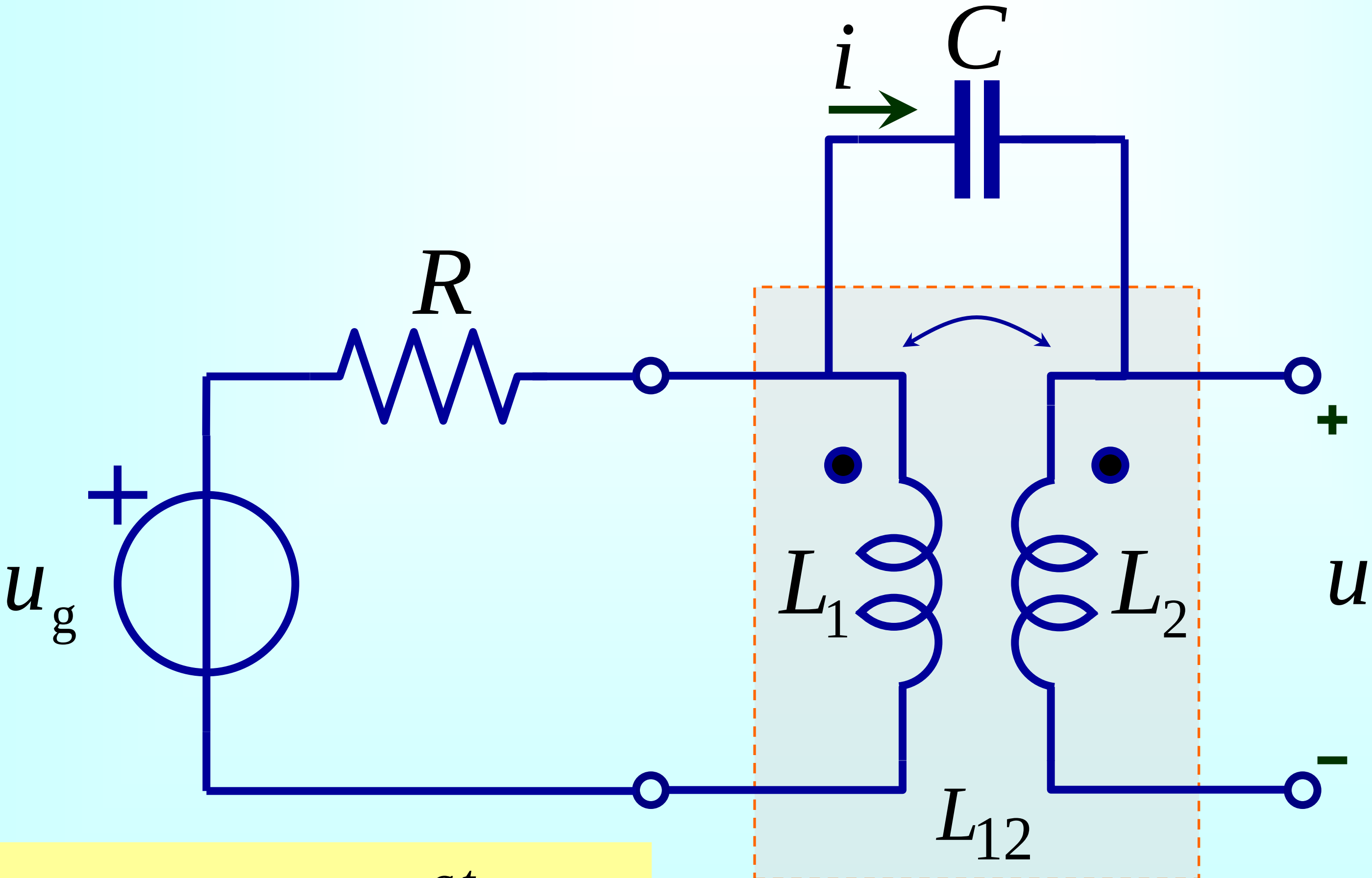
numzamene =

$$\left(L = \frac{1}{10000} \quad C = \frac{1}{1000000} \quad \mathrm{Um} = 1 \quad m = 1 \right)$$

```
fplot(t, subs(resenjeDiff.iL, lhs(numzamene), rhs(numzamene)),[0 0.002])
```



Одзив на експоненцијалну побуду



$L = 1\text{mH}$

$C = 1\mu\text{F}$

$R = 1\text{k}\Omega$

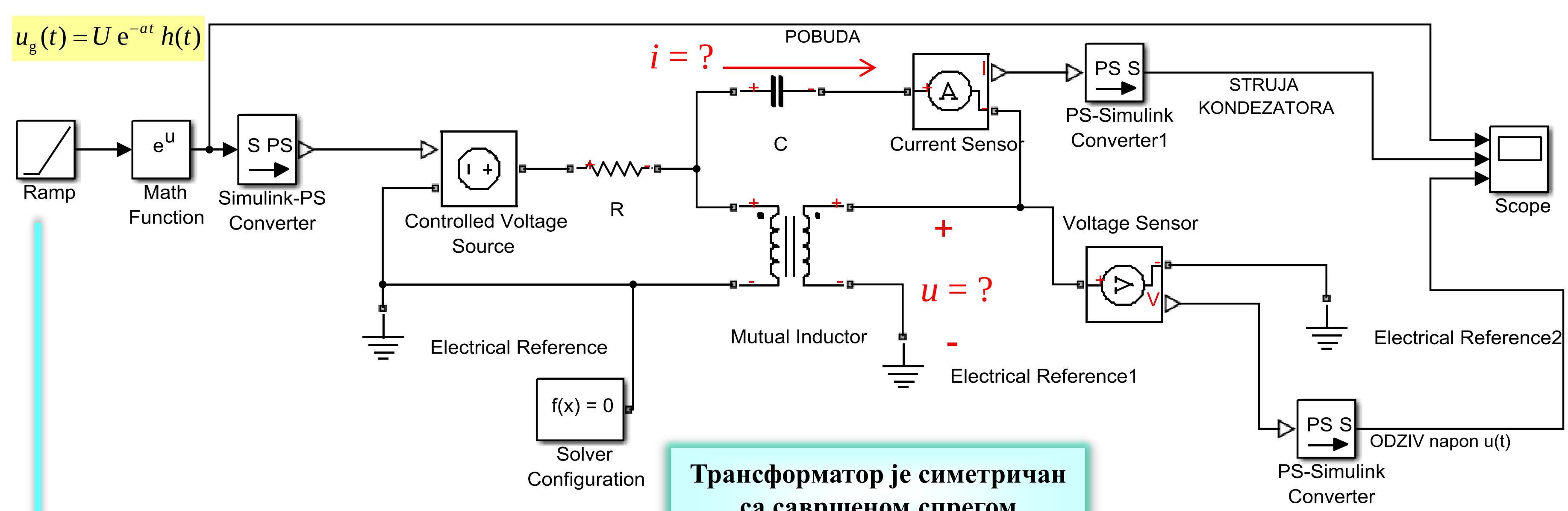
$$u_g(t) = U\,\mathrm{e}^{-at}\,h(t)$$

Трансформатор је симетричан са савршеном спрегом

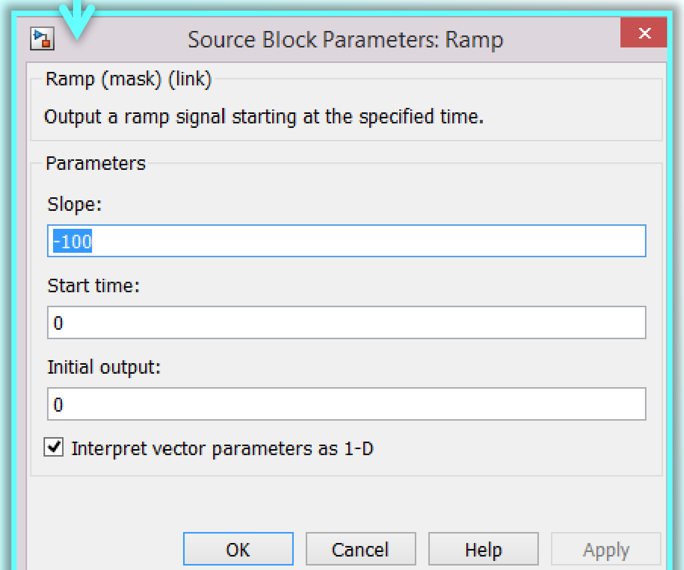
$U = 1\text{V},\ a = 100$

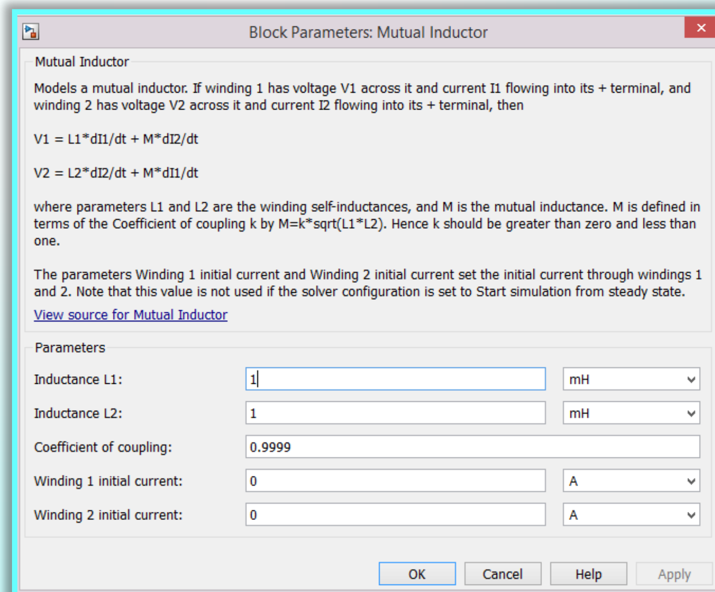
Задатак - експоненцијална побуда

- Параметри кола са слике су познати. Линеарни индуктивни трансформатор је симетричан са савршеном спрегом. Нема сакупљене енергије.
- Одредити струју кондензатора $i(t)$ и нацртати њен график.
- Одредити напон секундара $u(t)$ и нацртати његов график.



