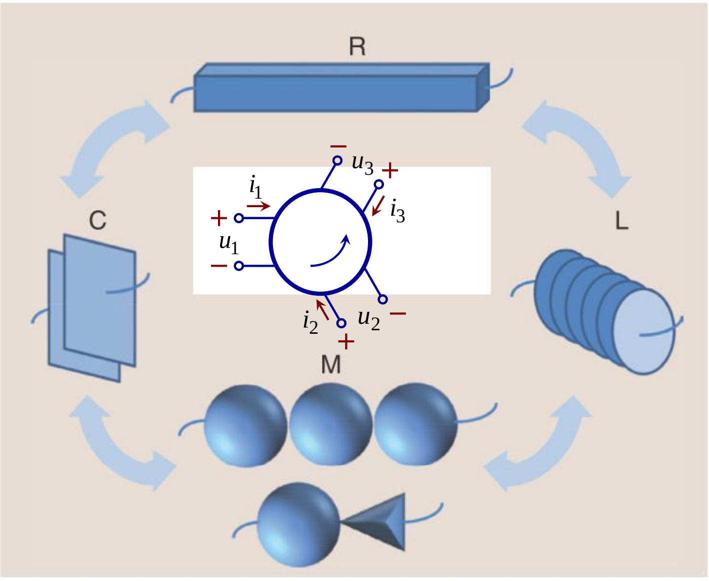
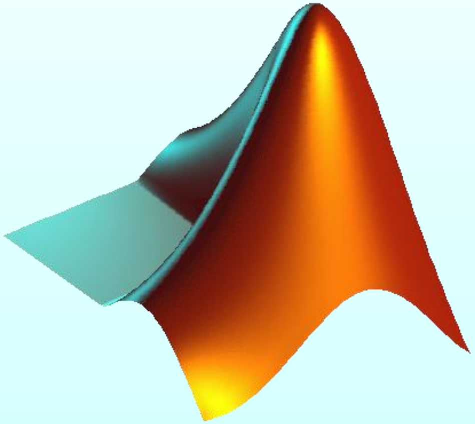
Практикум из рачунарске анализе кола

4. Временски домен. Трансформатори

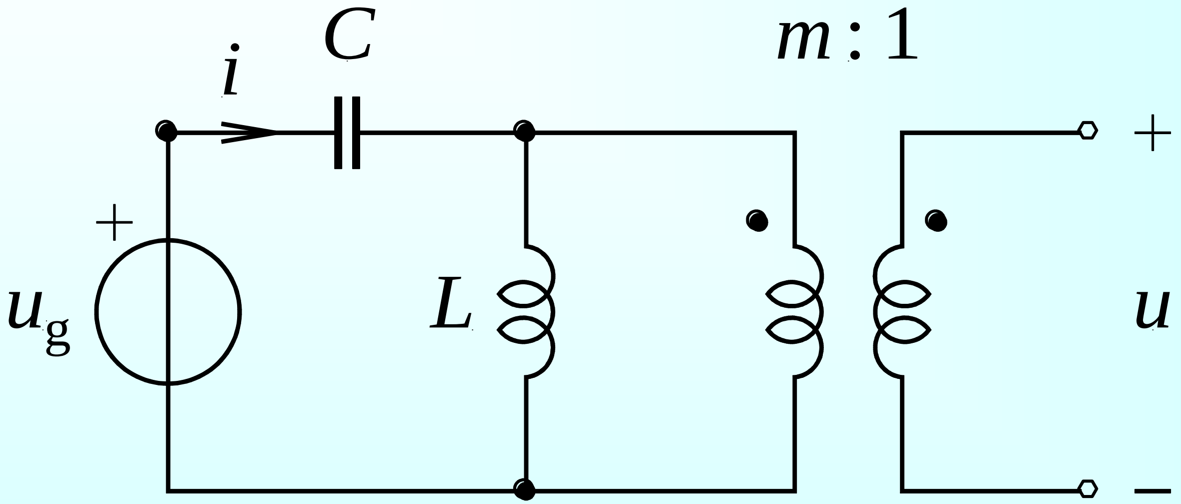




Милка Потребић Иваниш

Никола Баста

Одзив на простопериодичну побуду



$U_m = 1\text{V}$

$\omega = 10^5\ \text{rad/s}$

$L = 0.1\text{mH}$

$C = 1\mu\text{F}$

$m = 1$

$$u_g(t) = U_m \sin(\frac{1}{\sqrt{CL}} t)\, h(t)$$

Задатак - простопериодична побуда

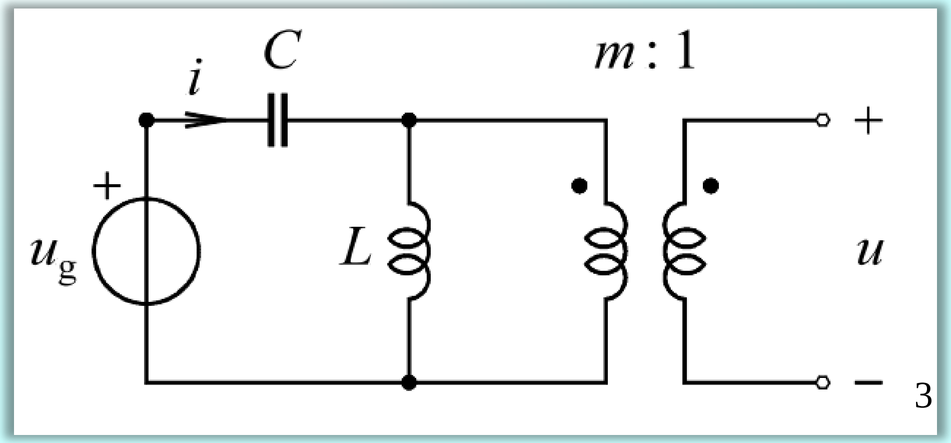
(5) Одредити једначину одзива за напон u електричног кола са слике. Вредности елемената и параметри побуде су познати,

$$u_{\mathrm{g}}(t) = U_{\mathrm{m}} \sin(\frac{1}{\sqrt{CL}} t)\,\mathrm{h}(t)\,.$$

Нема сакупљене енергије.

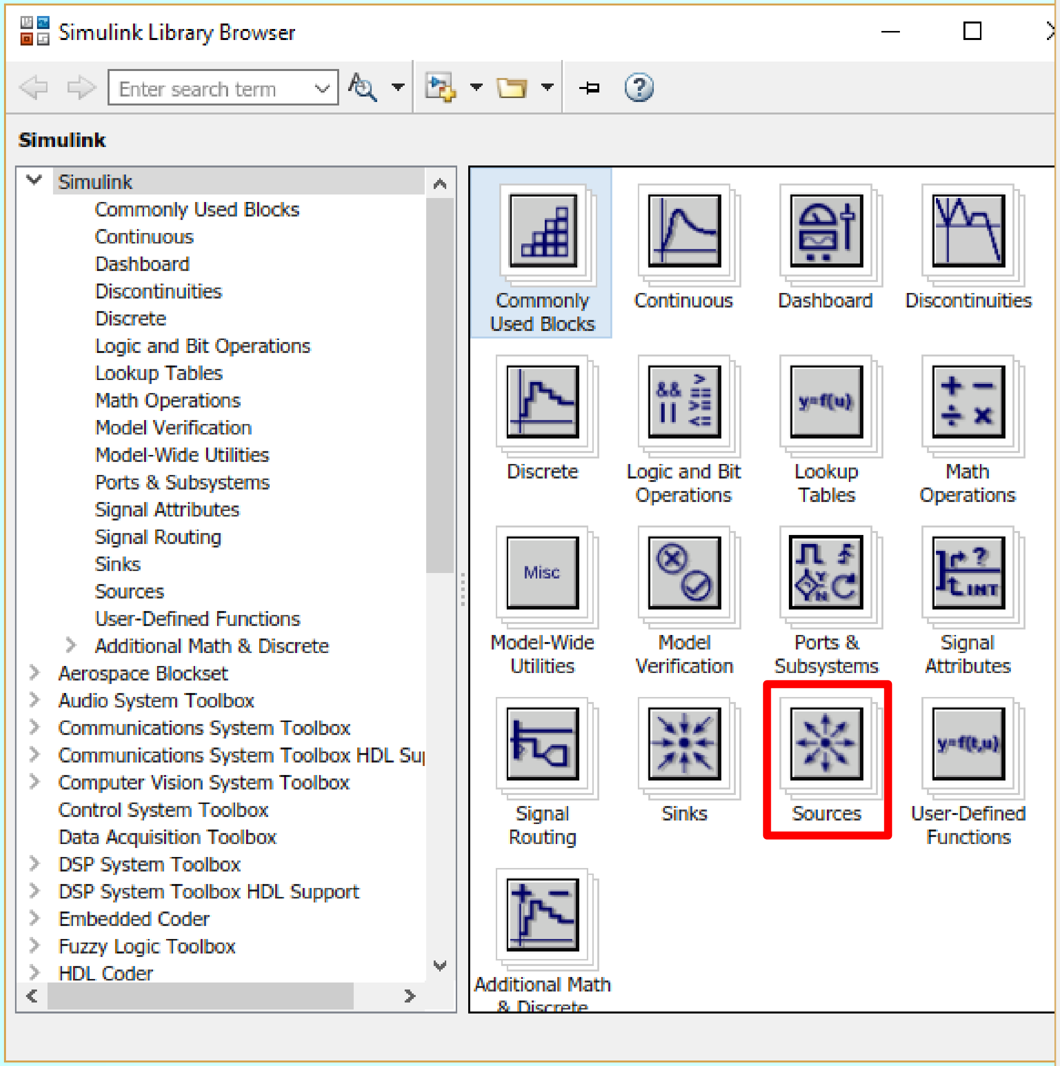
(5) Одредити струју извора, i, и нацртати график струје извора.

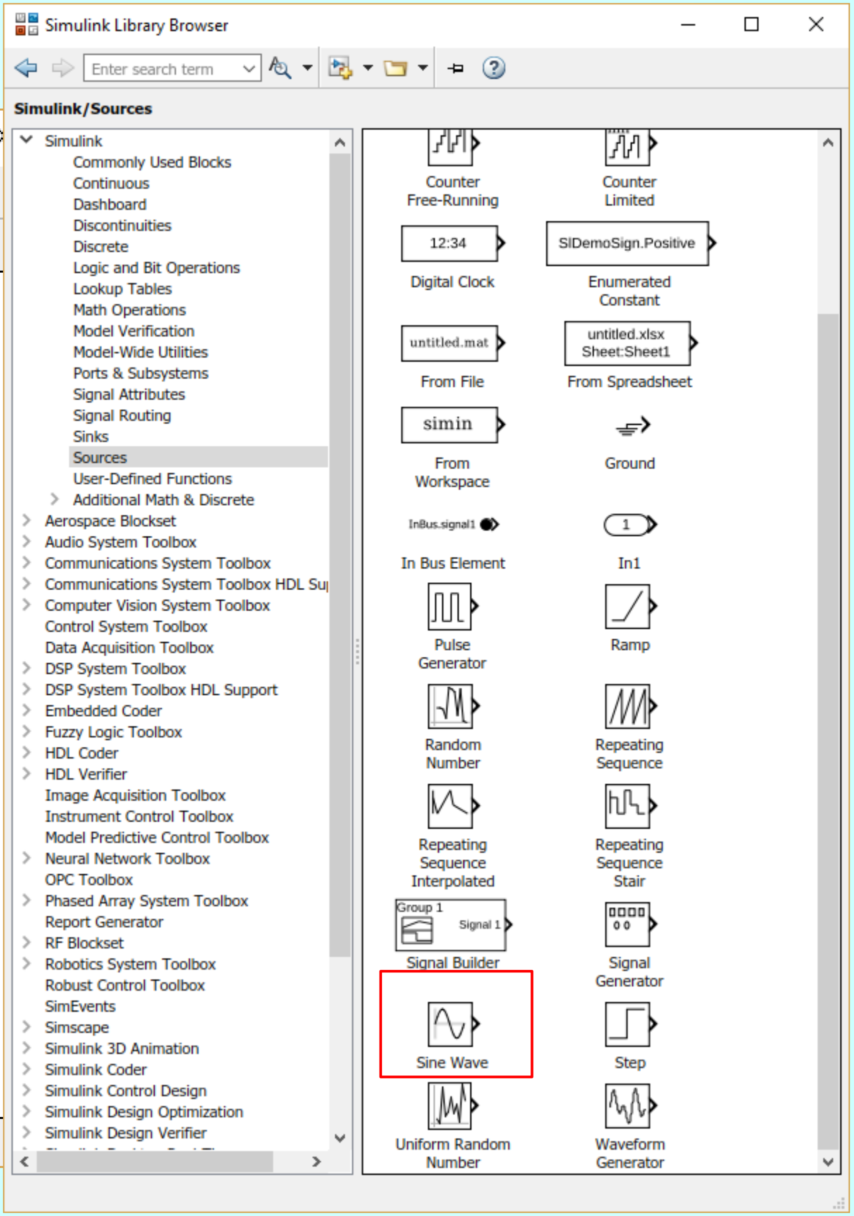
(5) Како гласе једначине стања кола? Написати их у матричном облику.