Трансформатор је симетричан са савршеном спрегом





$$L = 1\,\mathrm{mH}$$
$$C = 1\,\mu\mathrm{F}$$
$$R = 1\,\mathrm{k}\Omega$$

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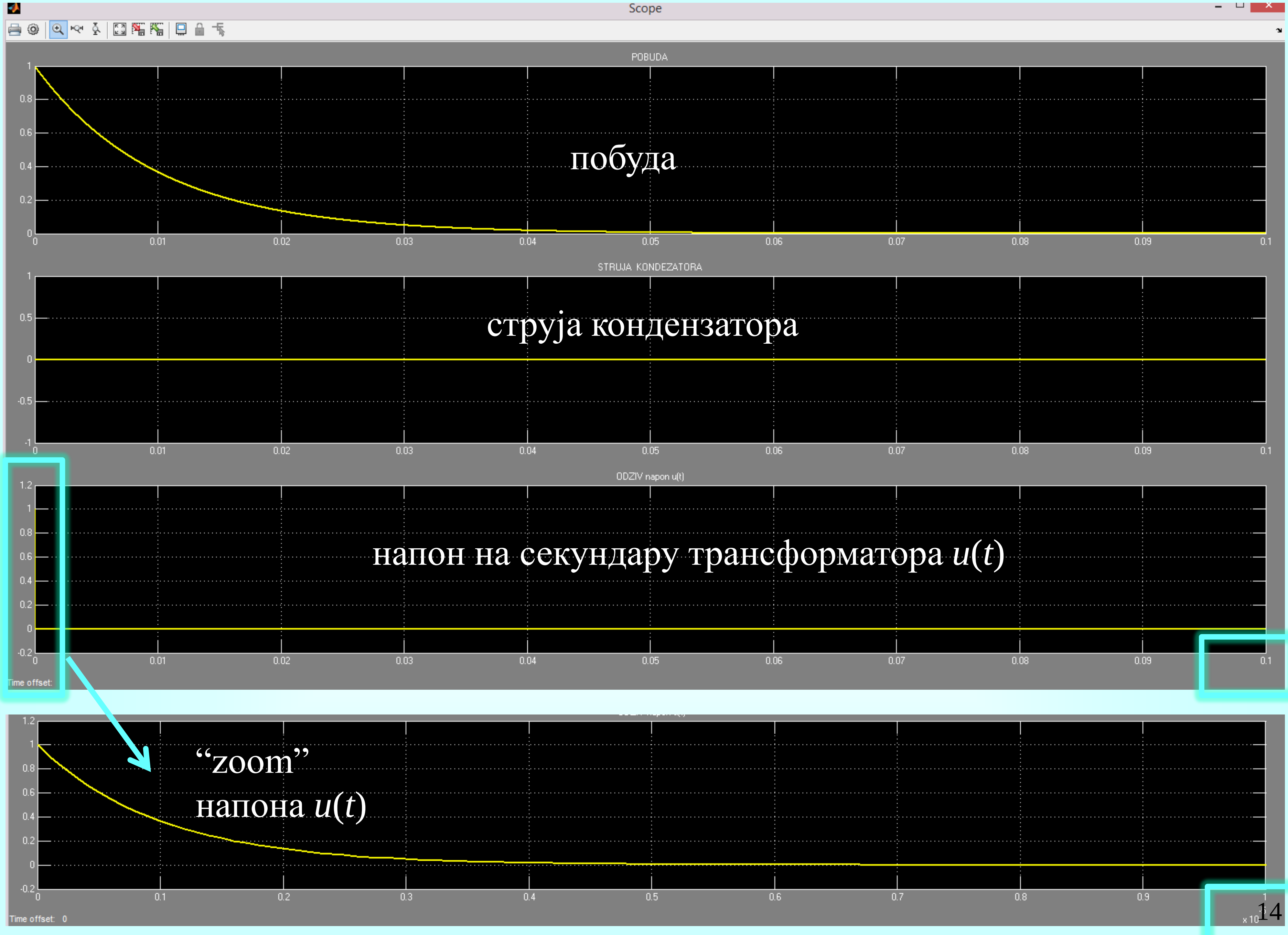
$$U = 1\,\mathrm{V},\ a = 100$$


Scope
POBUDA
побуда
STRUJA KONDEZATORA
струја кондензатора
ODZIV napon u(t)
напон на секундару трансформатора u(t)
Time offset:
“zoom”
напона u(t)
Time offset: 0
x 10

Једначине

MATLAB: Symbolic Math Toolbox

Jednacine

```
clear variables
syms C L L1 L12 L2 R U a DiL1 DiL2 DuC i iL1 iL2 iR u uC uL1 uR t ug
assume(0 < R & 0 < C & 0 < L1 & 0 < L2 & 0 < L12 & 0 < L & 0 < U & 0 < a)

zamene = [L1 == L, L2 == L, L12 == L]
```

zamene = $\left(L_1 = L \quad L_2 = L \quad L_{12} = L\right)$

```
pobuda = ug == U*exp(-(a*t))*heaviside(t)
```

pobuda = $\mathrm{ug} = U\,\mathrm{e}^{-a\,t}\,\mathrm{heaviside}(t)$

```
jednacine = [ug == uR + uL1,...
    uR == R * iR,...
    uL1 == L1 * DiL1 + L12 * DiL2,...
    u == L2 * DiL2 + L12 * DiL1,...
    uC == uL1 - u,...
    i == C * DuC,...
    iR == i + iL1,...
    i == iL2]
```

jednacine = $\left(\mathrm{ug} = \mathrm{uL}_1 + \mathrm{uR} \quad \mathrm{uR} = R\,\mathrm{iR} \quad \mathrm{uL}_1 = \mathrm{DiL}_1\,L_1 + \mathrm{DiL}_2\,L_{12} \quad u = \mathrm{DiL}_2\,L_2 + \mathrm{DiL}_1\,L_{12} \quad \mathrm{uC} = \mathrm{uL}_1 - u \quad i = C\,\mathrm{DuC} \quad \mathrm{iR} = i + \mathrm{iL}_1 \quad i = \mathrm{iL}_2\right)$

```
promenljive = [uL1, u, uR, iR, i]
```

promenljive = $\left(\mathrm{uL}_1 \quad u \quad \mathrm{uR} \quad \mathrm{iR} \quad i\right)$

Једначине извода

MATLAB: Symbolic Math Toolbox

Jednacine izvoda

```
jednacineIzvoda = eliminate(jednacine, promenljive)
```

jednacineIzvoda = $\left[R\,\mathrm{iL}_1 - \mathrm{ug} + R\,\mathrm{iL}_2 + \mathrm{DiL}_1\,L_1 + \mathrm{DiL}_2\,L_{12}, C\,\mathrm{DuC} - \mathrm{iL}_2, \mathrm{uC} - \mathrm{DiL}_1\,L_1 + \mathrm{DiL}_2\,L_2 + \mathrm{DiL}_1\,L_{12} - \mathrm{DiL}_2\,L_{12}\right]$

```
JednacineIzvodaZamene = subs(jednacineIzvoda, lhs(zamene), rhs(zamene))
```

JednacineIzvodaZamene = $\left(R\,\mathrm{iL}_1 - \mathrm{ug} + R\,\mathrm{iL}_2 + \mathrm{DiL}_1\,L + \mathrm{DiL}_2\,L \quad C\,\mathrm{DuC} - \mathrm{iL}_2 \quad \mathrm{uC}\right)$

```
syms iL1(t) uC(t) iL2(t)
PromenljiveStanjaIzvodi = [uC == uC(t), DuC == diff(uC(t)), iL1 == iL1(t), DiL1 == diff(iL1(t)), iL2 == iL2(t), DiL2 == diff(iL2(t))]
```

PromenljiveStanjaIzvodi(t) =

$$\left(\mathrm{uC}(t) = \mathrm{uC}(t) \quad \mathrm{DuC} = \frac{\partial}{\partial t}\,\mathrm{uC}(t) \quad \mathrm{iL}_1(t) = \mathrm{iL}_1(t) \quad \mathrm{DiL}_1 = \frac{\partial}{\partial t}\,\mathrm{iL}_1(t) \quad \mathrm{iL}_2(t) = \mathrm{iL}_2(t) \quad \mathrm{DiL}_2 = \frac{\partial}{\partial t}\,\mathrm{iL}_2(t)\right)$$

```
JednacinePromenljiveStanja = subs(JednacineIzvodaZamene, [lhs(PromenljiveStanjaIzvodi) lhs(pobuda)], [rhs(PromenljiveStanjaIzvodi) rhs(pobuda)])
```

JednacinePromenljiveStanja =

$$\left(L\,\frac{\partial}{\partial t}\,\mathrm{iL}_1(t) + L\,\frac{\partial}{\partial t}\,\mathrm{iL}_2(t) + R\,\mathrm{iL}_1(t) + R\,\mathrm{iL}_2(t) - U\,\mathrm{e}^{-a\,t}\,\mathrm{heaviside}(t) \quad C\,\frac{\partial}{\partial t}\,\mathrm{uC}(t) - \mathrm{iL}_2(t) \quad \mathrm{uC}(t)\right)$$

```
promenljiveStanja = [iL1(t), iL2(t), uC(t)]
```

promenljiveStanja = $\left(\mathrm{iL}_1(t) \quad \mathrm{iL}_2(t) \quad \mathrm{uC}(t)\right)$

Елиминација сувишних једначина

Redukcija reda sistema

```
[noveJednacine, novePromenljive, Rstruct] = reduceRedundancies(JednacinePromenljiveStanja, promenljiveStanja)
```

noveJednacine =

$$e^{-a\,t}\left(L\,e^{a\,t}\,\frac{\partial}{\partial t}\,\mathrm{iL}_1(t) - U\,\mathrm{heaviside}(t) + R\,e^{a\,t}\,\mathrm{iL}_1(t)\right)$$

novePromenljive = $\mathrm{iL}_1(t)$

```
Rstruct = struct with fields:
      solvedEquations: [2×1 sym]
    constantVariables: [2×2 sym]
    replacedVariables: [0×2 sym]
       otherEquations: [0×1 sym]
```

MATLAB: Symbolic Math Toolbox

```
reseneJednacine = Rstruct.solvedEquations
```

reseneJednacine =

$$\begin{pmatrix} \mathrm{uC}(t) \\ -\mathrm{iL}_2(t) \end{pmatrix}$$

```
konstantnePromenljive = Rstruct.constantVariables
```

konstantnePromenljive =

$$\begin{pmatrix} \mathrm{uC}(t) & 0 \\ \mathrm{iL}_2(t) & 0 \end{pmatrix}$$

```
zamenjenePromenljive = Rstruct.replacedVariables
```

zamenjenePromenljive =

Empty sym: 0-by-2

```
drugeJednacine = Rstruct.otherEquations
```

drugeJednacine =

Empty sym: 0-by-1

Решавање диференцијалне једначине

MATLAB: Symbolic Math Toolbox

Resavanje diferencijalne jednacine

```
odziv_iL1 = dsolve(noveJednacine, iL1(0) == 0, 'IgnoreAnalyticConstraints', false)
```

odziv_iL1 =

$$\frac{U\,e^{-\frac{R\,t}{L}}\,e^{-a\,t}\,\mathrm{heaviside}(t)\,\left(e^{\frac{R\,t}{L}}-e^{a\,t}\right)}{R-L\,a}$$

Napon *u*(*t*)

```
odziv_u = subs(L12*diff(expand(odziv_iL1)), lhs(zamene), rhs(zamene))
```

odziv_u =

$$L\left(\frac{U\,e^{-a\,t}\,\delta(t)}{R-L\,a}-\frac{U\,e^{-\frac{R\,t}{L}}\,\delta(t)}{R-L\,a}-\frac{U\,a\,e^{-a\,t}\,\mathrm{heaviside}(t)}{R-L\,a}+\frac{R\,U\,e^{-\frac{R\,t}{L}}\,\mathrm{heaviside}(t)}{L\,(R-L\,a)}\right)$$

```
odziv_u_simp = simplify(odziv_u)
```

odziv_u_simp =

$$-L\left(\frac{U\,a\,e^{-a\,t}\,\mathrm{heaviside}(t)}{R-L\,a}-\frac{R\,U\,e^{-\frac{R\,t}{L}}\,\mathrm{heaviside}(t)}{L\,(R-L\,a)}\right)$$

```
vrednosti = [U==1, R==1000, C==1e-6, L==1e-3, a==100]
```

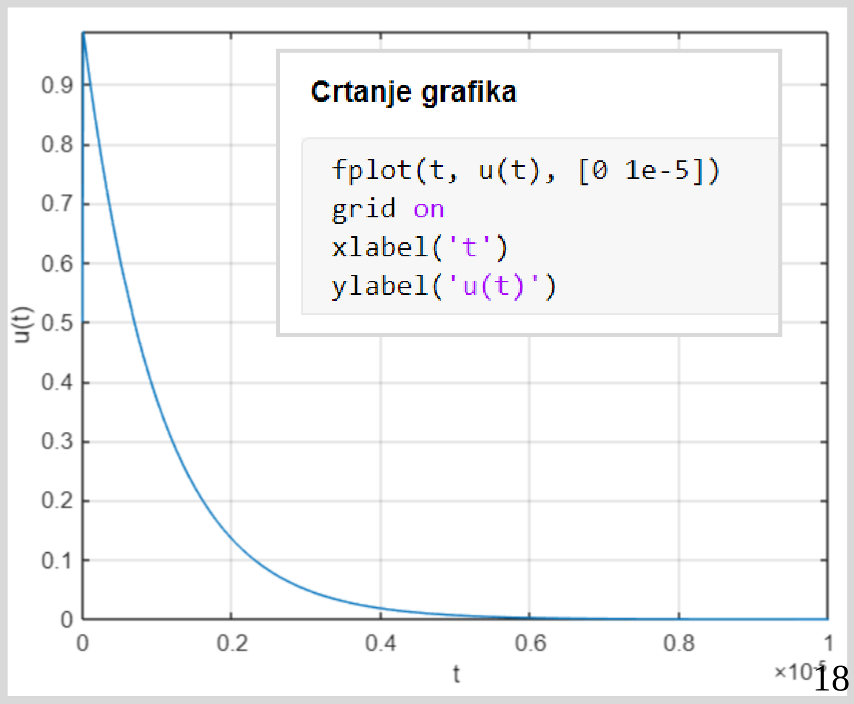
vrednosti =

$$\left(U=1\quad R=1000\quad C=\frac{1}{1000000}\quad L=\frac{1}{1000}\quad a=100\right)$$

```
u(t) = subs(odziv_u_simp, lhs(vrednosti), rhs(vrednosti))
```

u(t) =

$$\frac{10000\,e^{-1000000\,t}\,\mathrm{heaviside}(t)}{9999}-\frac{e^{-100\,t}\,\mathrm{heaviside}(t)}{9999}$$



Crtanje grafika

```
fplot(t, u(t), [0 1e-5])
grid on
xlabel('t')
ylabel('u(t)')
```

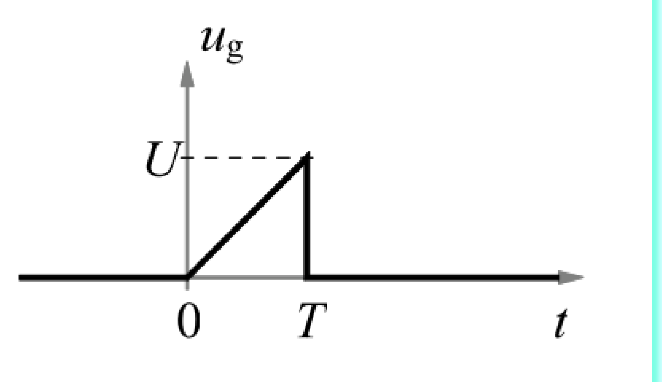
Одзив на троугаони импулс

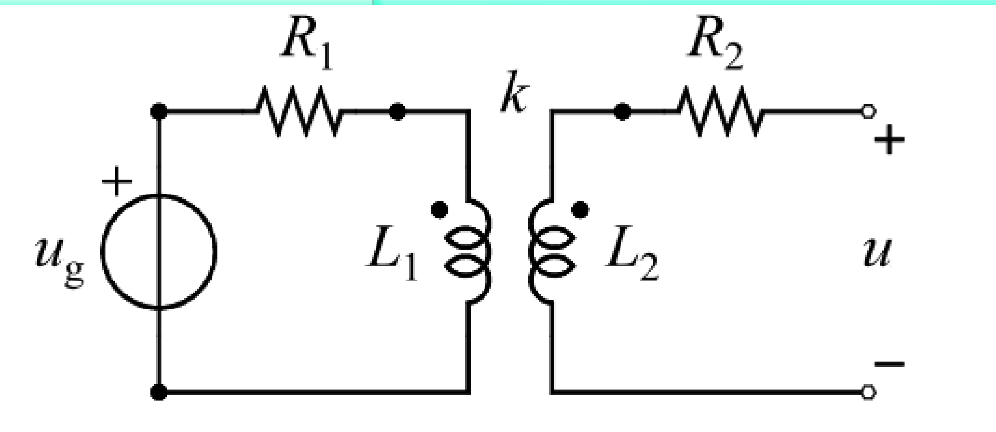
Параметри електричног кола са слике су познати. Побуда је дата на слици, $T = L/R$, $R_1 = R$, $R_2 = 2R$, $L_1 = L$, $L_2 = 4L$, $k = 1/2$.

(5) Одредити индициону функцију за напон отвореног секундара (одскочни одзив).

(5) Одредити напон отвореног секундара и

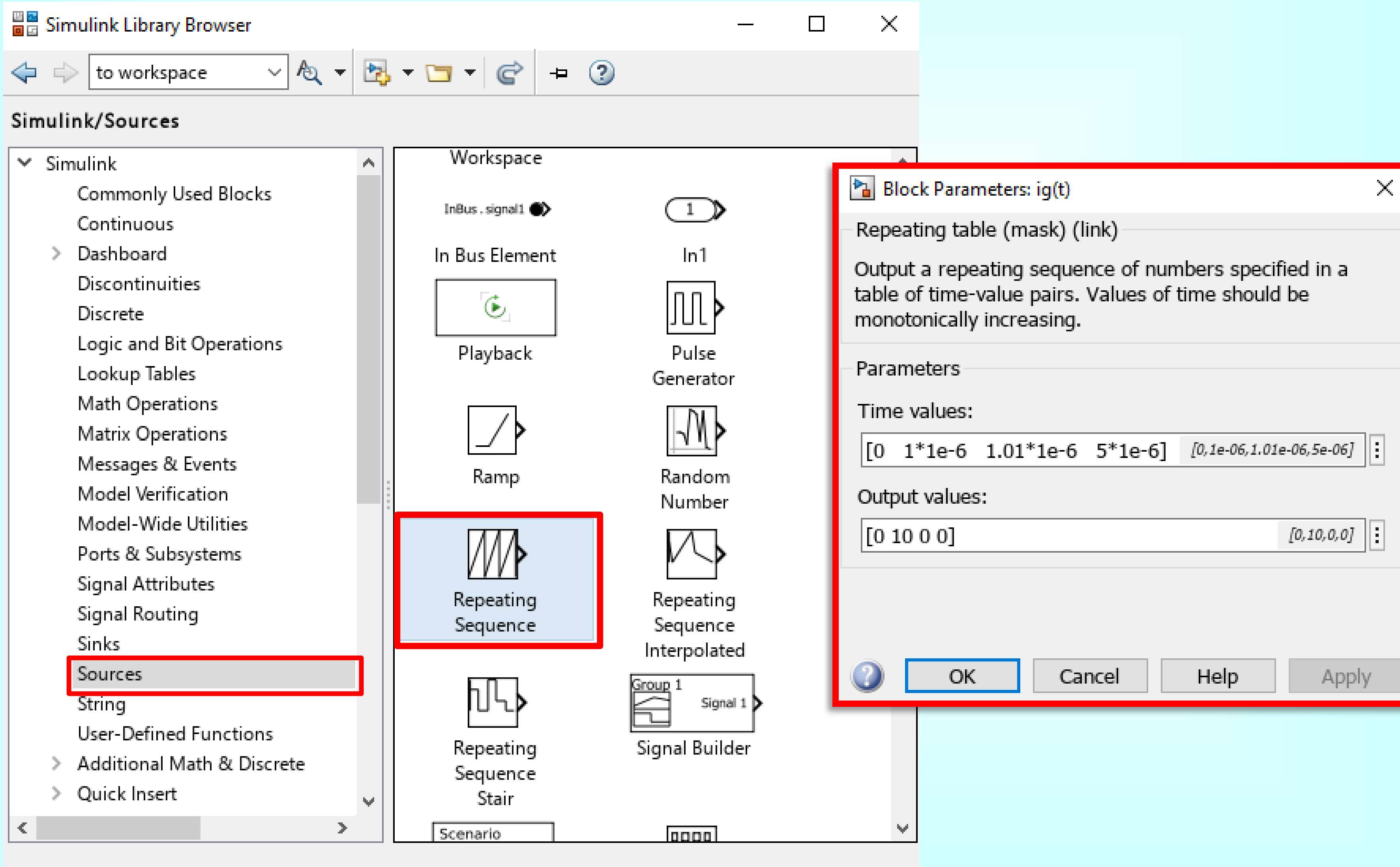
(5) нацртати његов график. Обележити осе графика, координатни почетак, пресеке и додире графика са осама, и тачке екстремума.