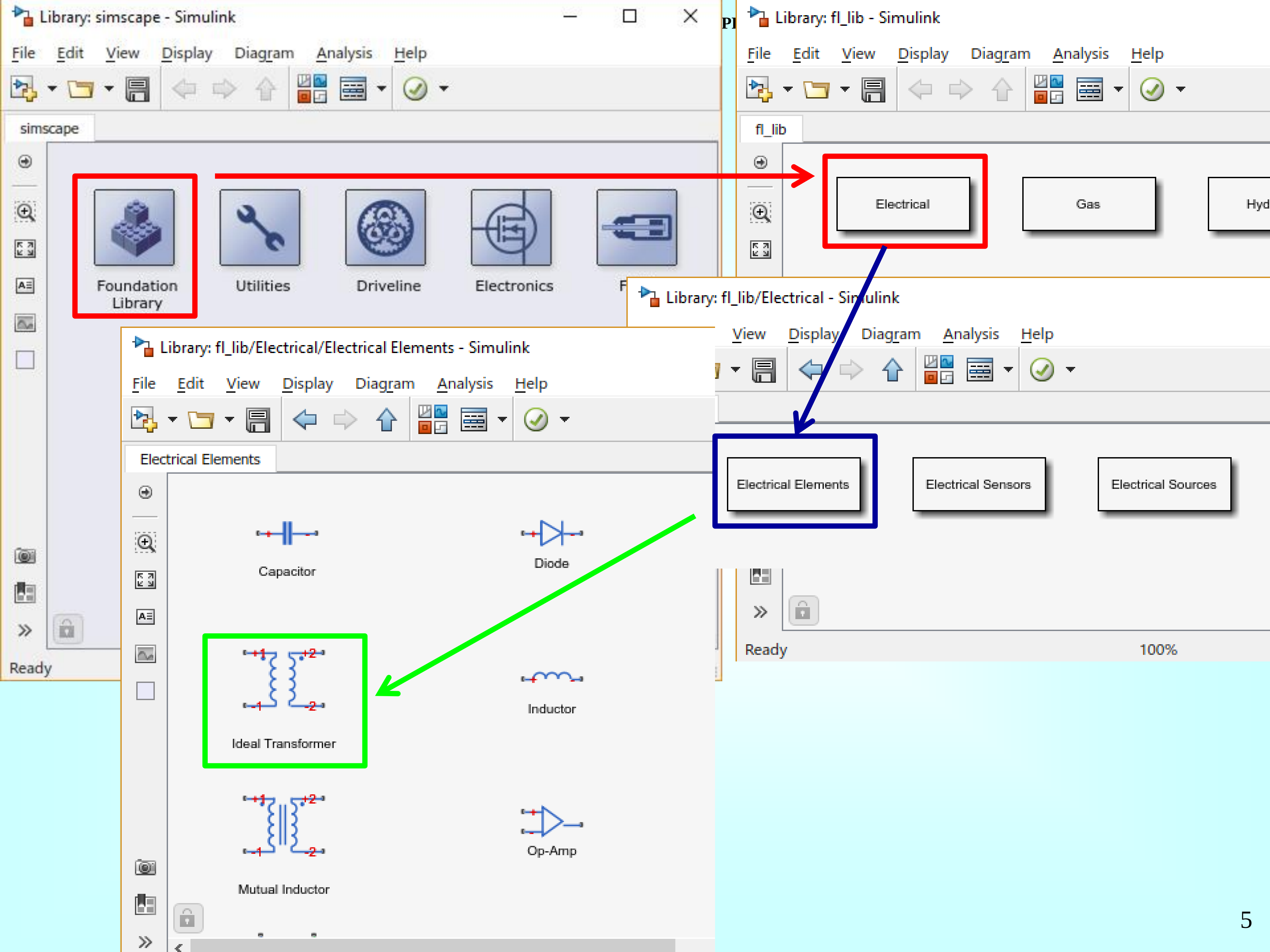


MATLAB: Simulink

Sources



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File Edit View Display Diagram Analysis Help
simscape
Foundation Library
Utilities
Driveline
Electronics
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Library: fl_lib - Simulink
File Edit View Display Diagram Analysis Help
fl_lib
Electrical
Gas
Hyd
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Electrical Elements
Electrical Sensors
Electrical Sources
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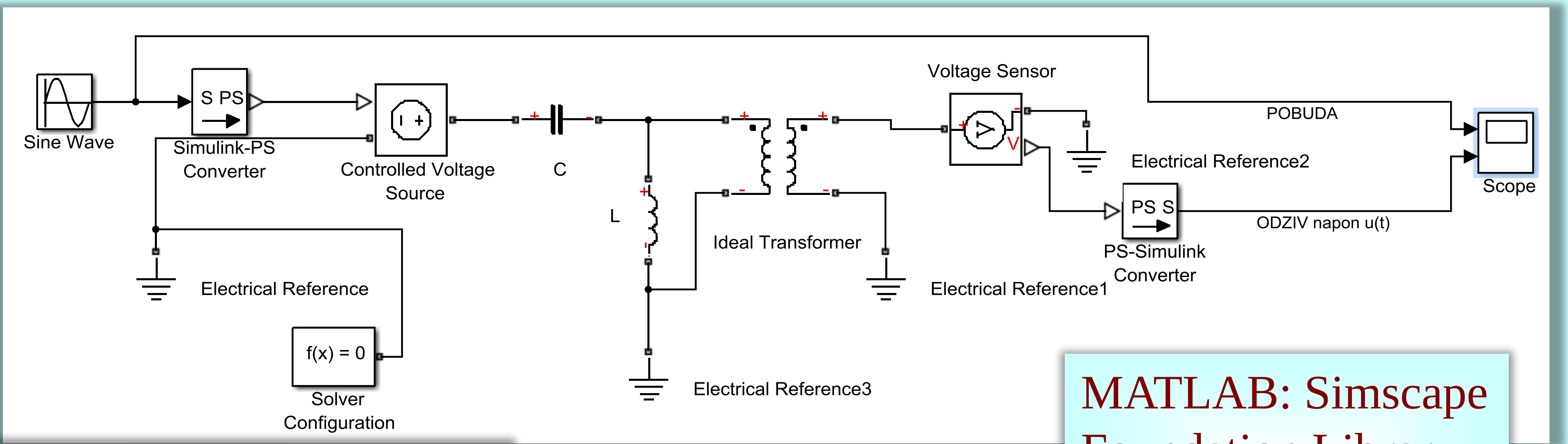
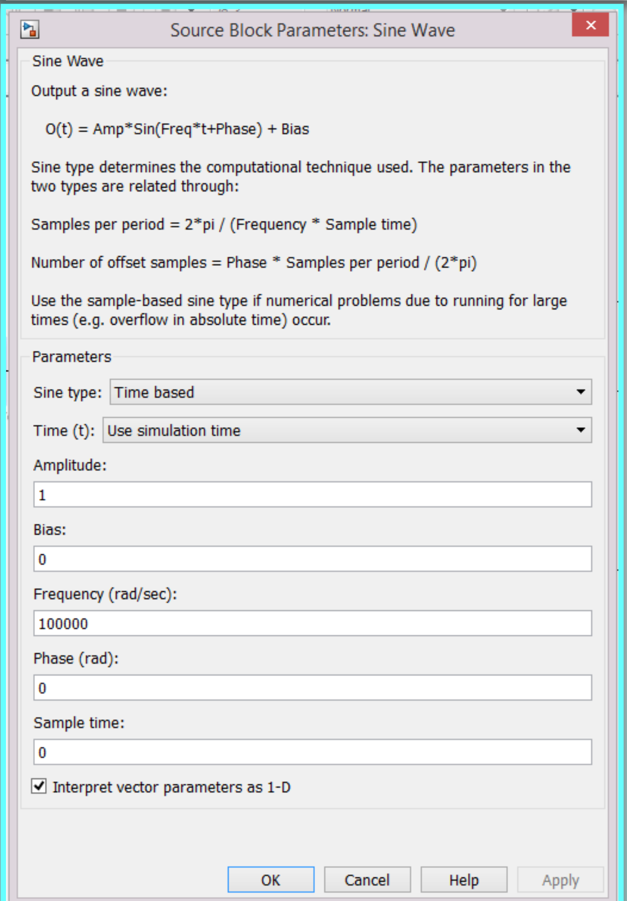
Electrical Elements
Capacitor
Diode
Ideal Transformer
Inductor
Mutual Inductor
Op-Amp



MATLAB: Simscape Foundation Library Utilities Simulink



Sine Wave

Output a sine wave:

O(t) = Amp*Sin(Freq*t+Phase) + Bias

Sine type determines the computational technique used. The parameters in the two types are related through:

Samples per period = 2*pi / (Frequency * Sample time)

Number of offset samples = Phase * Samples per period / (2*pi)

Use the sample-based sine type if numerical problems due to running for large times (e.g. overflow in absolute time) occur.

Parameters

Sine type: Time based

Time (t): Use simulation time

Amplitude: 1

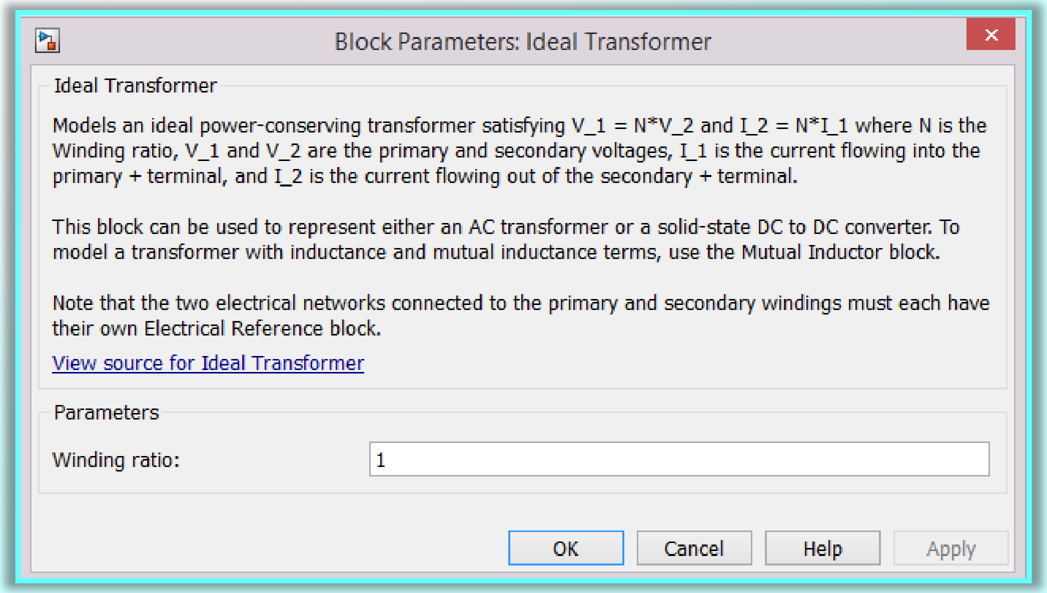
Bias: 0

Frequency (rad/sec): 100000

Phase (rad): 0

Sample time: 0

☑ Interpret vector parameters as 1-D



Ideal Transformer

Models an ideal power-conserving transformer satisfying V_1 = N*V_2 and I_2 = N*I_1 where N is the Winding ratio, V_1 and V_2 are the primary and secondary voltages, I_1 is the current flowing into the primary + terminal, and I_2 is the current flowing out of the secondary + terminal.

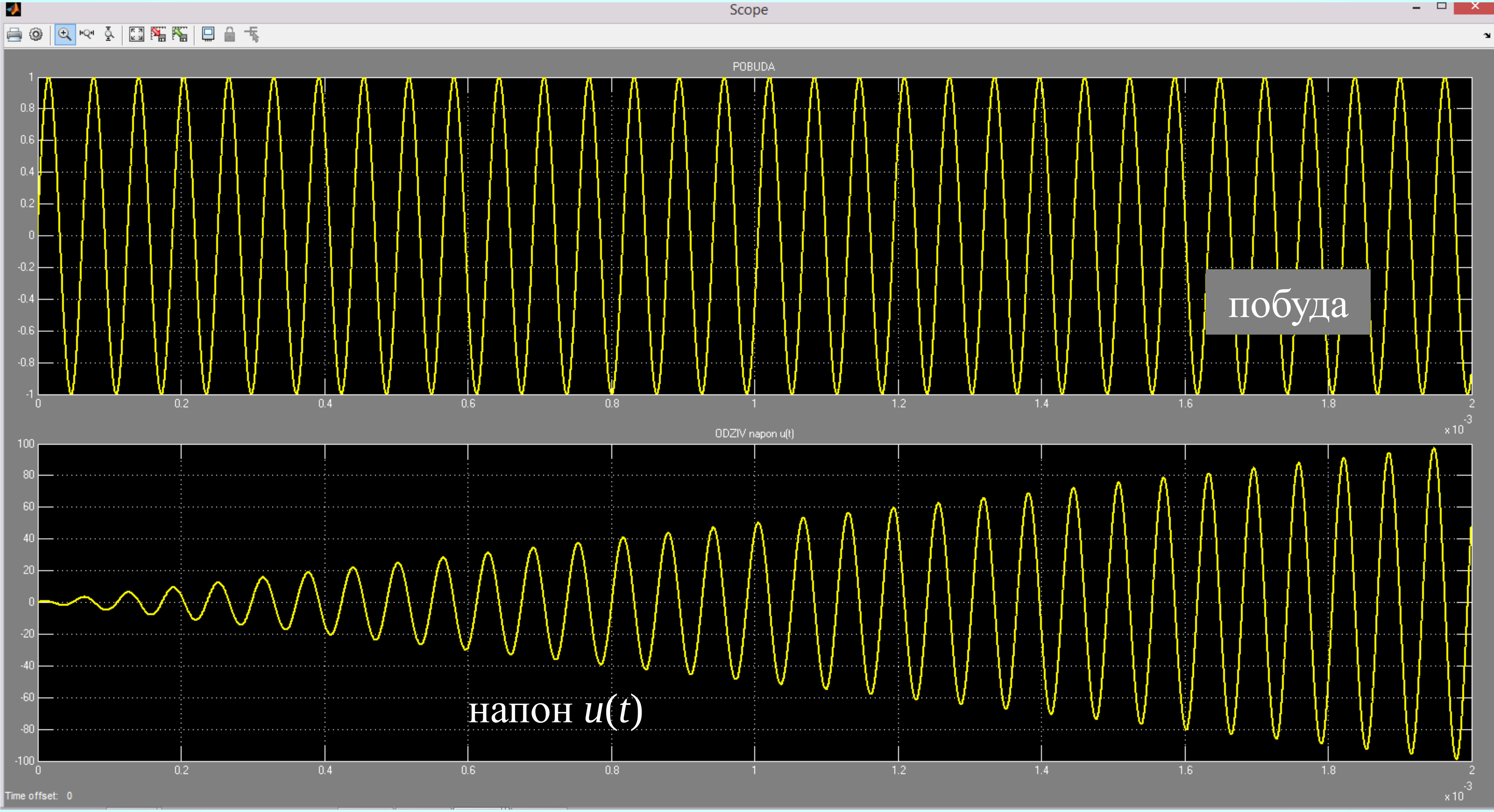
This block can be used to represent either an AC transformer or a solid-state DC to DC converter. To model a transformer with inductance and mutual inductance terms, use the Mutual Inductor block.

Note that the two electrical networks connected to the primary and secondary windings must each have their own Electrical Reference block.

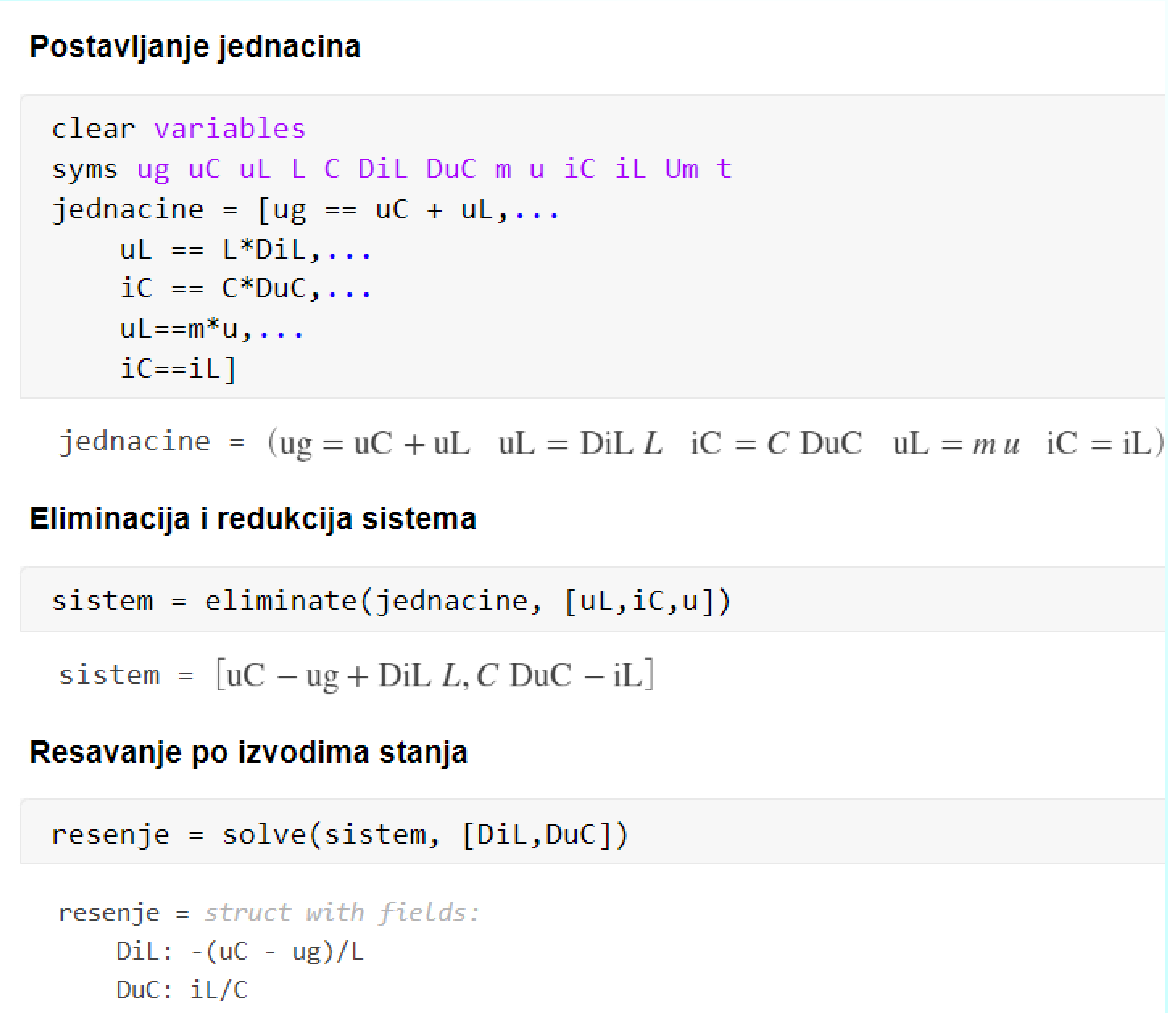
View source for Ideal Transformer

Parameters

Winding ratio: 1


Scope
POBUDA
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
x 10^-3
побуда
ODZIV napon u(t)
100
80
60
40
20
0
-20
-40
-60
-80
-100
напон u(t)
Time offset: 0

Једначине



Postavljanje jednacina

```
clear variables
syms ug uC uL L C DiL DuC m u iC iL Um t
jednacine = [ug == uC + uL,...
    uL == L*DiL,...
    iC == C*DuC,...
    uL==m*u,...
    iC==iL]
```

jednacine = $\left(\mathrm{ug} = \mathrm{uC} + \mathrm{uL} \quad \mathrm{uL} = \mathrm{DiL}\,L \quad \mathrm{iC} = C\,\mathrm{DuC} \quad \mathrm{uL} = m\,u \quad \mathrm{iC} = \mathrm{iL}\right)$

Eliminacija i redukcija sistema

```
sistem = eliminate(jednacine, [uL,iC,u])
```

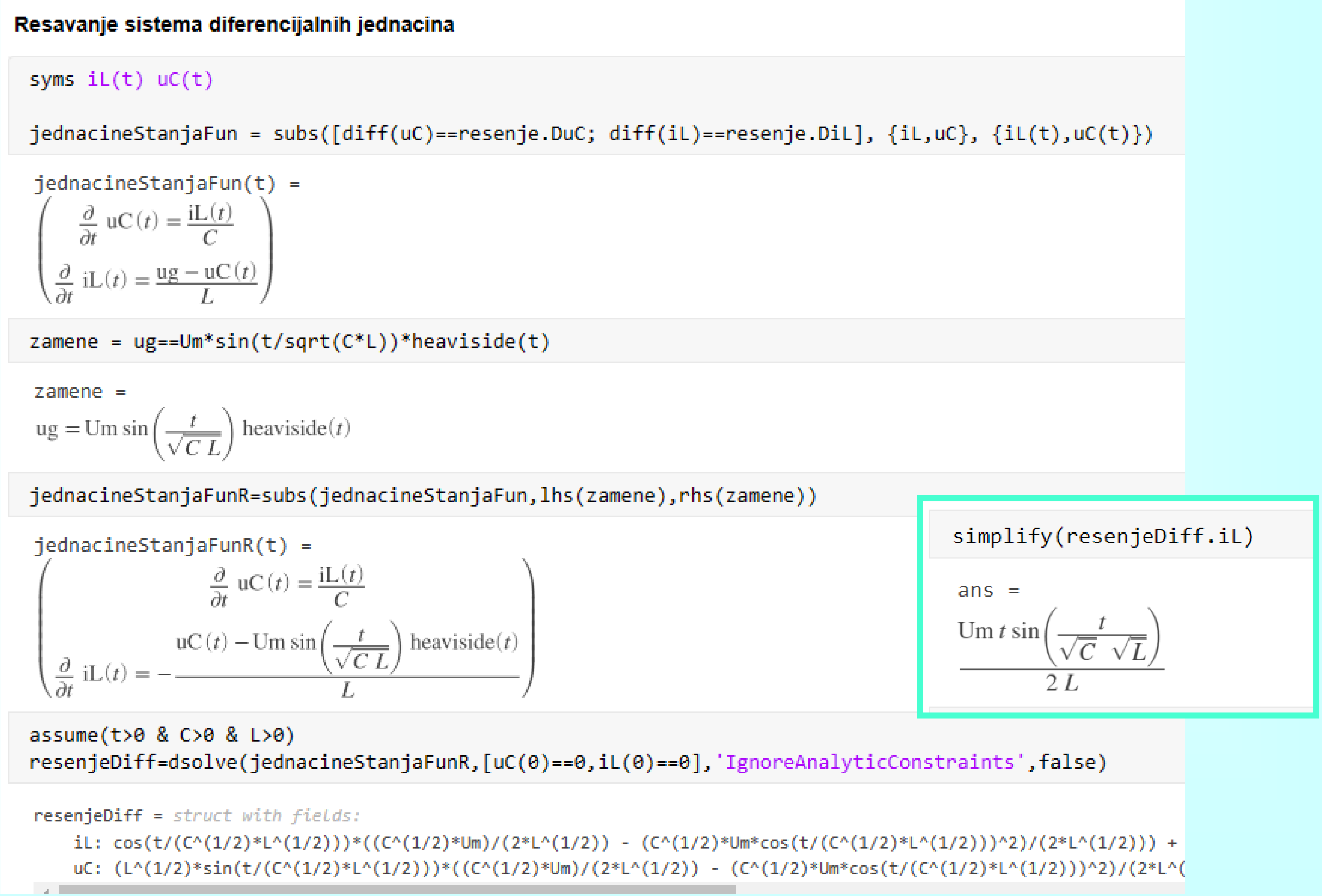
sistem = $\left[\mathrm{uC} - \mathrm{ug} + \mathrm{DiL}\,L, C\,\mathrm{DuC} - \mathrm{iL}\right]$

Resavanje po izvodima stanja

```
resenje = solve(sistem, [DiL,DuC])
```

```
resenje = struct with fields:
    DiL: -(uC - ug)/L
    DuC: iL/C
```

Решавање система



Resavanje sistema diferencijalnih jednacina

```
syms iL(t) uC(t)

jednacineStanjaFun = subs([diff(uC)==resenje.DuC; diff(iL)==resenje.DiL], {iL,uC}, {iL(t),uC(t)})
```

jednacineStanjaFun(t) =

$$\begin{pmatrix} \frac{\partial}{\partial t}\,\mathrm{uC}(t) = \frac{\mathrm{iL}(t)}{C} \\ \frac{\partial}{\partial t}\,\mathrm{iL}(t) = \frac{\mathrm{ug} - \mathrm{uC}(t)}{L} \end{pmatrix}$$

```
zamene = ug==Um*sin(t/sqrt(C*L))*heaviside(t)
```

zamene =

$$\mathrm{ug} = \mathrm{Um}\sin\left(\frac{t}{\sqrt{C\,L}}\right)\mathrm{heaviside}(t)$$

```
jednacineStanjaFunR=subs(jednacineStanjaFun,lhs(zamene),rhs(zamene))
```

jednacineStanjaFunR(t) =

$$\begin{pmatrix} \frac{\partial}{\partial t}\,\mathrm{uC}(t) = \frac{\mathrm{iL}(t)}{C} \\ \frac{\partial}{\partial t}\,\mathrm{iL}(t) = -\frac{\mathrm{uC}(t) - \mathrm{Um}\sin\left(\frac{t}{\sqrt{C\,L}}\right)\mathrm{heaviside}(t)}{L} \end{pmatrix}$$

```
assume(t>0 & C>0 & L>0)
resenjeDiff=dsolve(jednacineStanjaFunR,[uC(0)==0,iL(0)==0],'IgnoreAnalyticConstraints',false)
```

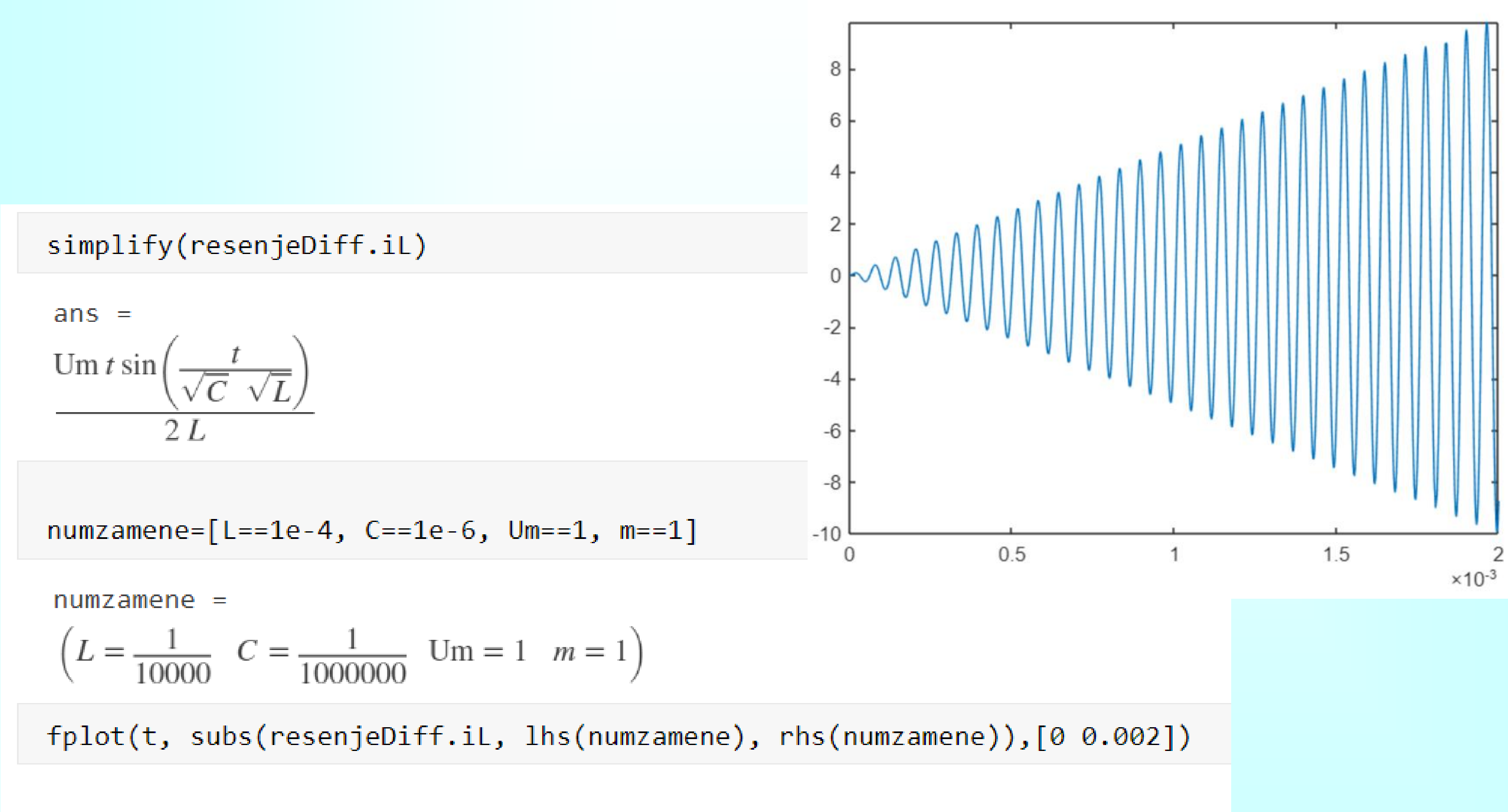
```
resenjeDiff = struct with fields:
    iL: cos(t/(C^(1/2)*L^(1/2)))*((C^(1/2)*Um)/(2*L^(1/2)) - (C^(1/2)*Um*cos(t/(C^(1/2)*L^(1/2)))^2)/(2*L^(1/2))) +
    uC: (L^(1/2)*sin(t/(C^(1/2)*L^(1/2)))*((C^(1/2)*Um)/(2*L^(1/2)) - (C^(1/2)*Um*cos(t/(C^(1/2)*L^(1/2)))^2)/(2*L^(
```

```
simplify(resenjeDiff.iL)
```