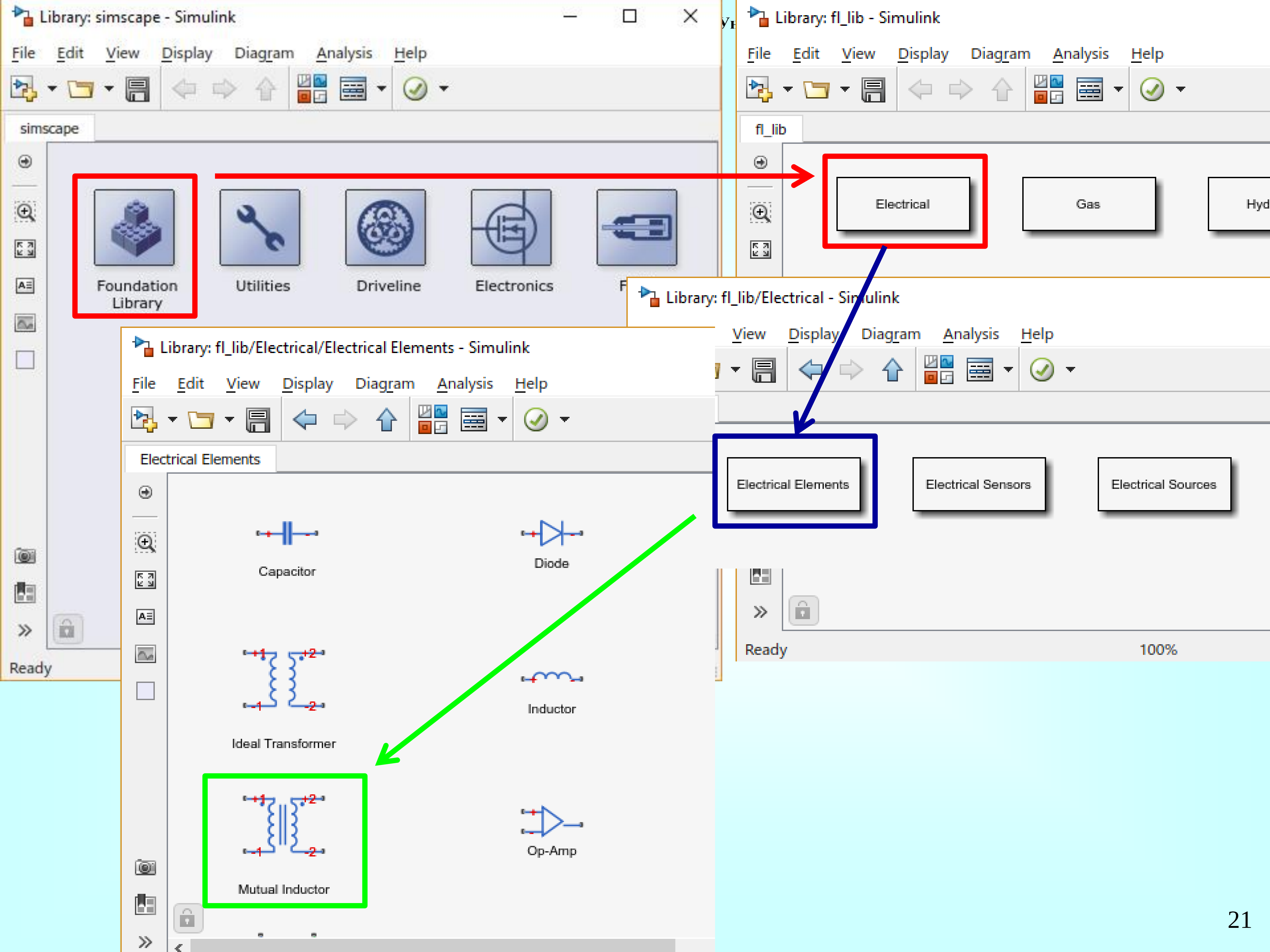




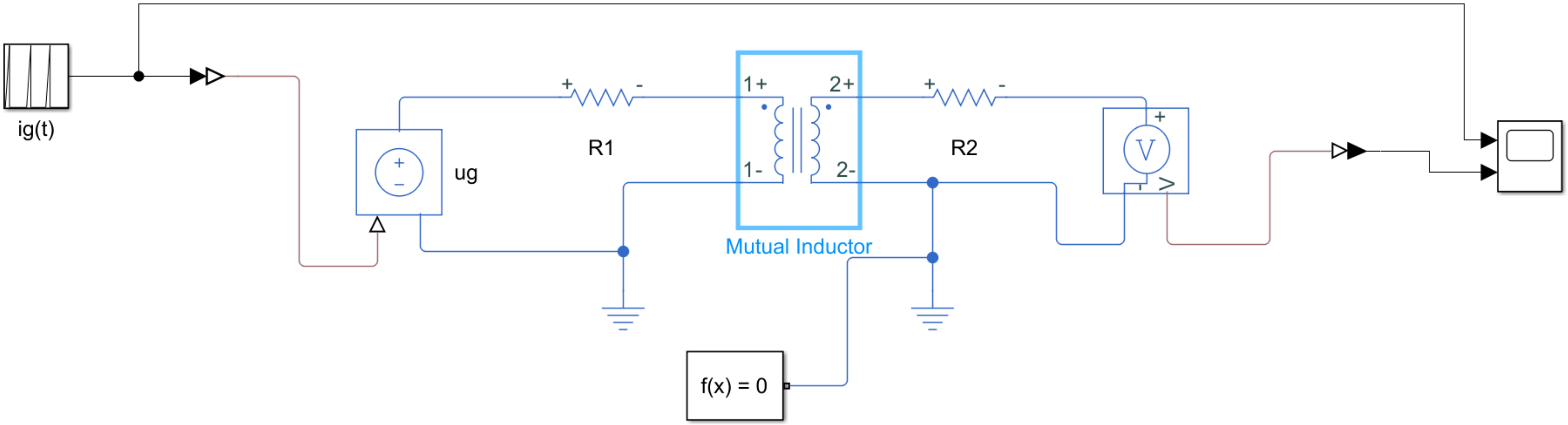
MATLAB: Simulink

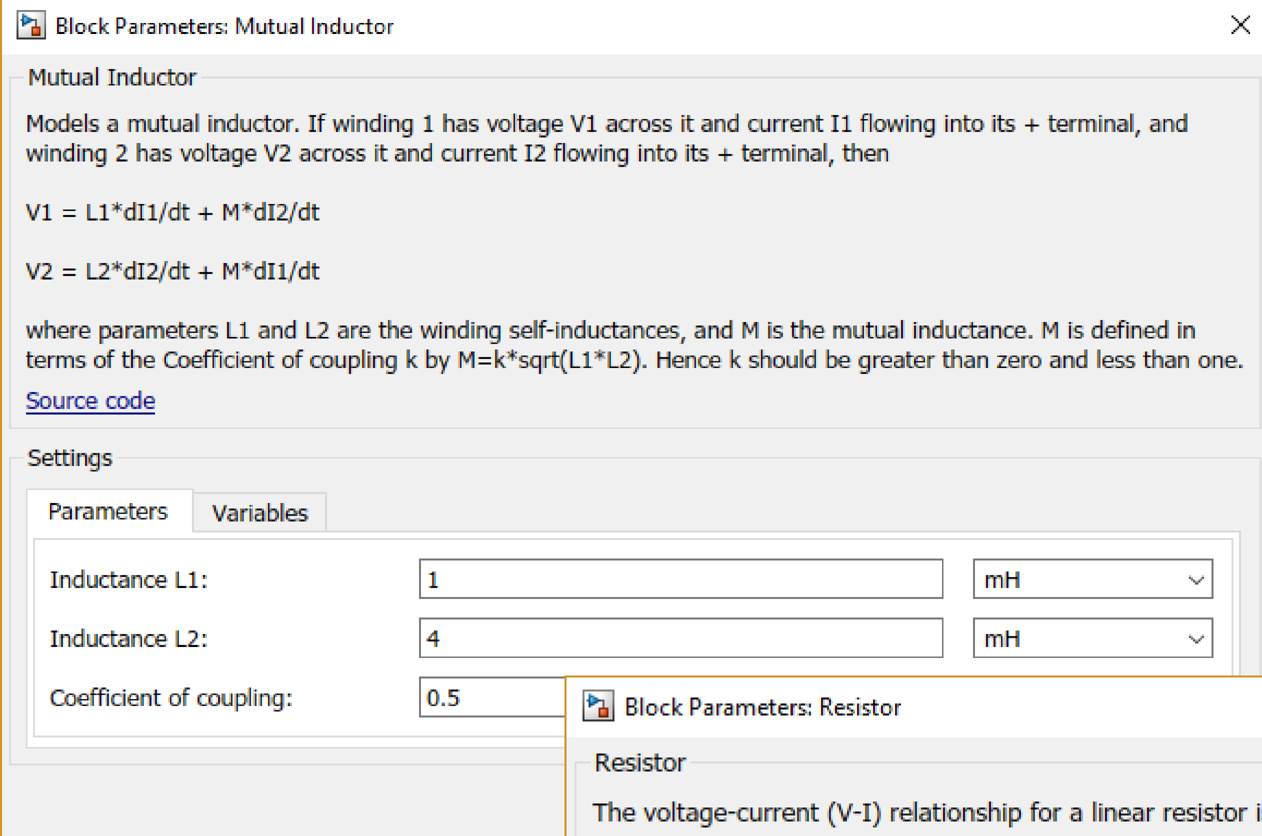
Sources

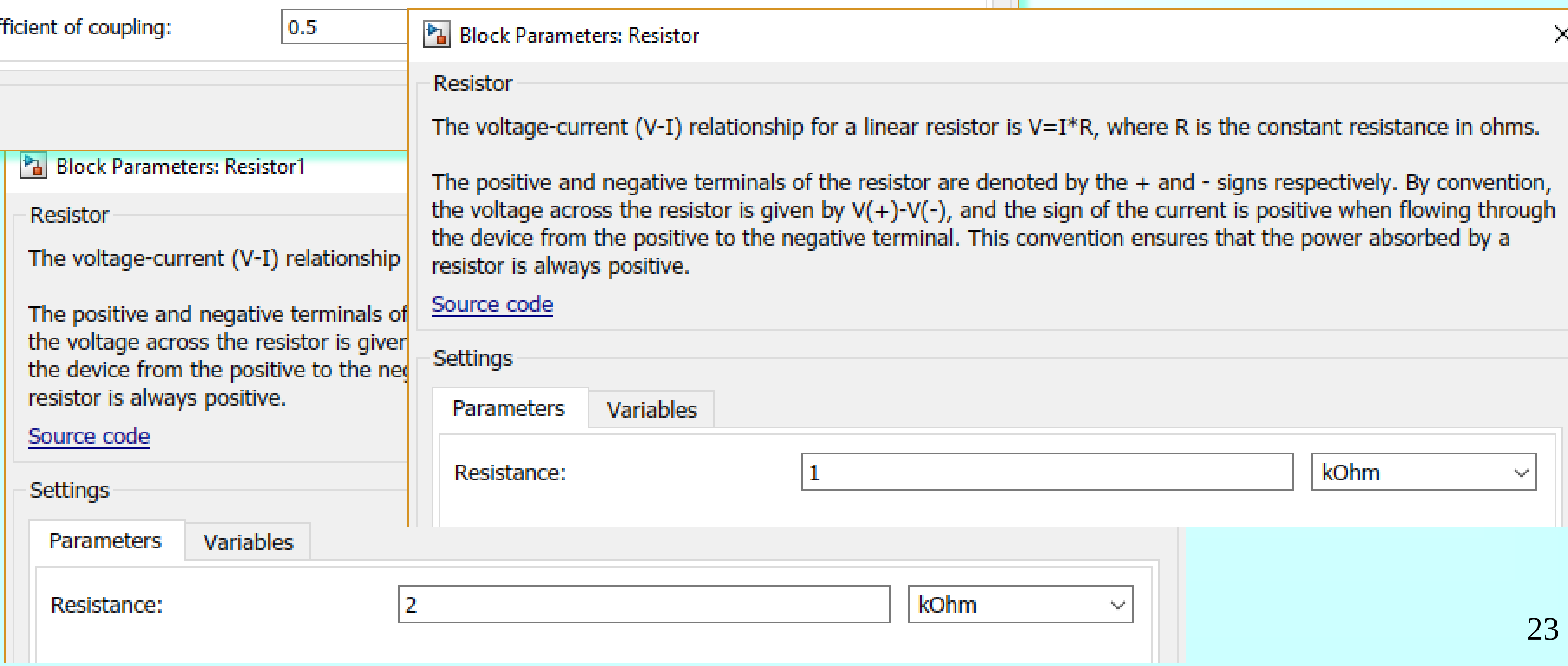


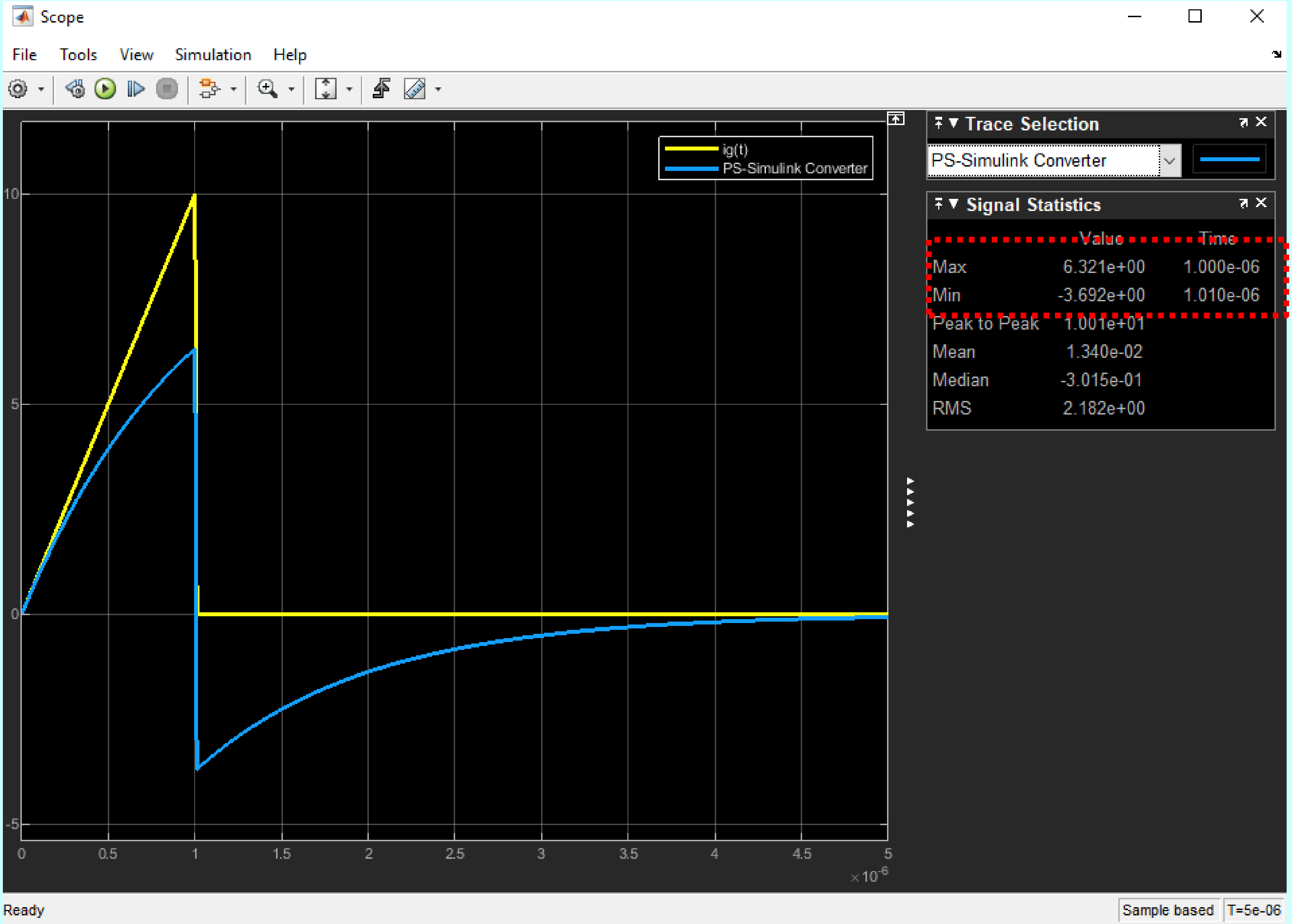

Library: simscape - Simulink
File Edit View Display Diagram Analysis Help
simscape
Foundation Library
Utilities
Driveline
Electronics
Ready
Library: fl_lib - Simulink
File Edit View Display Diagram Analysis Help
fl_lib
Electrical
Gas
Library: fl_lib/Electrical - Simulink
View Display Diagram Analysis Help
Electrical Elements
Electrical Sensors
Electrical Sources
Ready
100%
Library: fl_lib/Electrical/Electrical Elements - Simulink
File Edit View Display Diagram Analysis Help
Electrical Elements
Capacitor
Diode
Ideal Transformer
Inductor
Mutual Inductor
Op-Amp

Sastavljanje kola




Block Parameters: Mutual Inductor
Mutual Inductor
Models a mutual inductor. If winding 1 has voltage V1 across it and current I1 flowing into its + terminal, and winding 2 has voltage V2 across it and current I2 flowing into its + terminal, then
V1 = L1*dI1/dt + M*dI2/dt
V2 = L2*dI2/dt + M*dI1/dt
where parameters L1 and L2 are the winding self-inductances, and M is the mutual inductance. M is defined in terms of the Coefficient of coupling k by M=k*sqrt(L1*L2). Hence k should be greater than zero and less than one.
Source code
Settings
Parameters
Variables
Inductance L1:
1
mH
Inductance L2:
4
mH
Coefficient of coupling:
0.5


Block Parameters: Resistor
Resistor
The voltage-current (V-I) relationship for a linear resistor is V=I*R, where R is the constant resistance in ohms.
The positive and negative terminals of the resistor are denoted by the + and - signs respectively. By convention, the voltage across the resistor is given by V(+)-V(-), and the sign of the current is positive when flowing through the device from the positive to the negative terminal. This convention ensures that the power absorbed by a resistor is always positive.
Source code
Settings
Parameters
Variables
Resistance:
1
kOhm
Block Parameters: Resistor1
Resistor
The voltage-current (V-I) relationship
The positive and negative terminals of
the voltage across the resistor is given
the device from the positive to the neg
resistor is always positive.
Source code
Settings
Parameters
Variables
Resistance:
2
kOhm



Scope
File Tools View Simulation Help
ig(t)
PS-Simulink Converter
10
5
0
-5
0 0.5 1 1.5 2 2.5 3 3.5 4 4.5 5
×10^-6
Trace Selection
PS-Simulink Converter
Signal Statistics
Value Time
Max 6.321e+00 1.000e-06
Min -3.692e+00 1.010e-06
Peak to Peak 1.001e+01
Mean 1.340e-02
Median -3.015e-01
RMS 2.182e+00
Ready
Sample based
T=5e-06

Једначине

Postavljanje jednacina

```
clear variables
syms Di1 L1 L2 L U L12 R R1 R2 i1 u1 u2 uR1 ug T t
jednacine = [ug == uR1 + u1, u1 == L1 * Di1, u2 == L12 * Di1, uR1 == R1 * i1]
```

jednacine = $\left(\mathrm{ug} = u_1 + \mathrm{uR}_1 \quad u_1 = \mathrm{Di}_1\, L_1 \quad u_2 = \mathrm{Di}_1\, L_{12} \quad \mathrm{uR}_1 = R_1\, i_1\right)$