ans =

$$\frac{\mathrm{Um}\,t\sin\left(\frac{t}{\sqrt{C}\,\sqrt{L}}\right)}{2\,L}$$

График функције одзива



```
simplify(resenjeDiff.iL)
```

ans =

$$\frac{\mathrm{Um}\, t \sin\left(\frac{t}{\sqrt{C}\ \sqrt{L}}\right)}{2\, L}$$

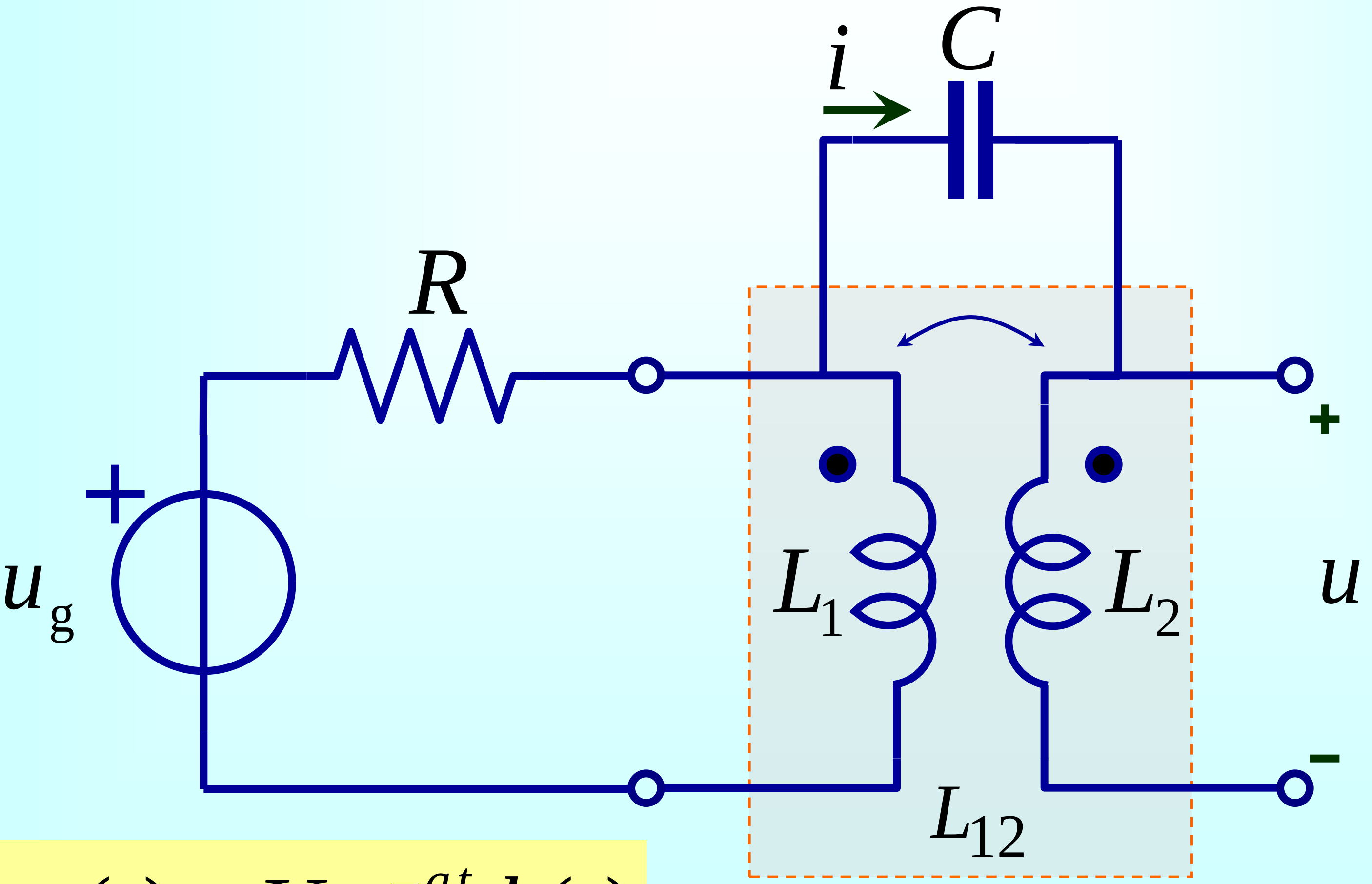
```
numzamene=[L==1e-4, C==1e-6, Um==1, m==1]
```

numzamene =

$$\left(L = \frac{1}{10000} \quad C = \frac{1}{1000000} \quad \mathrm{Um} = 1 \quad m = 1\right)$$

```
fplot(t, subs(resenjeDiff.iL, lhs(numzamene), rhs(numzamene)),[0 0.002])
```

Одзив на експоненцијалну побуду



$L = 1\text{mH}$

$C = 1\mu\text{F}$

$R = 1\text{k}\Omega$

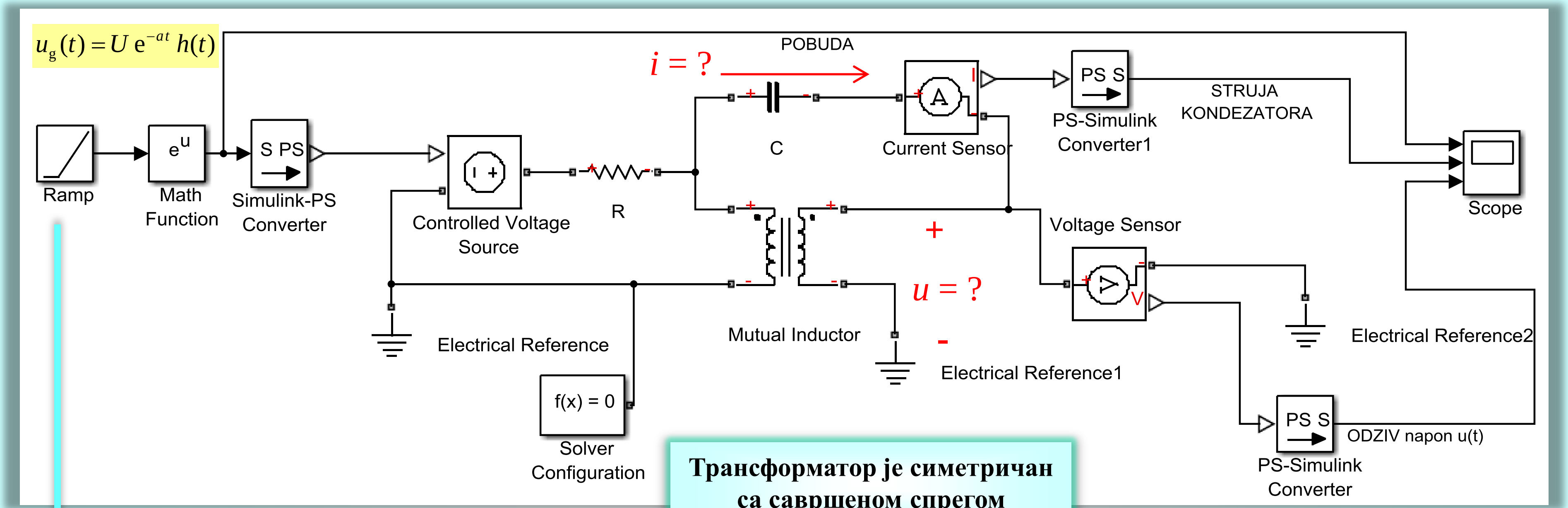
$$u_g(t) = U\,\mathrm{e}^{-at}\,h(t)$$

Трансформатор је симетричан са савршеном спрегом

$U = 1\text{V},\ a = 100$

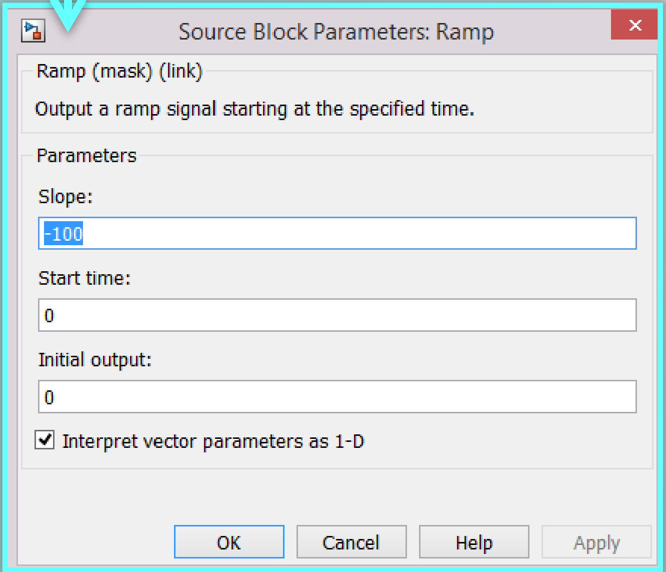
Задатак - експоненцијална побуда

- Параметри кола са слике су познати. Линеарни индуктивни трансформатор је симетричан са савршеном спрегом. Нема сакупљене енергије.
- Одредити струју кондензатора $i(t)$ и нацртати њен график.
- Одредити напон секундара $u(t)$ и нацртати његов график.



$$u_g(t) = U\,\mathrm{e}^{-at}\,h(t)$$

Трансформатор је симетричан са савршеном спрегом



Source Block Parameters: Ramp

Ramp (mask) (link)

Output a ramp signal starting at the specified time.

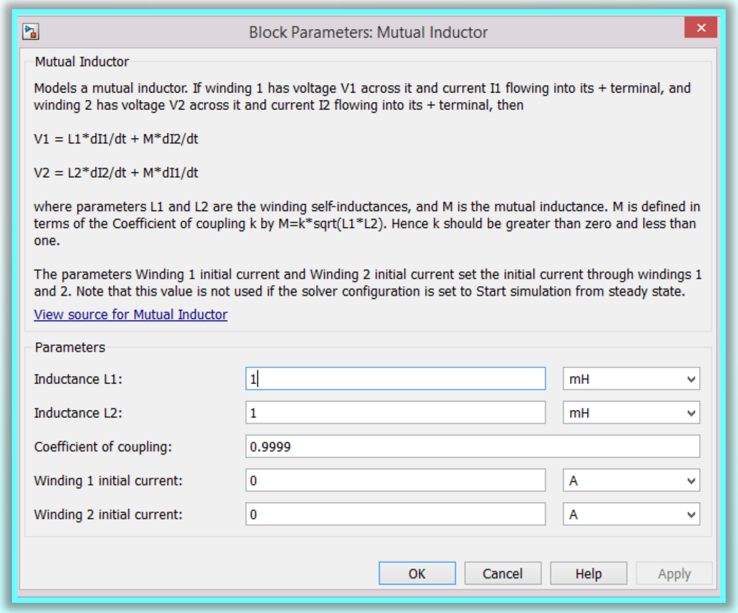
Parameters

Slope: -100

Start time: 0

Initial output: 0

☑ Interpret vector parameters as 1-D



Block Parameters: Mutual Inductor

Mutual Inductor

Models a mutual inductor. If winding 1 has voltage V1 across it and current I1 flowing into its + terminal, and winding 2 has voltage V2 across it and current I2 flowing into its + terminal, then

V1 = L1*dI1/dt + M*dI2/dt

V2 = L2*dI2/dt + M*dI1/dt

where parameters L1 and L2 are the winding self-inductances, and M is the mutual inductance. M is defined in terms of the Coefficient of coupling k by M=k*sqrt(L1*L2). Hence k should be greater than zero and less than one.

The parameters Winding 1 initial current and Winding 2 initial current set the initial current through windings 1 and 2. Note that this value is not used if the solver configuration is set to Start simulation from steady state.

View source for Mutual Inductor

Parameters

Inductance L1:	1	mH
Inductance L2:	1	mH
Coefficient of coupling:	0.9999	
Winding 1 initial current:	0	A
Winding 2 initial current:	0	A

$$L = 1\,\mathrm{mH}$$

$$C = 1\,\mu\mathrm{F}$$

$$R = 1\,\mathrm{k}\Omega$$

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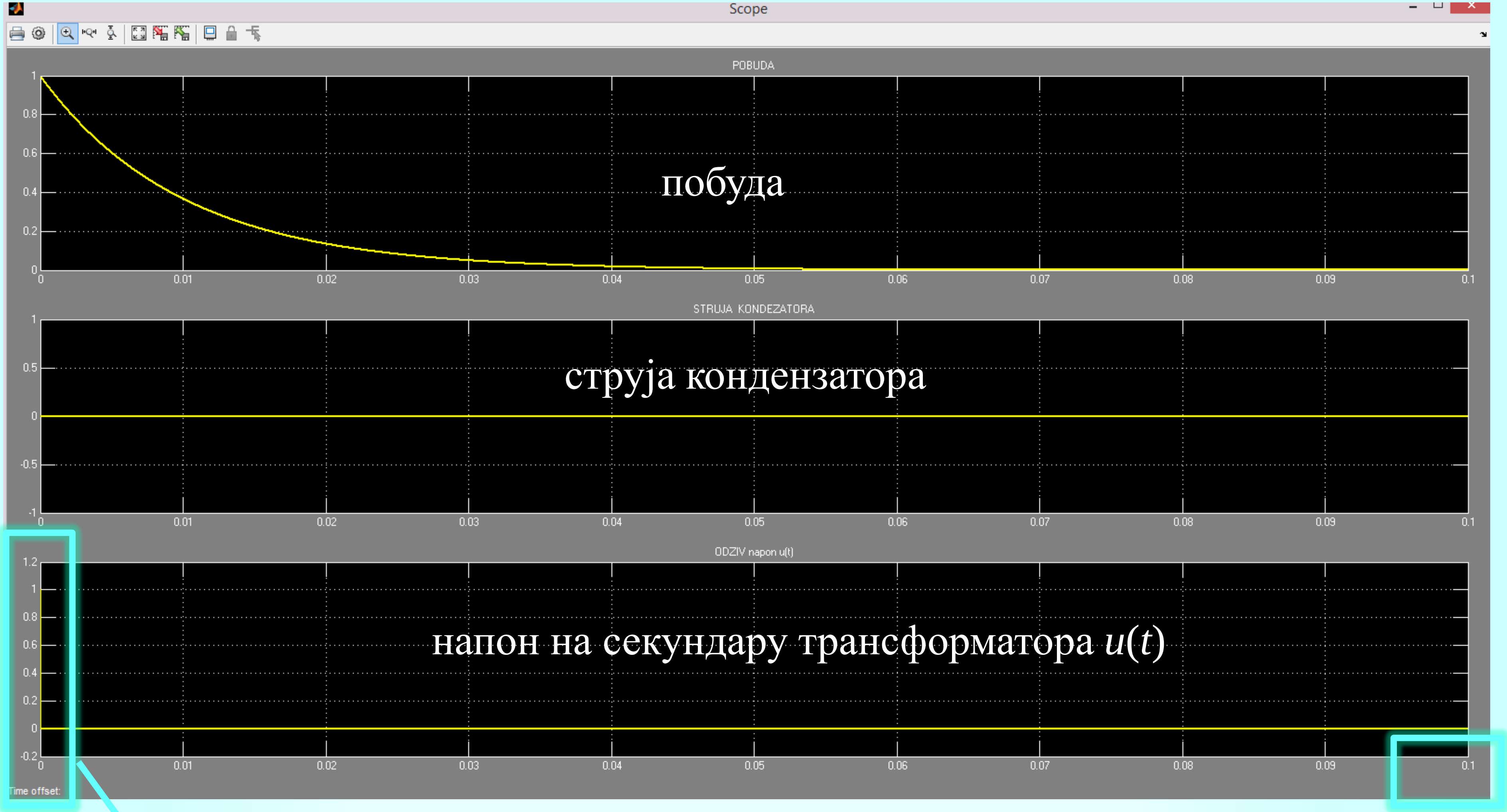
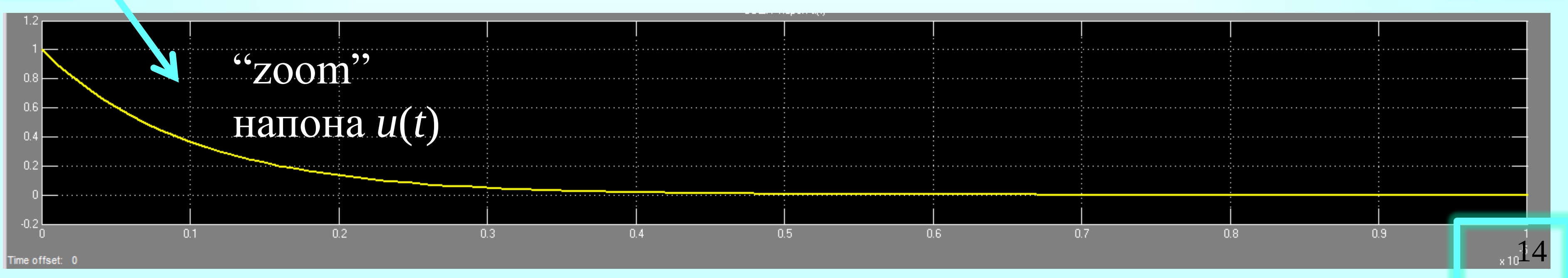
$$U = 1\,\mathrm{V},\ a = 100$$



побуда

струја кондензатора

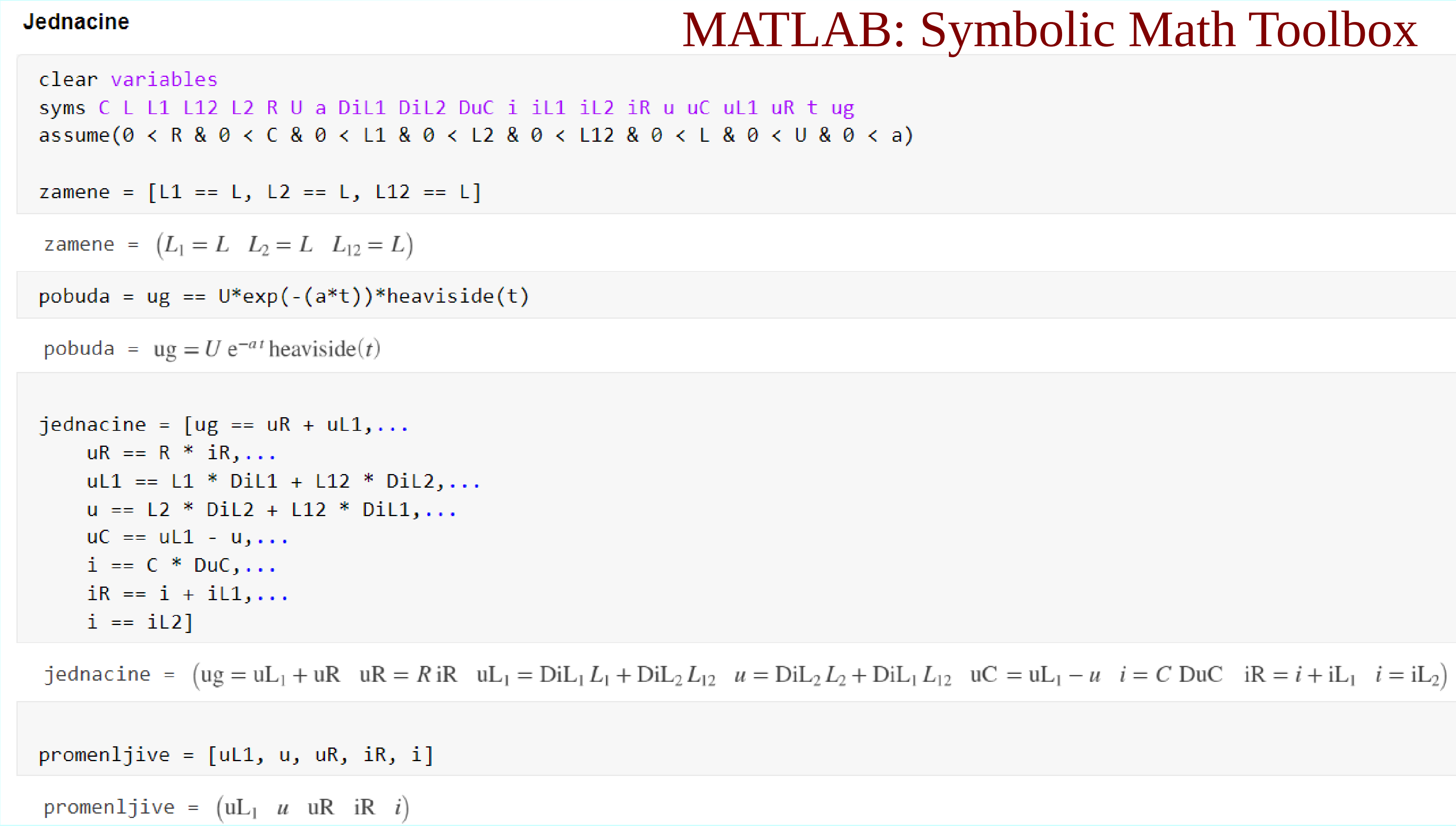
напон на секундару трансформатора $u(t)$



“zoom”
напона $u(t)$

Једначине

MATLAB: Symbolic Math Toolbox



Jednacine

```
clear variables
syms C L L1 L12 L2 R U a DiL1 DiL2 DuC i iL1 iL2 iR u uC uL1 uR t ug
assume(0 < R & 0 < C & 0 < L1 & 0 < L2 & 0 < L12 & 0 < L & 0 < U & 0 < a)

zamene = [L1 == L, L2 == L, L12 == L]
```

zamene = $\left(L_1 = L \quad L_2 = L \quad L_{12} = L\right)$

```
pobuda = ug == U*exp(-(a*t))*heaviside(t)
```

pobuda = $\mathrm{ug} = U\,\mathrm{e}^{-a\,t}\,\mathrm{heaviside}(t)$

```
jednacine = [ug == uR + uL1,...
    uR == R * iR,...
    uL1 == L1 * DiL1 + L12 * DiL2,...
    u == L2 * DiL2 + L12 * DiL1,...
    uC == uL1 - u,...
    i == C * DuC,...
    iR == i + iL1,...
    i == iL2]
```

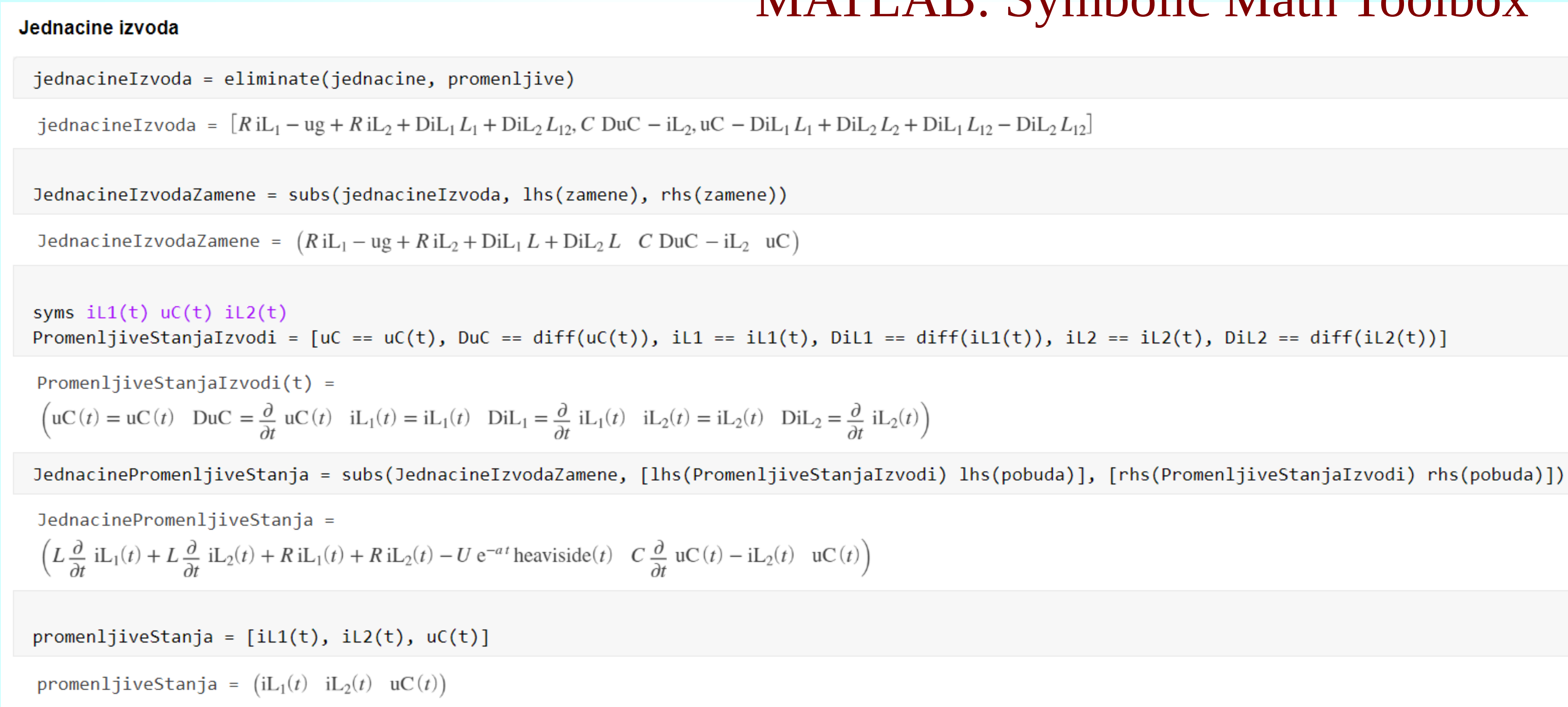
jednacine = $\left(\mathrm{ug} = \mathrm{uL}_1 + \mathrm{uR} \quad \mathrm{uR} = R\,\mathrm{iR} \quad \mathrm{uL}_1 = \mathrm{DiL}_1\,L_1 + \mathrm{DiL}_2\,L_{12} \quad u = \mathrm{DiL}_2\,L_2 + \mathrm{DiL}_1\,L_{12} \quad \mathrm{uC} = \mathrm{uL}_1 - u \quad i = C\,\mathrm{DuC} \quad \mathrm{iR} = i + \mathrm{iL}_1 \quad i = \mathrm{iL}_2\right)$

```
promenljive = [uL1, u, uR, iR, i]
```

promenljive = $\left(\mathrm{uL}_1 \quad u \quad \mathrm{uR} \quad \mathrm{iR} \quad i\right)$

Једначине извода

MATLAB: Symbolic Math Toolbox



Jednacine izvoda

```
jednacineIzvoda = eliminate(jednacine, promenljive)
```

jednacineIzvoda = $\left[R\,\mathrm{iL}_1 - \mathrm{ug} + R\,\mathrm{iL}_2 + \mathrm{DiL}_1\,L_1 + \mathrm{DiL}_2\,L_{12}, C\;\mathrm{DuC} - \mathrm{iL}_2, \mathrm{uC} - \mathrm{DiL}_1\,L_1 + \mathrm{DiL}_2\,L_2 + \mathrm{DiL}_1\,L_{12} - \mathrm{DiL}_2\,L_{12}\right]$

```
JednacineIzvodaZamene = subs(jednacineIzvoda, lhs(zamene), rhs(zamene))
```

JednacineIzvodaZamene = $\left(R\,\mathrm{iL}_1 - \mathrm{ug} + R\,\mathrm{iL}_2 + \mathrm{DiL}_1\,L + \mathrm{DiL}_2\,L \quad C\;\mathrm{DuC} - \mathrm{iL}_2 \quad \mathrm{uC}\right)$

```
syms iL1(t) uC(t) iL2(t)
PromenljiveStanjaIzvodi = [uC == uC(t), DuC == diff(uC(t)), iL1 == iL1(t), DiL1 == diff(iL1(t)), iL2 == iL2(t), DiL2 == diff(iL2(t))]
```

PromenljiveStanjaIzvodi(t) =

$$\left(\mathrm{uC}(t) = \mathrm{uC}(t) \quad \mathrm{DuC} = \frac{\partial}{\partial t}\,\mathrm{uC}(t) \quad \mathrm{iL}_1(t) = \mathrm{iL}_1(t) \quad \mathrm{DiL}_1 = \frac{\partial}{\partial t}\,\mathrm{iL}_1(t) \quad \mathrm{iL}_2(t) = \mathrm{iL}_2(t) \quad \mathrm{DiL}_2 = \frac{\partial}{\partial t}\,\mathrm{iL}_2(t)\right)$$

```
JednacinePromenljiveStanja = subs(JednacineIzvodaZamene, [lhs(PromenljiveStanjaIzvodi) lhs(pobuda)], [rhs(PromenljiveStanjaIzvodi) rhs(pobuda)])
```

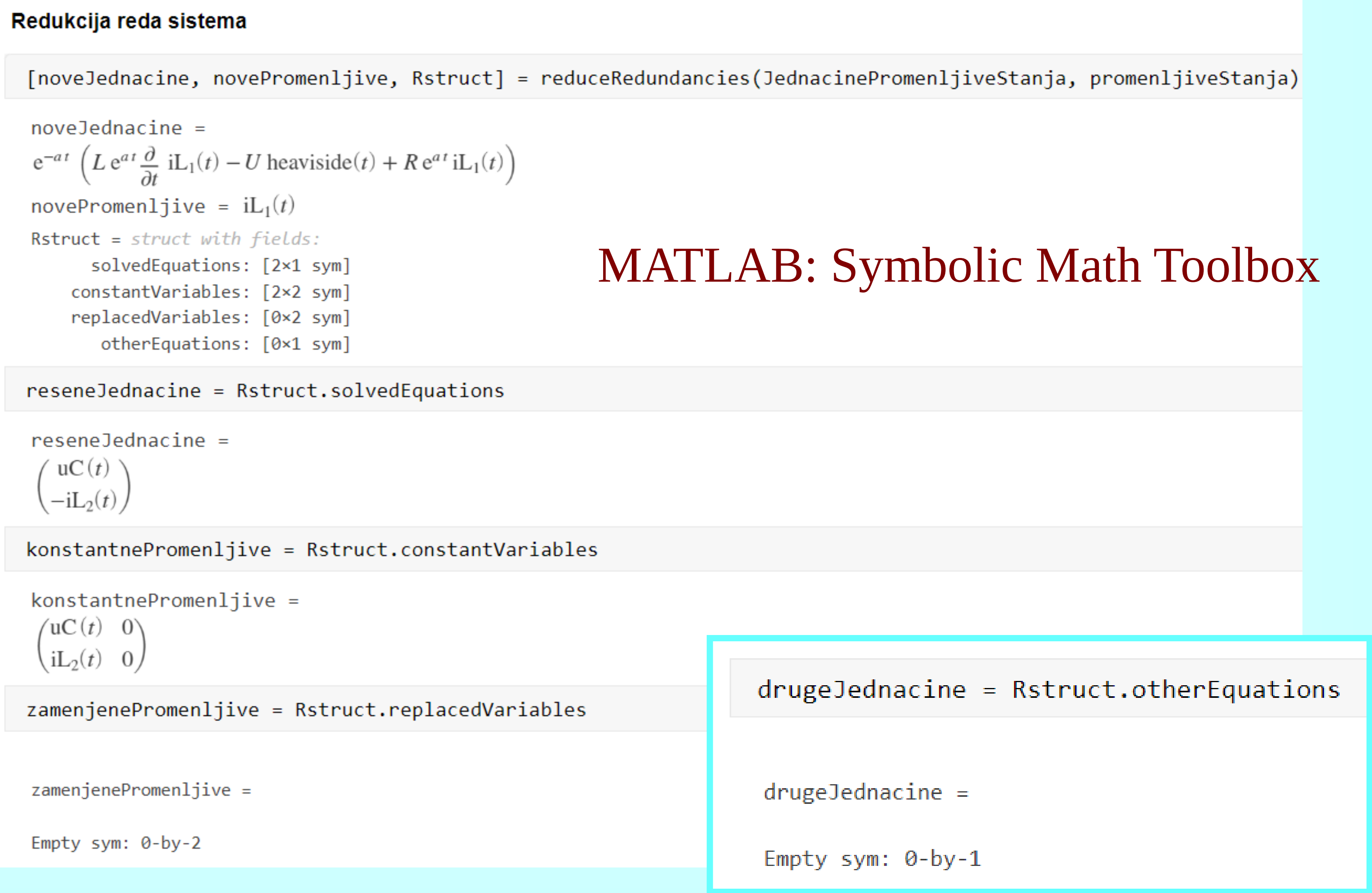
JednacinePromenljiveStanja =

$$\left(L\,\frac{\partial}{\partial t}\,\mathrm{iL}_1(t) + L\,\frac{\partial}{\partial t}\,\mathrm{iL}_2(t) + R\,\mathrm{iL}_1(t) + R\,\mathrm{iL}_2(t) - U\,\mathrm{e}^{-a\,t}\,\mathrm{heaviside}(t) \quad C\,\frac{\partial}{\partial t}\,\mathrm{uC}(t) - \mathrm{iL}_2(t) \quad \mathrm{uC}(t)\right)$$

```
promenljiveStanja = [iL1(t), iL2(t), uC(t)]
```

promenljiveStanja = $\left(\mathrm{iL}_1(t) \quad \mathrm{iL}_2(t) \quad \mathrm{uC}(t)\right)$