Jednacine stanja

```
jednacineIzvoda = eliminate(jednacine, [u1, u2, uR1])
```

jednacineIzvoda = $\left[\mathrm{ug} - R_1\, i_1 - \mathrm{Di}_1\, L_1\right]$

```
assume([L1, L2, L12, U, R1, t], 'real')
assumeAlso(0 < R1 & 0 < L1 & 0 < L2 & 0 < L12 & 0 < L & 0 < U)

jednacineStanja = solve(jednacineIzvoda, Di1)
```

jednacineStanja =

$$\frac{\mathrm{ug} - R_1\, i_1}{L_1}$$

MATLAB: Symbolic Math Toolbox

Решавање диференцијалне једначине

Resavanje diferencijalne jednacine

```
syms i1(t) g(t)
promenljiveStanja = i1 == i1(t)
```

promenljiveStanja(t) = $i_1(t) = i_1(t)$

```
zamene = [R1 == R, R2 == 2*R, L1 == L, L2 == 4*L, L12 == (1/2)*sqrt(L*4*L)]
```

zamene = $\left(R_1 = R \quad R_2 = 2\,R \quad L_1 = L \quad L_2 = 4\,L \quad L_{12} = L\right)$

```
pobuda = ug == heaviside(t)
```

pobuda = $\mathrm{ug} = \mathrm{heaviside}(t)$

```
jednacineStanjaFun = subs(diff(i1(t)) == jednacineStanja,...
                          lhs([promenljiveStanja pobuda zamene]),...
                          rhs([promenljiveStanja pobuda zamene]))
```

jednacineStanjaFun =

$$\frac{\partial}{\partial t}\, i_1(t) = \frac{\mathrm{heaviside}(t) - R\, i_1(t)}{L}$$

```
resenjeDiff = dsolve(jednacineStanjaFun, i1(0) == 0, 'IgnoreAnalyticConstraints', false)
```

resenjeDiff =

$$\frac{e^{-\frac{R\,t}{L}}\,\mathrm{heaviside}(t)\,\left(e^{\frac{R\,t}{L}} - 1\right)}{R}$$

MATLAB: Symbolic Math Toolbox

Гринова функција и конволуциони интеграл

Grinova funkcija

```
g_i1(t) = simplify(diff(resenjeDiff))
```

g_i1(t) =

$$\frac{e^{-\frac{R\,t}{L}}\,\mathrm{heaviside}(t)}{L}$$

Konvolucioni integral

```
syms tau t
zameneLRT = [T == L/R, L12 == L]
```

zameneLRT =

$$\left(T=\frac{L}{R}\quad L_{12}=L\right)$$

```
assume((0 < t)& (t< T))
i1_1(t) = simplify(int( (U*tau/T)*g_i1(t-tau), tau, 0, t))
```

i1_1(t) =

$$\frac{U\,\left(R\,t-L+L\,e^{-\frac{R\,t}{L}}\right)}{R^2\,T}$$

```
assume((T > 0) & (T < t))
i1_2(t) = simplify(  int((U*tau/T)*g_i1(t-tau), tau, 0, T))
```

i1_2(t) =

$$\frac{L\,U\,e^{-\frac{R\,t}{L}}}{R^2\,T}-\frac{U\,e^{\frac{R\,(T-t)}{L}}\,(L-R\,T)}{R^2\,T}$$

MATLAB: Symbolic Math Toolbox

Конволуциони интеграл (наставак)

```
u2_1(t) = simplify(subs(L12*diff(i1_1(t)), lhs(zameneLRT), rhs(zameneLRT)))
```

u2_1(t) =

$$U - U\,e^{-\frac{R\,t}{L}}$$

```
u2_2(t) = simplify(subs(L12*diff(i1_2(t)), lhs(zameneLRT), rhs(zameneLRT)) )
```

u2_2(t) =

$$-U\,e^{-\frac{R\,t}{L}}$$

```
assume(t,'clear')
u2(t) = (piecewise((t<0), 0, (0<=t)&(t<T), u2_1(t), (t>=T), u2_2(t) ))
```

u2(t) =

$$\begin{cases} 0 & \text{if } t < 0 \\ U - U\,e^{-\frac{R\,t}{L}} & \text{if } t < T \wedge 0 \le t \\ -U\,e^{-\frac{R\,t}{L}} & \text{if } T \le t \end{cases}$$

MATLAB: Symbolic Math Toolbox

График функције одзива

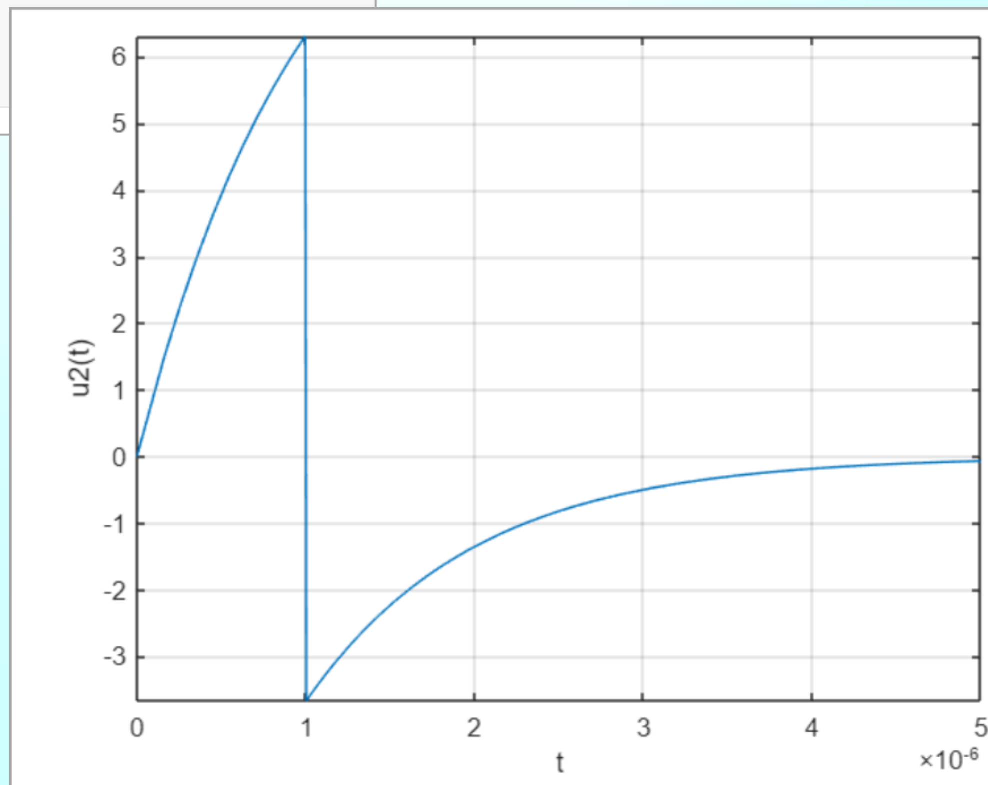
Crtanje grafika

```
vrednosti = [R == 1000, L == 1e-3, T == 1e-6, U == 10]
```

vrednosti =

$$\left(R = 1000 \quad L = \frac{1}{1000} \quad T = \frac{1}{1000000} \quad U = 10\right)$$