Елиминација сувишних једначина



Redukcija reda sistema

```
[noveJednacine, novePromenljive, Rstruct] = reduceRedundancies(JednacinePromenljiveStanja, promenljiveStanja)
```

noveJednacine =

$$e^{-a\,t}\left(L\,e^{a\,t}\,\frac{\partial}{\partial t}\,\mathrm{iL}_1(t) - U\,\mathrm{heaviside}(t) + R\,e^{a\,t}\,\mathrm{iL}_1(t)\right)$$

novePromenljive = $\mathrm{iL}_1(t)$

```
Rstruct = struct with fields:
       solvedEquations: [2×1 sym]
     constantVariables: [2×2 sym]
     replacedVariables: [0×2 sym]
        otherEquations: [0×1 sym]
```

```
reseneJednacine = Rstruct.solvedEquations
```

reseneJednacine =

$$\begin{pmatrix} \mathrm{uC}(t) \\ -\mathrm{iL}_2(t) \end{pmatrix}$$

```
konstantnePromenljive = Rstruct.constantVariables
```

konstantnePromenljive =

$$\begin{pmatrix} \mathrm{uC}(t) & 0 \\ \mathrm{iL}_2(t) & 0 \end{pmatrix}$$

```
zamenjenePromenljive = Rstruct.replacedVariables
```

```
zamenjenePromenljive =

Empty sym: 0-by-2
```

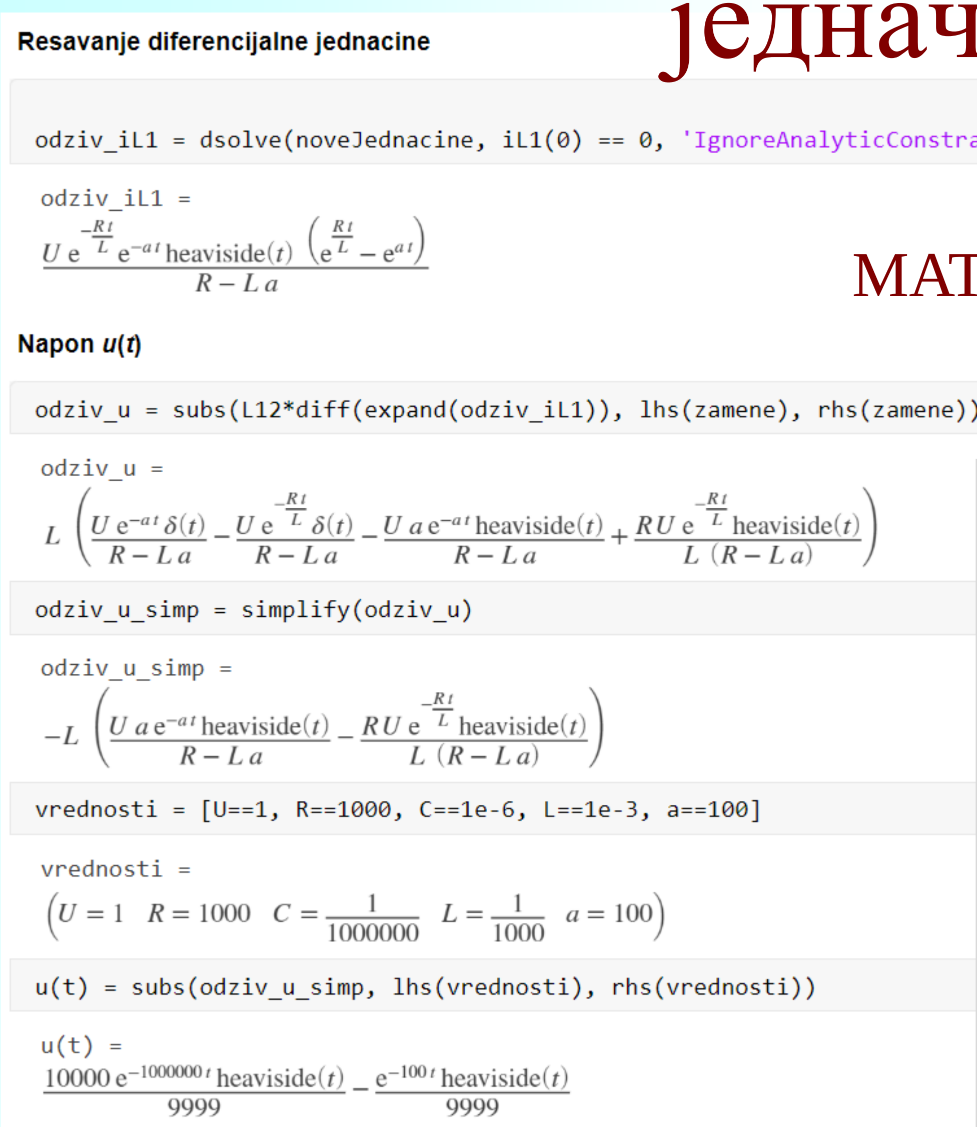
MATLAB: Symbolic Math Toolbox

```
drugeJednacine = Rstruct.otherEquations
```

```
drugeJednacine =

Empty sym: 0-by-1
```

Решавање диференцијалне једначине

MATLAB: Symbolic Math Toolbox



Resavanje diferencijalne jednacine

```
odziv_iL1 = dsolve(noveJednacine, iL1(0) == 0, 'IgnoreAnalyticConstraints', false)
```

odziv_iL1 =

$$\frac{U\,\mathrm{e}^{-\frac{R\,t}{L}}\,\mathrm{e}^{-a\,t}\,\mathrm{heaviside}(t)\,\left(\mathrm{e}^{\frac{R\,t}{L}}-\mathrm{e}^{a\,t}\right)}{R-L\,a}$$

Napon *u*(*t*)

```
odziv_u = subs(L12*diff(expand(odziv_iL1)), lhs(zamene), rhs(zamene))
```

odziv_u =

$$L\,\left(\frac{U\,\mathrm{e}^{-a\,t}\,\delta(t)}{R-L\,a}-\frac{U\,\mathrm{e}^{-\frac{R\,t}{L}}\,\delta(t)}{R-L\,a}-\frac{U\,a\,\mathrm{e}^{-a\,t}\,\mathrm{heaviside}(t)}{R-L\,a}+\frac{R\,U\,\mathrm{e}^{-\frac{R\,t}{L}}\,\mathrm{heaviside}(t)}{L\,(R-L\,a)}\right)$$

```
odziv_u_simp = simplify(odziv_u)
```

odziv_u_simp =

$$-L\,\left(\frac{U\,a\,\mathrm{e}^{-a\,t}\,\mathrm{heaviside}(t)}{R-L\,a}-\frac{R\,U\,\mathrm{e}^{-\frac{R\,t}{L}}\,\mathrm{heaviside}(t)}{L\,(R-L\,a)}\right)$$

```
vrednosti = [U==1, R==1000, C==1e-6, L==1e-3, a==100]
```

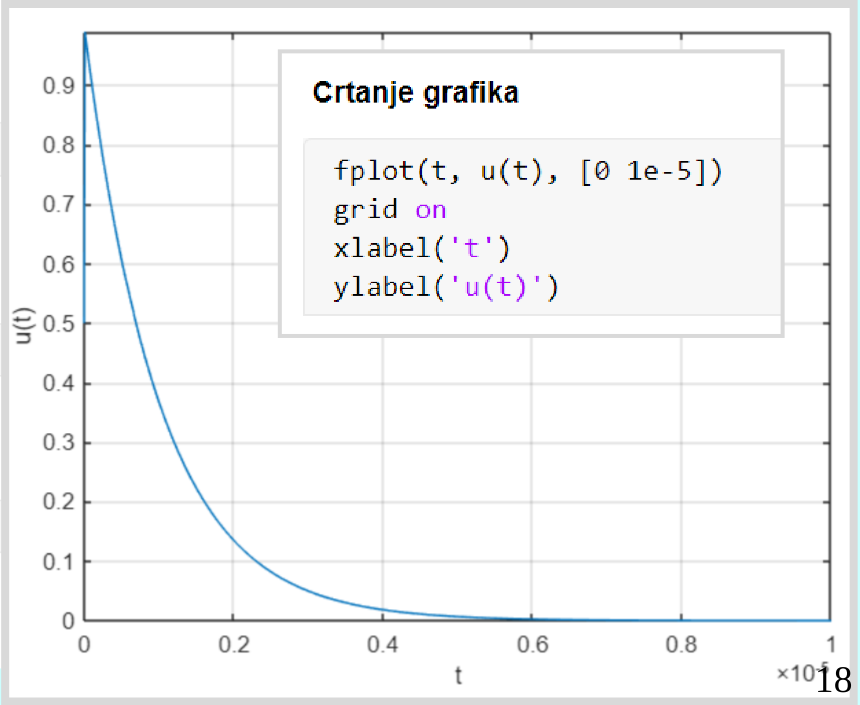
vrednosti =

$$\left(U=1 \quad R=1000 \quad C=\frac{1}{1000000} \quad L=\frac{1}{1000} \quad a=100\right)$$

```
u(t) = subs(odziv_u_simp, lhs(vrednosti), rhs(vrednosti))
```

u(t) =

$$\frac{10000\,\mathrm{e}^{-1000000\,t}\,\mathrm{heaviside}(t)}{9999}-\frac{\mathrm{e}^{-100\,t}\,\mathrm{heaviside}(t)}{9999}$$



Crtanje grafika

```
fplot(t, u(t), [0 1e-5])
grid on
xlabel('t')
ylabel('u(t)')
```

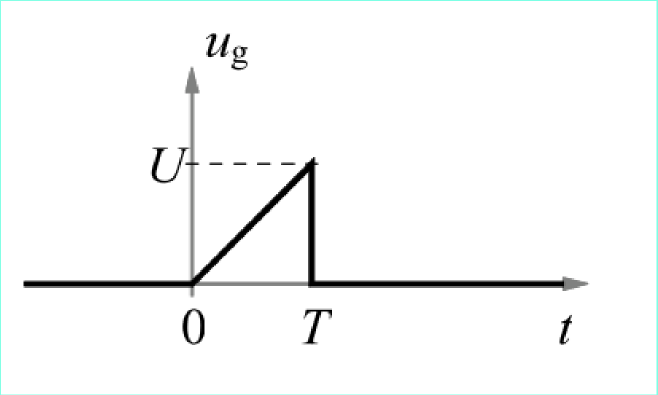
Одзив на троугаони импулс

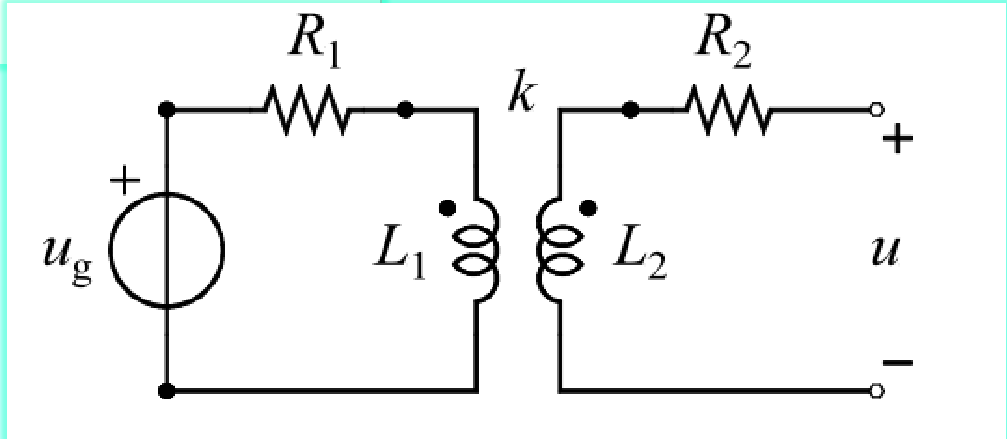
Параметри електричног кола са слике су познати. Побуда је дата на слици, $T = L/R$, $R_1 = R$, $R_2 = 2R$, $L_1 = L$, $L_2 = 4L$, $k = 1/2$.

(5) Одредити индициону функцију за напон отвореног секундара (одскочни одзив).