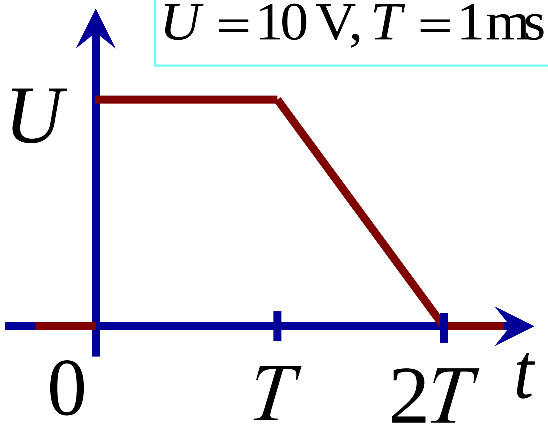
```
fplot(t, subs(u2(t), lhs(vrednosti), rhs(vrednosti)), [0 5e-6] )
grid on
xlabel('t')
ylabel('u2(t)')
```

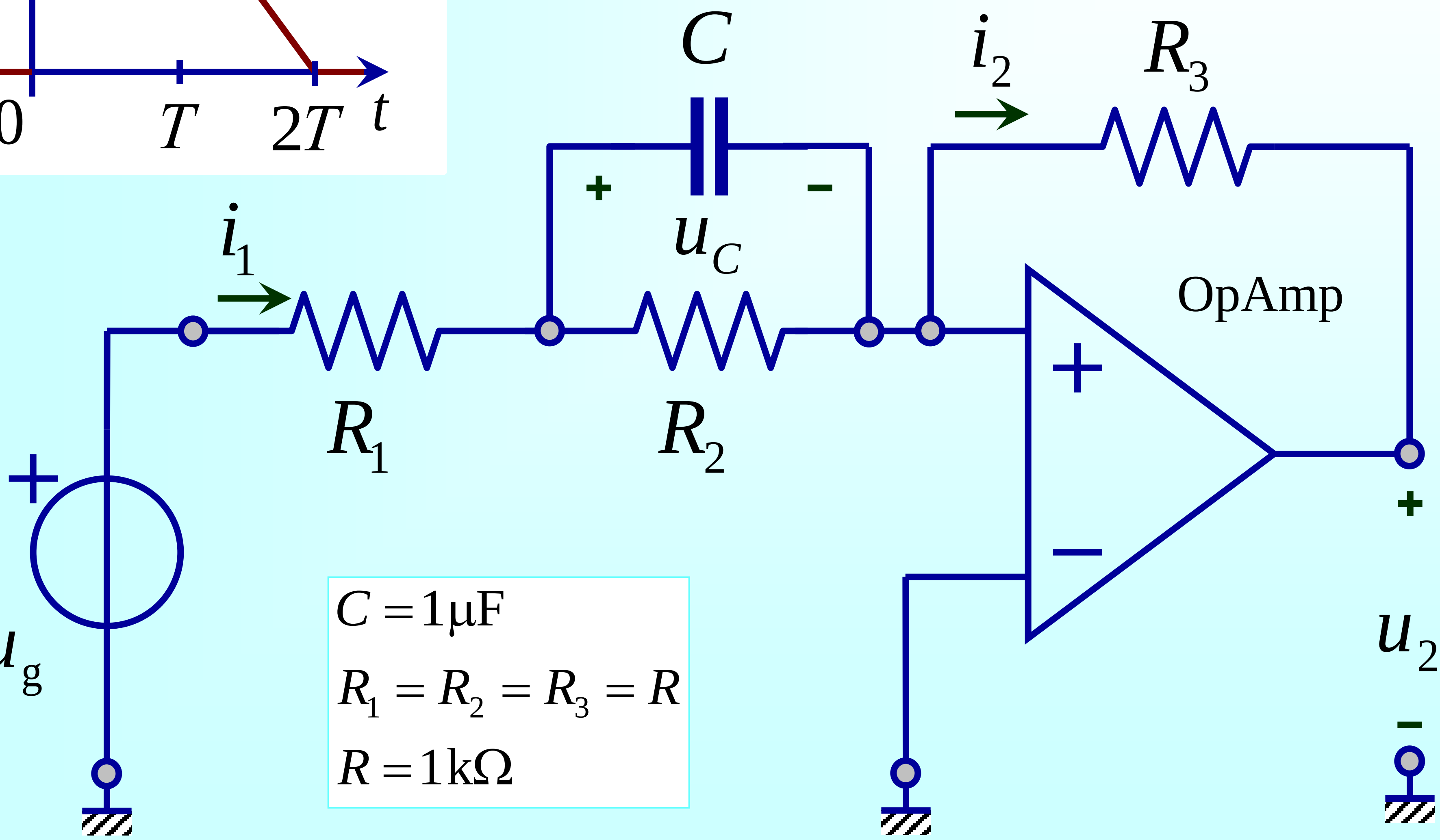


MATLAB: Symbolic Math Toolbox

Одзив на сложену побуду

$U = 10\,\mathrm{V}, T = 1\,\mathrm{ms}$





$C = 1\,\mu\mathrm{F}$

$R_1 = R_2 = R_3 = R$

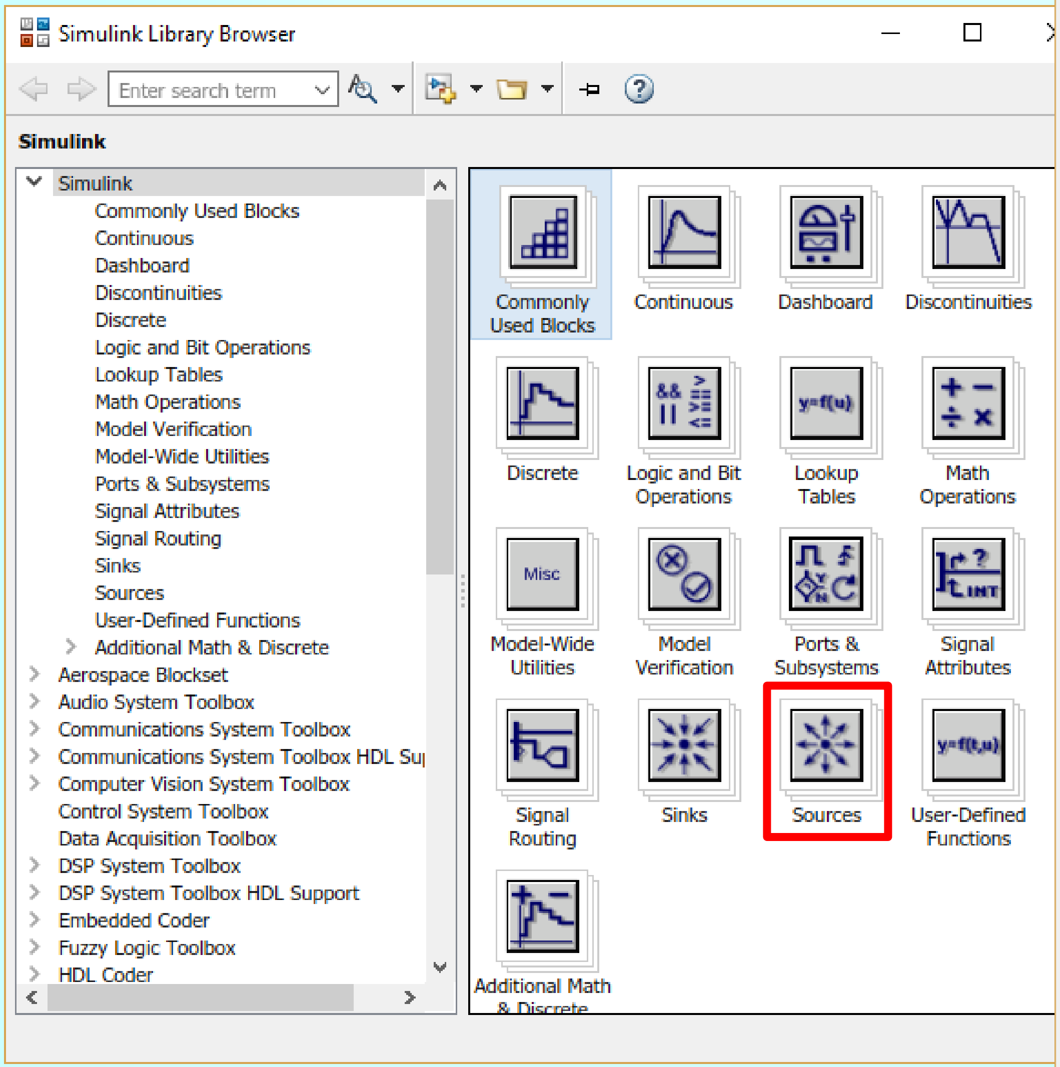
$R = 1\,\mathrm{k\Omega}$

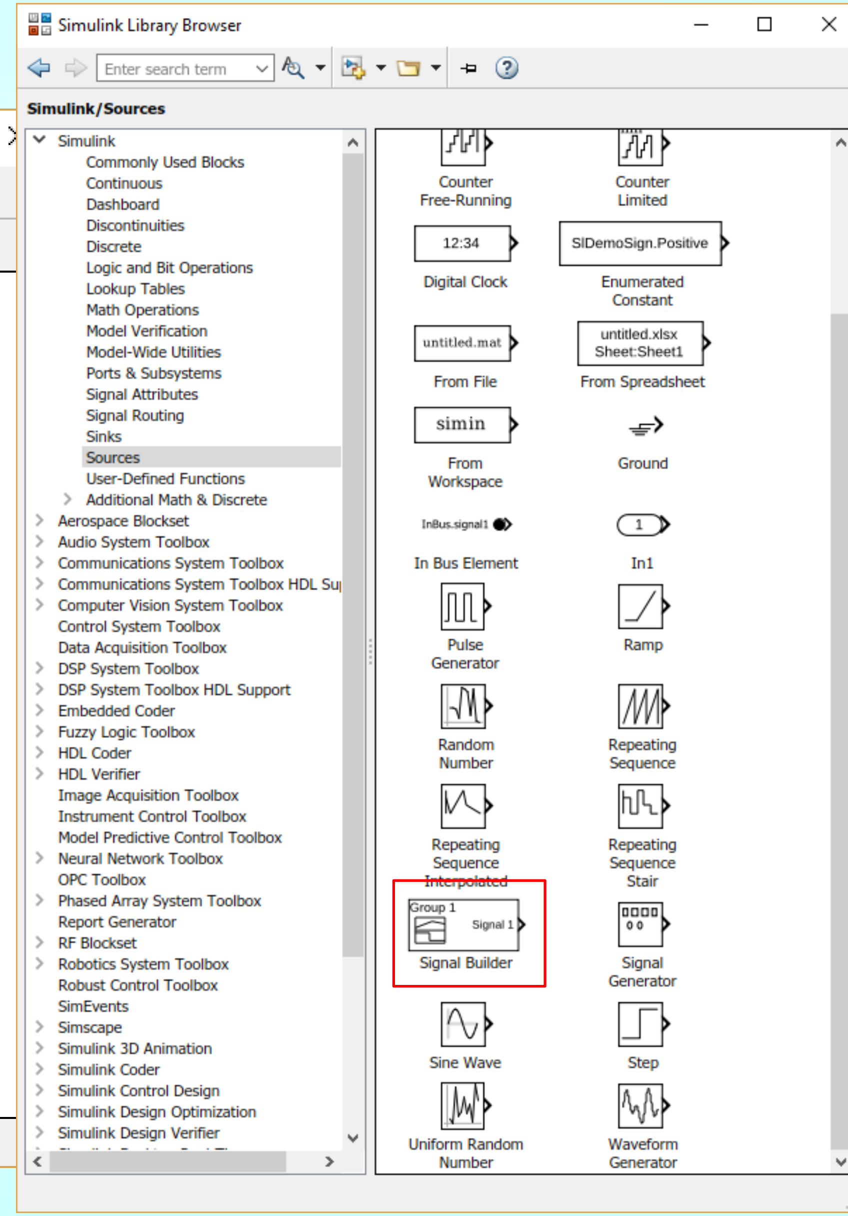
Задатак - сложена побуда

- Параметри кола са слике су познати: $R_1 = R_2 = R_3 = R$. Нема сакупљене енергије.
- Применом конволуционог интеграла одредити напон кондензатора $u_C(t)$, а затим и излазни напон $u_2(t)$ и нацртати његов график.
- Напон генератора се мења са временом као на дијаграму, при чему је $T = RC$.

MATLAB: Simulink

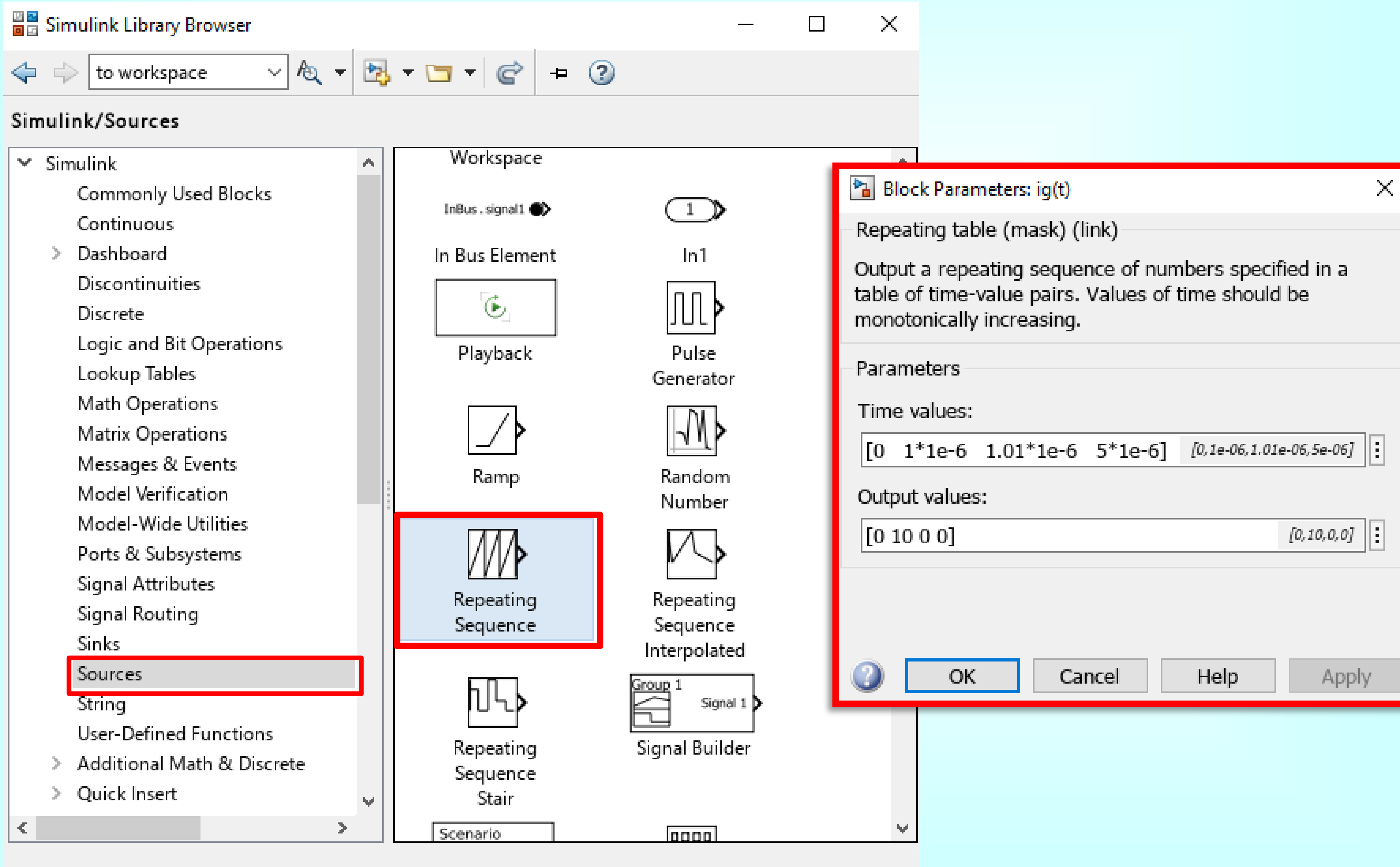
Sources

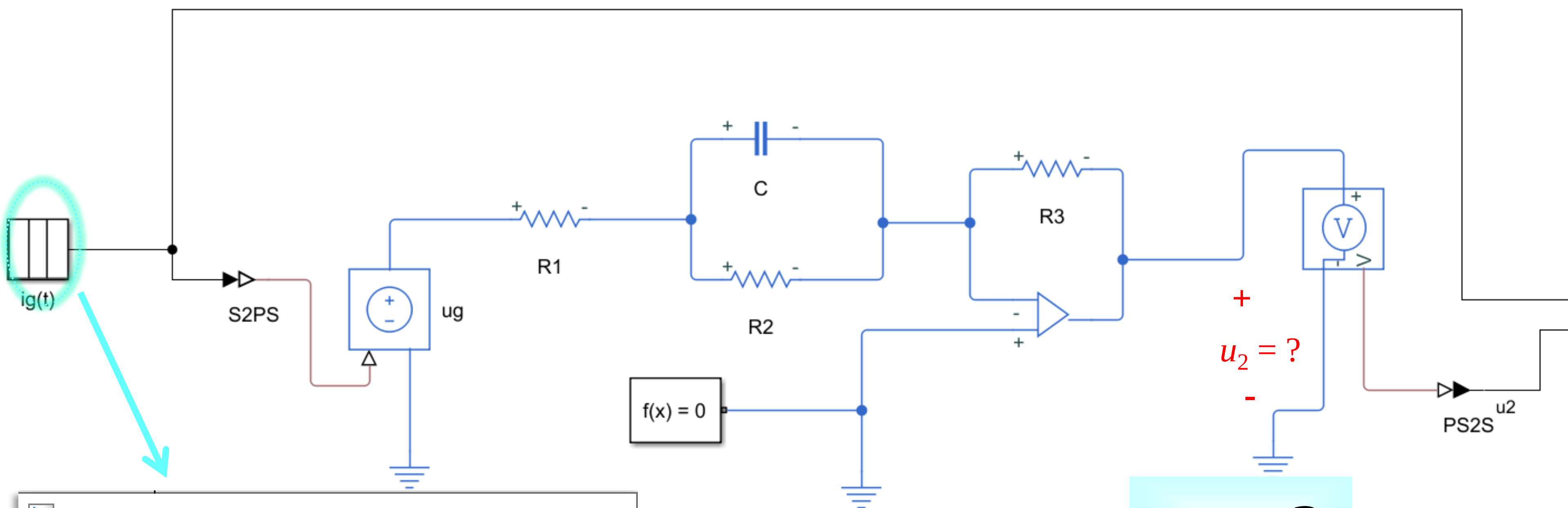


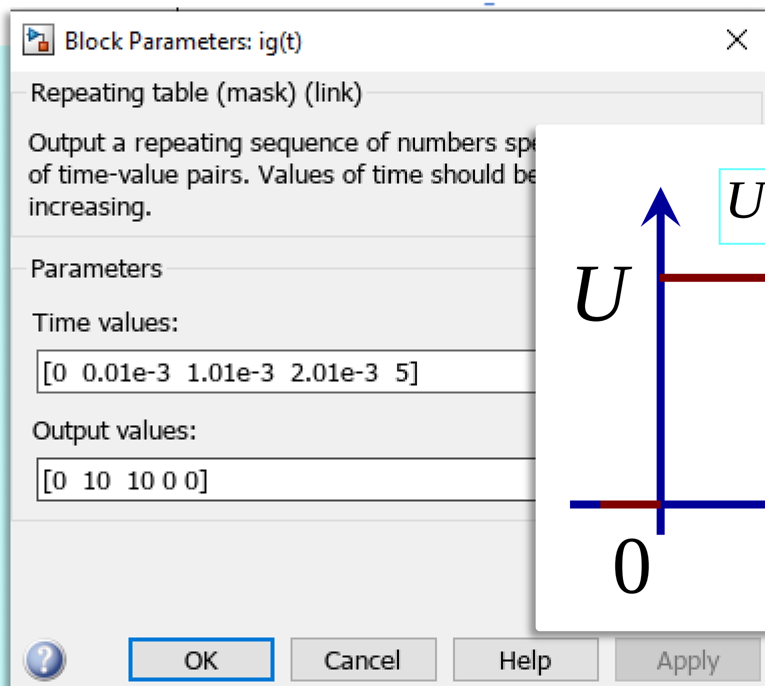


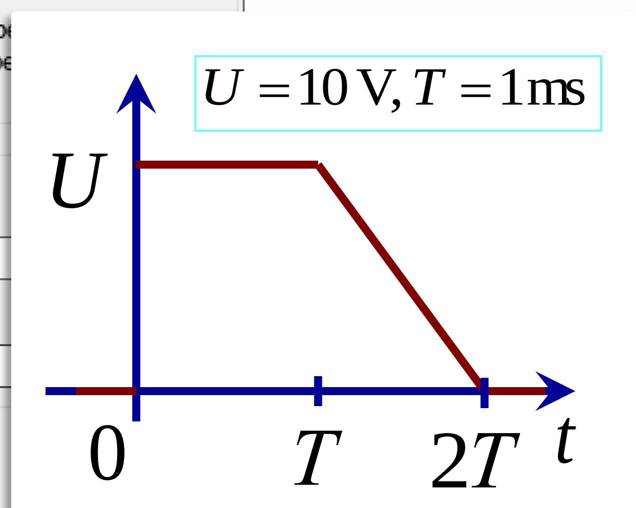
MATLAB: Simulink

Sources





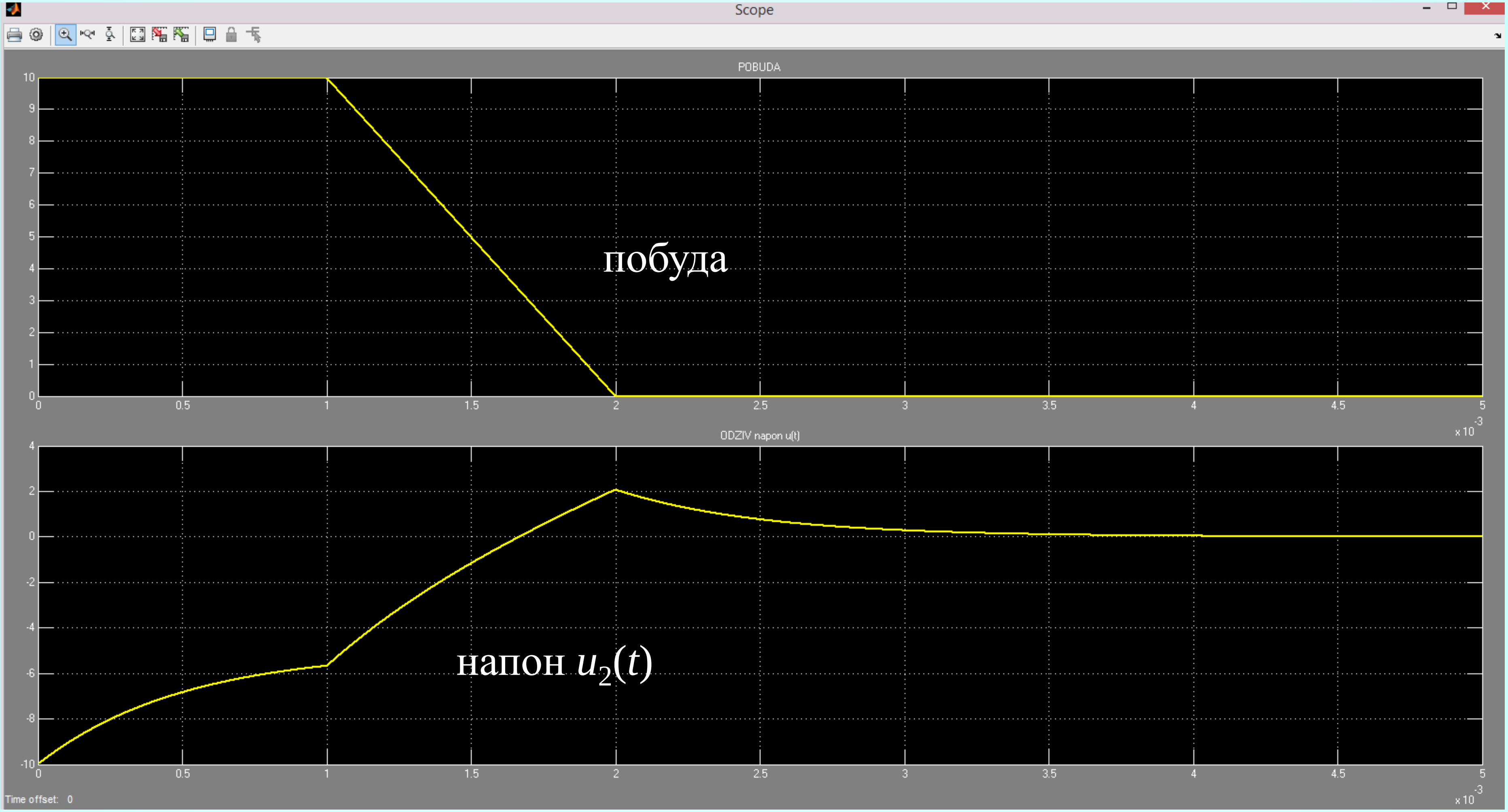




$$u_2 = ?$$

$$R_1 = R_2 = R_3 = 1\,\mathrm{k\Omega}$$
$$C_1 = 1\,\mu\mathrm{F}$$

MATLAB: Simscape
Foundation Library
Utilities
Simulink


Scope
POBUDA
побуда
ODZIV napon u(t)
напон $u_2(t)$
Time offset: 0
x 10^-3

Једначине

```
clear variables
syms C R R1 R2 R3 T U t
assume(0 < R1 & 0 < R2 & 0 < R3 & 0 < R & 0 < C & 0 < U & 0 < T)

syms DuC i1 i2 iC iR2 u uC uR1 uR2 ug
jednacine = [ug == uR1 + uC,...
    uR1 == R1 * i1,...
    iC == C * DuC,...
    uR2 == R2 * iR2,...
    uR2 == uC,...
    i1 == iR2 + iC,...
    i2 == i1,...
    u == -(R3 * i2)]
```

jednacine = $\left(\mathrm{ug} = \mathrm{uC} + \mathrm{uR}_1 \quad \mathrm{uR}_1 = R_1\, i_1 \quad \mathrm{iC} = C\ \mathrm{DuC} \quad \mathrm{uR}_2 = R_2\, \mathrm{iR}_2 \quad \mathrm{uR}_2 = \mathrm{uC} \quad i_1 = \mathrm{iC} + \mathrm{iR}_2 \quad i_2 = i_1 \quad u = -R_3\, i_2\right)$

```
jednacineIzvoda = eliminate(jednacine, [i1, iC, uR1, iR2, uR2, i2, u])
```

jednacineIzvoda = $\left[R_1\,\mathrm{uC} + R_2\,\mathrm{uC} - R_2\,\mathrm{ug} + C\ \mathrm{DuC}\ R_1\, R_2\right]$

```
jednacineStanje = solve(jednacineIzvoda, DuC)
```

jednacineStanje =

$$-\frac{R_1\,\mathrm{uC} + R_2\,\mathrm{uC} - R_2\,\mathrm{ug}}{C\, R_1\, R_2}$$

Решавање диференцијалне једначине

Rešavanje diferencijalne jednačine

```
syms uC(t)
PromenljiveStanjaIzvodi = [uC == uC(t), DuC == diff(uC(t), t)]
```

PromenljiveStanjaIzvodi(t) =

$$\left(\mathrm{uC}(t) = \mathrm{uC}(t) \quad \mathrm{DuC} = \frac{\partial}{\partial t}\,\mathrm{uC}(t)\right)$$

```
zamene = [R1 == R, R2 == R, R3 == R, T == R * C]
```

zamene = $\left(R_1 = R \quad R_2 = R \quad R_3 = R \quad T = C\,R\right)$

```
pobuda = ug == heaviside(t)
```

pobuda = $\mathrm{ug} = \mathrm{heaviside}(t)$

```
JS = subs(jednacineStanje, lhs([PromenljiveStanjaIzvodi pobuda  zamene]), rhs([PromenljiveStanjaIzvodi pobuda  zamene]))
```

JS =

$$\frac{R\,\mathrm{heaviside}(t) - 2\,R\,\mathrm{uC}(t)}{C\,R^2}$$

```
f_uC = dsolve(diff(uC(t)) == JS, uC(0)==0, 'IgnoreAnalyticConstraints', false)
```

f_uC =

$$\frac{\mathrm{e}^{-\frac{2t}{C\,R}}\,\mathrm{heaviside}(t)\left(\mathrm{e}^{\frac{2t}{C\,R}} - 1\right)}{2}$$

Гринова функција и побуда

Grinova funkcija

```
g_uC(t) = simplify(diff(f_uC,t))
```

g_uC(t) =

$$\frac{e^{-\frac{2\,t}{C\,R}}\,\mathrm{heaviside}(t)}{C\,R}$$

Generatori

```
syms U
ug_1(t) = U;
ug_2(t) = -U*(t-2*T)/T;
ug_3(t) = sym(0);
ug_tot(t) = piecewise((0<t<=T),ug_1(t), (T<t)&(t<=2*T),ug_2(t), (2*T<t),ug_3(t) )
```

ug_tot(t) =

$$\begin{cases} U & \text{if } t \leq T \wedge 0 < t \\ \frac{U\,(2\,T - t)}{T} & \text{if } T < t \wedge t \leq 2\,T \\ 0 & \text{if } 2\,T < t \end{cases}$$

Конволуциони интеграл (1)

Konvolucioni integral

```
syms tau
syms uC_1(t)
assume((0 < t)&(t<=T));
uC_1(t) = int(ug_1(tau) * g_uC(t-tau), tau, 0, t)
```

uC_1(t) =