(5) Одредити напон отвореног секундара и

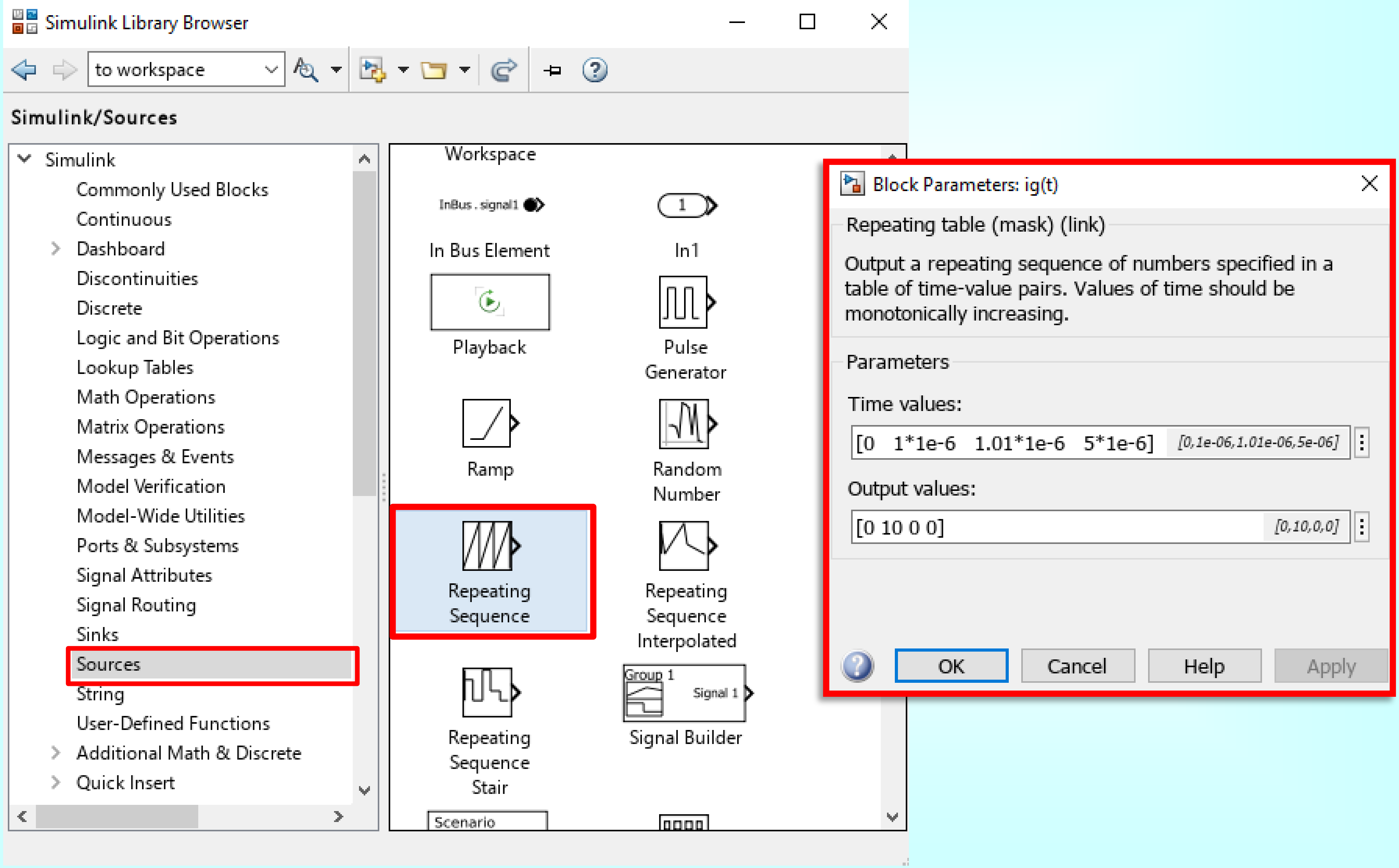
(5) нацртати његов график. Обележити осе графика, координатни почетак, пресеке и додире графика са осама, и тачке екстремума.





MATLAB: Simulink

Sources



Block Parameters: ig(t)

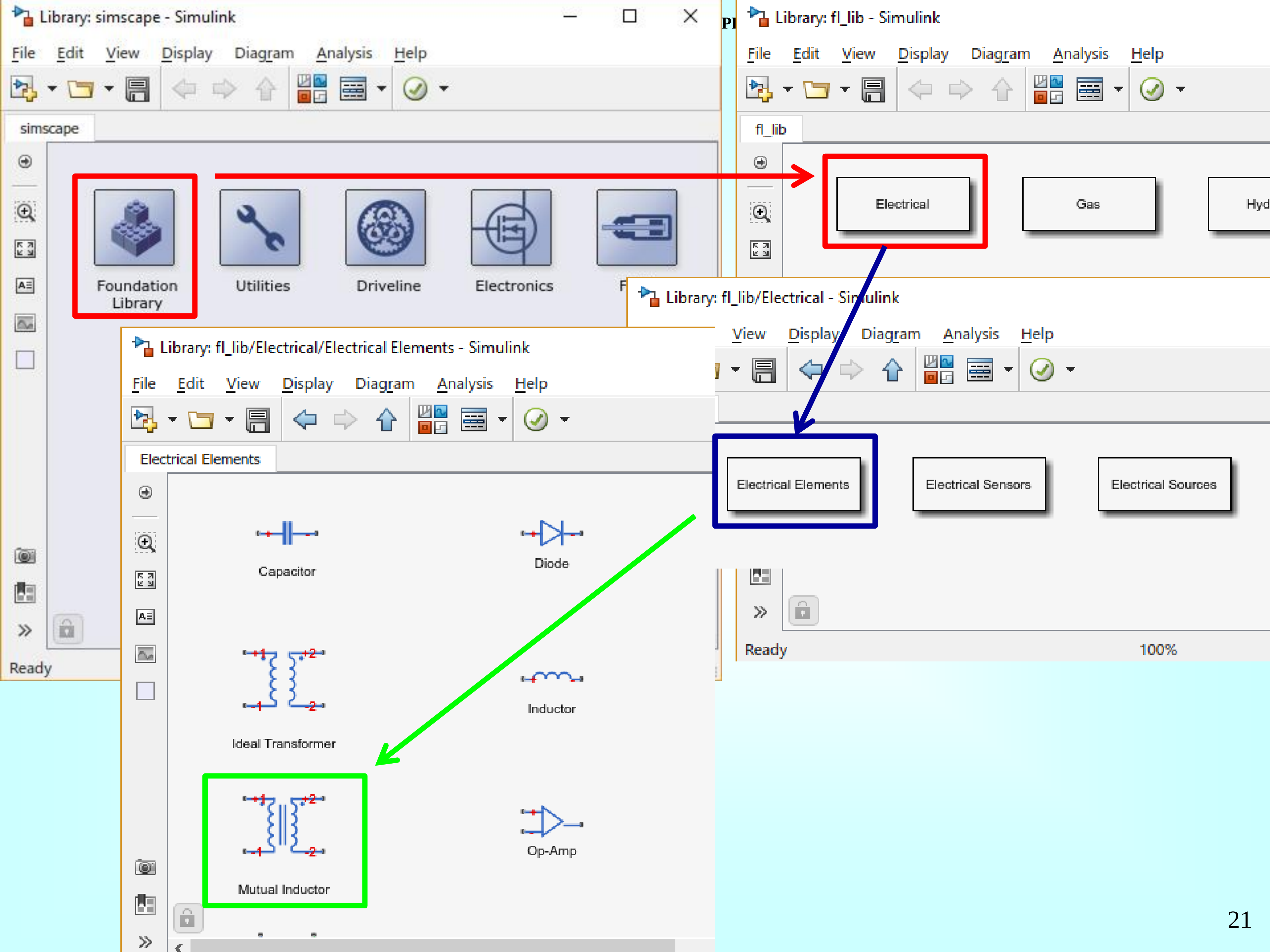
Repeating table (mask) (link)

Output a repeating sequence of numbers specified in a table of time-value pairs. Values of time should be monotonically increasing.

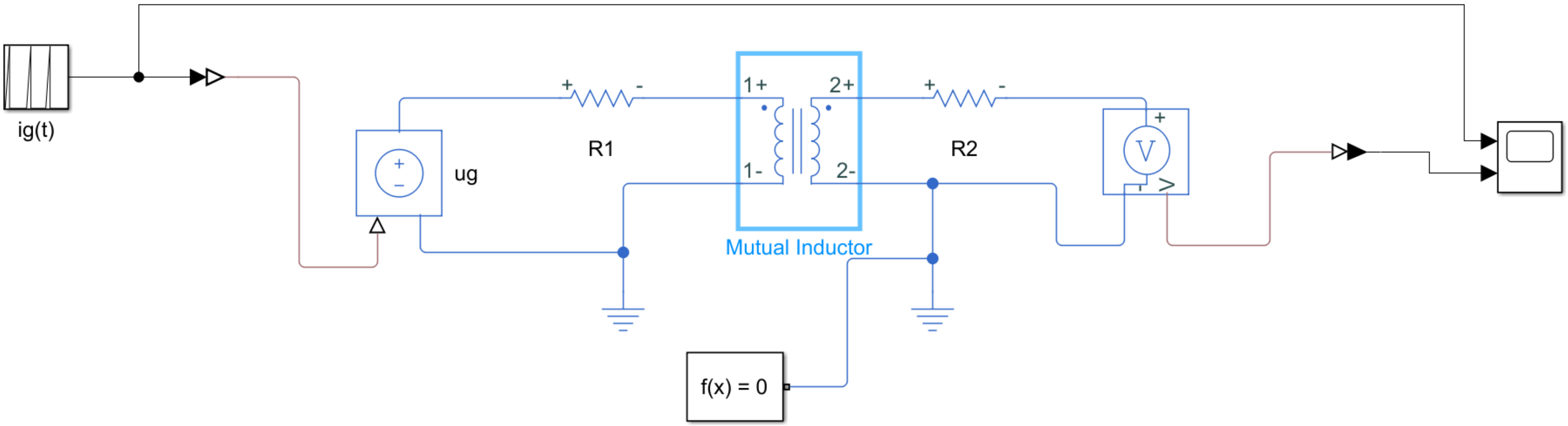
Parameters

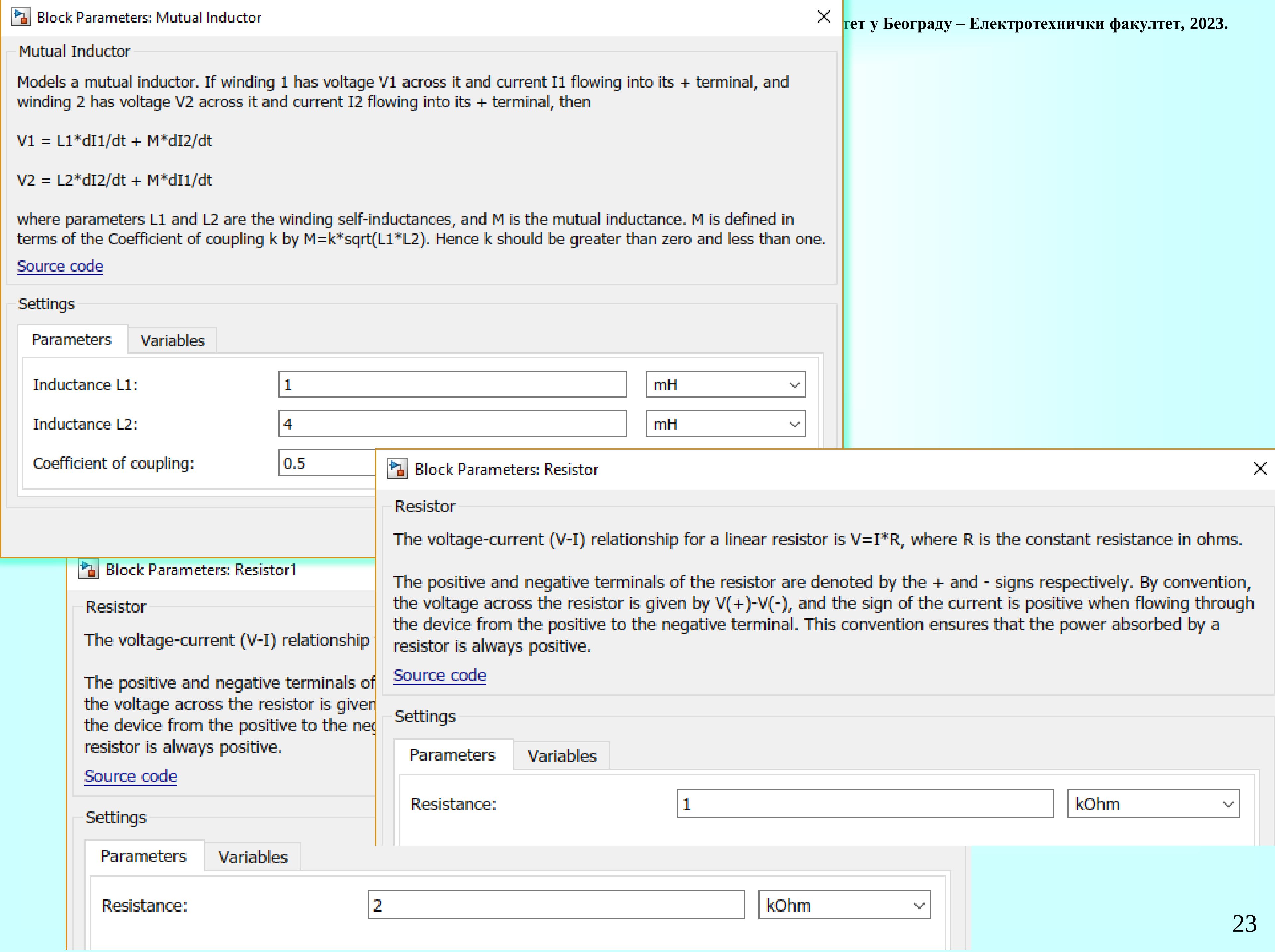
Time values: [0 1*1e-6 1.01*1e-6 5*1e-6] [0,1e-06,1.01e-06,5e-06]

Output values: [0 10 0 0] [0,10,0,0]


Library: simscape - Simulink
File Edit View Display Diagram Analysis Help
simscape
Foundation Library
Utilities
Driveline
Electronics
Ready
Library: fl_lib - Simulink
File Edit View Display Diagram Analysis Help
fl_lib
Electrical
Gas
Hyd
Library: fl_lib/Electrical - Simulink
View Display Diagram Analysis Help
Electrical Elements
Electrical Sensors
Electrical Sources
Ready
100%
Library: fl_lib/Electrical/Electrical Elements - Simulink
File Edit View Display Diagram Analysis Help
Electrical Elements
Capacitor
Diode
Ideal Transformer
Inductor
Mutual Inductor
Op-Amp

Sastavljanje kola





Block Parameters: Mutual Inductor

Mutual Inductor

Models a mutual inductor. If winding 1 has voltage V1 across it and current I1 flowing into its + terminal, and winding 2 has voltage V2 across it and current I2 flowing into its + terminal, then

V1 = L1*dI1/dt + M*dI2/dt

V2 = L2*dI2/dt + M*dI1/dt

where parameters L1 and L2 are the winding self-inductances, and M is the mutual inductance. M is defined in terms of the Coefficient of coupling k by M=k*sqrt(L1*L2). Hence k should be greater than zero and less than one.

Source code

Settings

Parameters | Variables

Inductance L1:	1	mH
Inductance L2:	4	mH
Coefficient of coupling:	0.5	

Block Parameters: Resistor

Resistor

The voltage-current (V-I) relationship for a linear resistor is V=I*R, where R is the constant resistance in ohms.

The positive and negative terminals of the resistor are denoted by the + and - signs respectively. By convention, the voltage across the resistor is given by V(+)-V(-), and the sign of the current is positive when flowing through the device from the positive to the negative terminal. This convention ensures that the power absorbed by a resistor is always positive.