$$-\frac{U\left(e^{-\frac{2t}{CR}}-1\right)}{2}$$

```
syms i2_1(t) i1_1(t)
i1_1(t) = subs(1/R * (ug_1(t) - uC_1(t)), lhs(zamene), rhs(zamene));
i2_1(t) = i1_1(t);
u2_1(t) = simplify( -(R * i2_1(t)) )
```

u2_1(t) =

$$-\frac{U}{2}-\frac{U\,e^{-\frac{2t}{CR}}}{2}$$

Конволуциони интеграл (2)

```
syms uC_2(t)
assume((0 < T)&(T < t) & (t <= 2*T));
uC_2(t) = int(ug_1(tau) * g_uC(t - tau), tau, sym(0), T) + int(ug_2(tau) * g_uC(t - tau), tau, T, t)
```

uC_2(t) =

$$\frac{U\,e^{-\frac{2t}{CR}}\left(e^{\frac{2T}{CR}}-1\right)}{2}-\frac{U\left(2\,e^{\frac{2T-2t}{CR}}-4\right)}{4}-\frac{U\left(2t-CR+CR\,e^{\frac{2T-2t}{CR}}\right)}{4T}$$

```
simplify(subs(uC_2(t), lhs(zamene), rhs(zamene)))
```

ans =

$$-\frac{2Ut-5CRU+CRU\,e^{2-\frac{2t}{CR}}+2CRU\,e^{-\frac{2t}{CR}}}{4CR}$$

```
i1_2(t) = subs(1/R * (ug_2(t) - uC_2(t)), lhs(zamene), rhs(zamene));
i2_2(t) = i1_2(t);
u2_2(t) = simplify( -(R * i2_2(t)) )
```

u2_2(t) =

$$-\frac{3CRU-2Ut+CRU\,e^{2-\frac{2t}{CR}}+2CRU\,e^{-\frac{2t}{CR}}}{4CR}$$

Конволуциони интеграл (3)

```
syms uC_3(t)
assume((2*T < t) & (0 < T));
uC_3(t) = int(ug_1(tau) * g_uC(t - tau), tau, 0, T) + int(ug_2(tau) * g_uC(t - tau), tau, T, 2*T)
```

uC_3(t) =

$$\frac{U\,e^{-\frac{2t}{C R}}\left(e^{\frac{2T}{C R}}-1\right)}{2}-\frac{U\,e^{\frac{2T}{C R}}\,e^{-\frac{2t}{C R}}\left(2T+C R-C R\,e^{\frac{2T}{C R}}\right)}{4T}$$

```
i1_3(t) = subs(1/R * (ug_3(t) - uC_3(t)), lhs(zamene), rhs(zamene));
i2_3(t) = i1_3(t);
syms u2_3(t)
u2_3(t) = simplify( -(R * i2_3(t))  )
```

u2_3(t) =

$$\frac{U\,e^{-\frac{2t}{C R}}\left(e^{2}-1\right)}{2}-\frac{U\,e^{2}\,e^{-\frac{2t}{C R}}\left(3\,C R-C R\,e^{2}\right)}{4\,C R}$$

```
simplify(u2_3(t))
```

ans =

$$\frac{U\,e^{-\frac{2t}{C R}}\left(e^{2}-1\right)}{2}-\frac{U\,e^{2}\,e^{-\frac{2t}{C R}}\left(3\,C R-C R\,e^{2}\right)}{4\,C R}$$

Вредности

```
vrednosti = [R == 1e3, C == 1e-6, U == 10, T == 1e-3]
```

vrednosti =

$$\left(R = 1000 \quad C = \frac{1}{1000000} \quad U = 10 \quad T = \frac{1}{1000}\right)$$

```
u2(t) = simplify(subs(piecewise((0 < t) <= T, u2_1(t),...
                      (T < t)&(t<=2*T), u2_2(t),...
                       (2*T < t), u2_3(t)), lhs(vrednosti), rhs(vrednosti)))
```

u2(t) =

$$\begin{cases} -\sigma_1 - 5 & \text{if } t \in \left(0, \frac{1}{1000}\right] \\ 5000\,t - \sigma_1 - \frac{5\,e^{2-2000\,t}}{2} - \frac{15}{2} & \text{if } t \in \left(\frac{1}{1000}, \frac{1}{500}\right] \\ 5\,e^{-2000\,t}\,(e^2 - 1) + 2500\,e^{-2000\,t}\,e^2\left(\frac{e^2}{1000} - \frac{3}{1000}\right) & \text{if } \frac{1}{500} < t \end{cases}$$

where

$$\sigma_1 = 5\,e^{-2000\,t}$$

```
ug(t) = subs(ug_tot(t), lhs(vrednosti),rhs(vrednosti))
```

ug(t) =

$$\begin{cases} 10 & \text{if } t \in \left(0, \frac{1}{1000}\right] \\ 20 - 10000\,t & \text{if } t \in \left(\frac{1}{1000}, \frac{1}{500}\right] \\ 0 & \text{if } \frac{1}{500} < t \end{cases}$$

Цртање графика одзива и побуде

Crtanje grafika

```
figure
fplot( t, u2(t), [0 5e-3])
hold on
fplot( t, ug(t), [0 5e-3])
hold off
grid on
xlabel('t')
ylabel('u2(t)')
legend({'u2(t)','ug(t)'})
```