Source code

Settings

Parameters | Variables

Resistance:	1	kOhm

Block Parameters: Resistor1

Resistor

The voltage-current (V-I) relationship

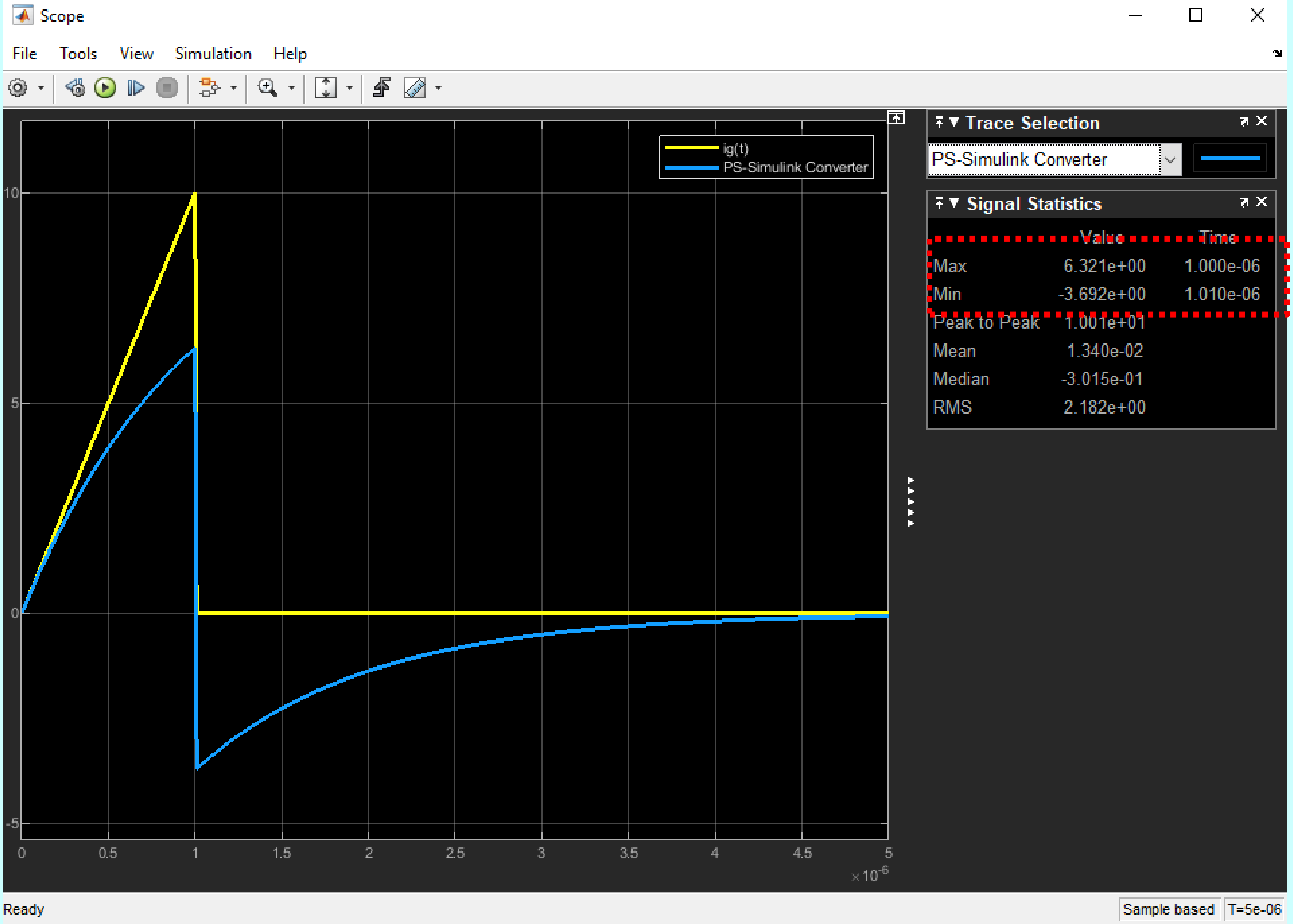
The positive and negative terminals of
the voltage across the resistor is given
the device from the positive to the neg
resistor is always positive.

Source code

Settings

Parameters | Variables

Resistance:	2	kOhm



Trace Selection: PS-Simulink Converter

Signal Statistics

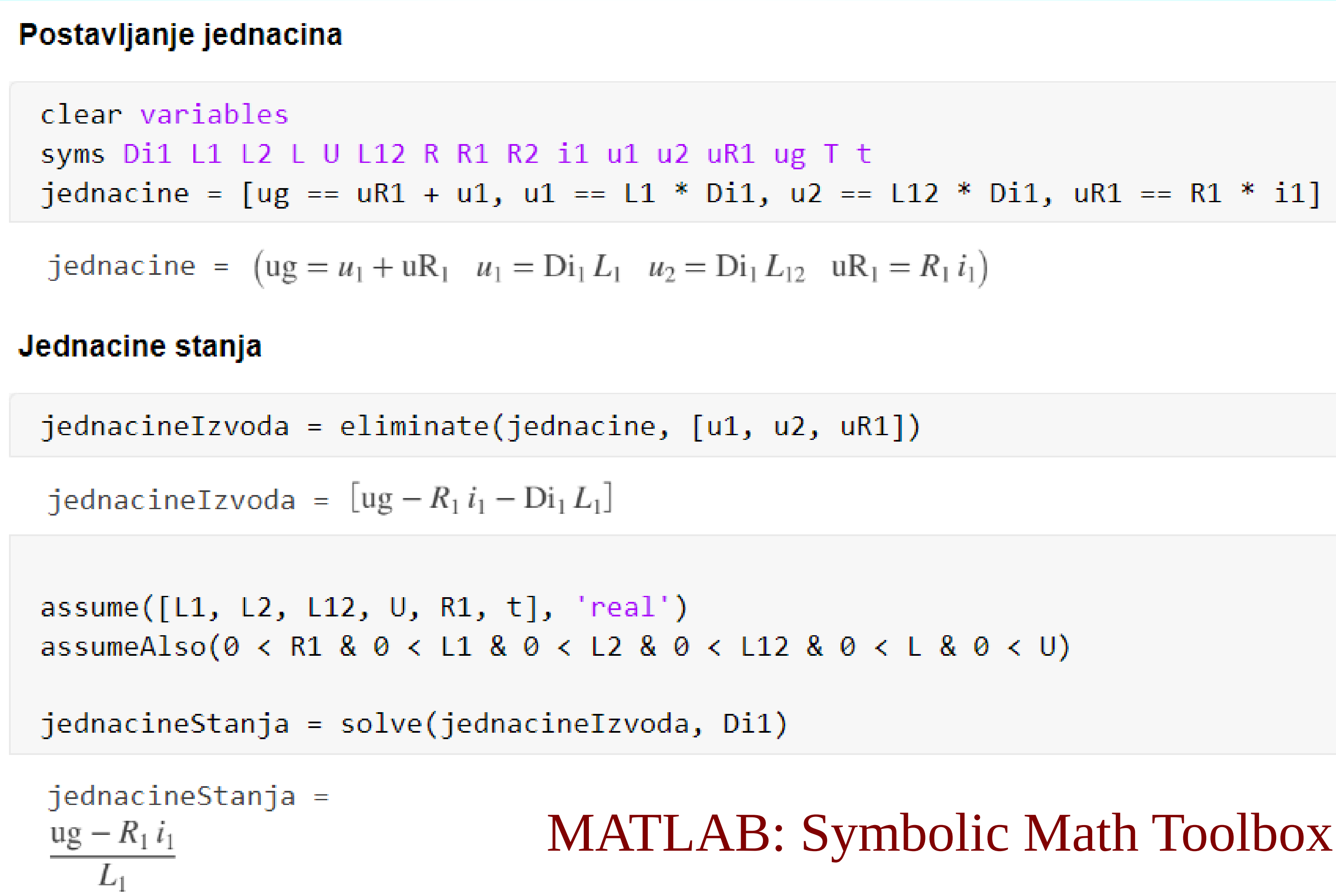
	Value	Time
Max	6.321e+00	1.000e-06
Min	-3.692e+00	1.010e-06
Peak to Peak	1.001e+01	
Mean	1.340e-02	
Median	-3.015e-01	
RMS	2.182e+00	

Ready

Sample based

T=5e-06

Једначине



Postavljanje jednacina

```
clear variables
syms Di1 L1 L2 L U L12 R R1 R2 i1 u1 u2 uR1 ug T t
jednacine = [ug == uR1 + u1, u1 == L1 * Di1, u2 == L12 * Di1, uR1 == R1 * i1]
```

jednacine = $\left(\mathrm{ug} = u_1 + \mathrm{uR}_1 \quad u_1 = \mathrm{Di}_1\, L_1 \quad u_2 = \mathrm{Di}_1\, L_{12} \quad \mathrm{uR}_1 = R_1\, i_1\right)$

Jednacine stanja

```
jednacineIzvoda = eliminate(jednacine, [u1, u2, uR1])
```

jednacineIzvoda = $\left[\mathrm{ug} - R_1\, i_1 - \mathrm{Di}_1\, L_1\right]$

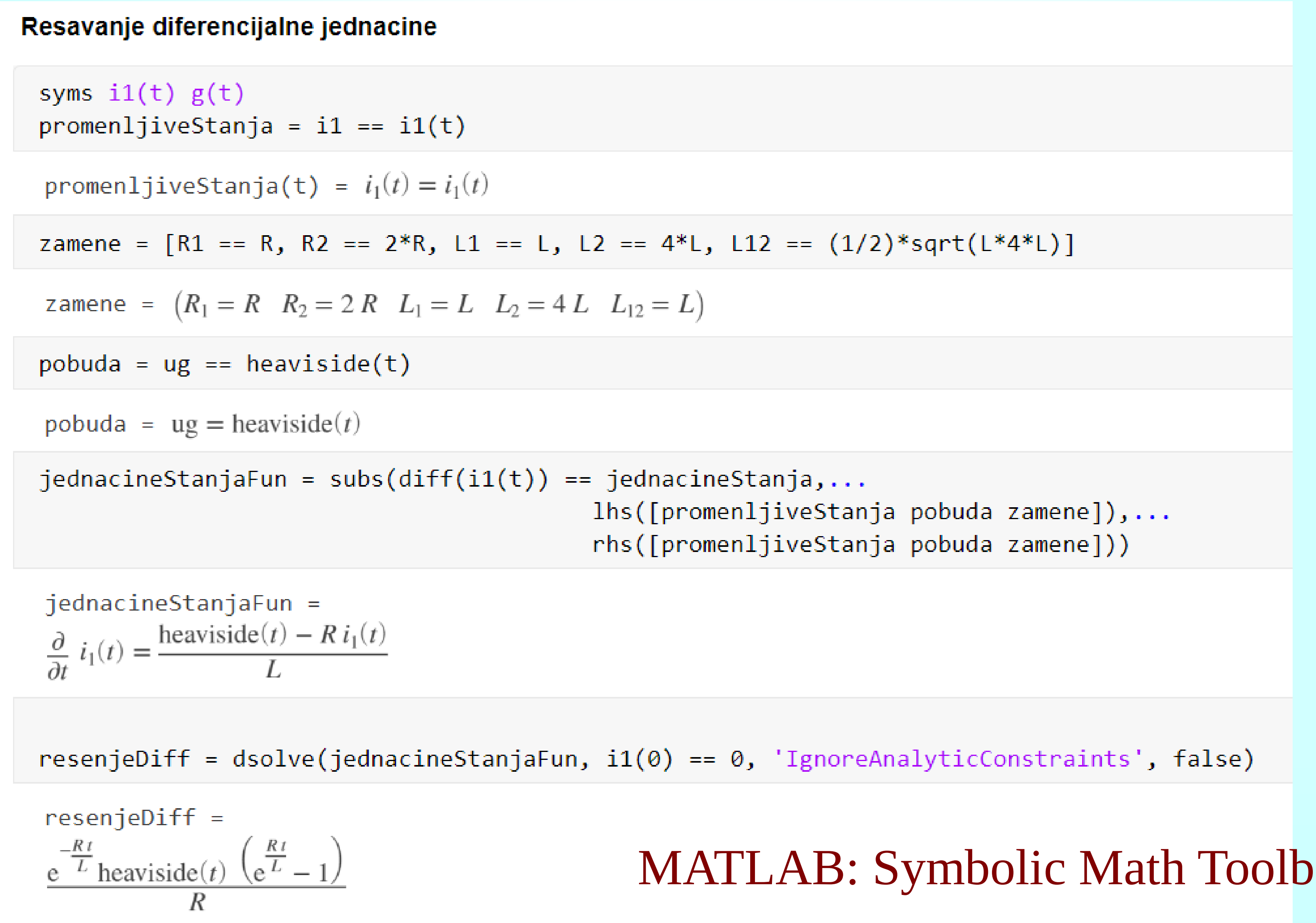
```
assume([L1, L2, L12, U, R1, t], 'real')
assumeAlso(0 < R1 & 0 < L1 & 0 < L2 & 0 < L12 & 0 < L & 0 < U)

jednacineStanja = solve(jednacineIzvoda, Di1)
```

jednacineStanja =

$$\frac{\mathrm{ug} - R_1\, i_1}{L_1}$$

MATLAB: Symbolic Math Toolbox

Решавање диференцијалне једначине



Resavanje diferencijalne jednacine

```
syms i1(t) g(t)
promenljiveStanja = i1 == i1(t)
```

promenljiveStanja(t) = $i_1(t) = i_1(t)$

```
zamene = [R1 == R, R2 == 2*R, L1 == L, L2 == 4*L, L12 == (1/2)*sqrt(L*4*L)]
```

zamene = $\left(R_1 = R \quad R_2 = 2\,R \quad L_1 = L \quad L_2 = 4\,L \quad L_{12} = L\right)$

```
pobuda = ug == heaviside(t)
```

pobuda = $\mathrm{ug} = \mathrm{heaviside}(t)$

```
jednacineStanjaFun = subs(diff(i1(t)) == jednacineStanja,...
                                        lhs([promenljiveStanja pobuda zamene]),...
                                        rhs([promenljiveStanja pobuda zamene]))
```

jednacineStanjaFun =

$$\frac{\partial}{\partial t}\, i_1(t) = \frac{\mathrm{heaviside}(t) - R\, i_1(t)}{L}$$

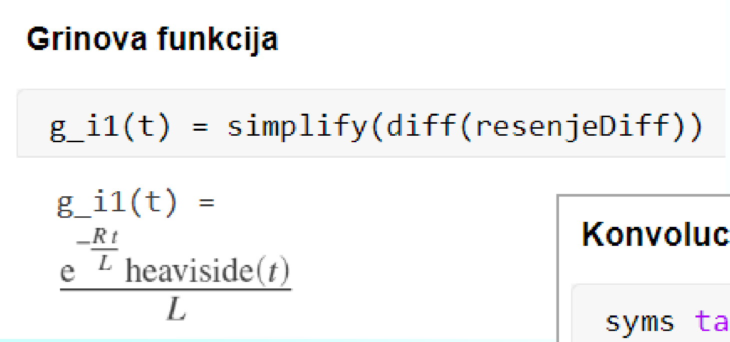
```
resenjeDiff = dsolve(jednacineStanjaFun, i1(0) == 0, 'IgnoreAnalyticConstraints', false)
```

resenjeDiff =

$$\frac{\mathrm{e}^{-\frac{R\,t}{L}}\,\mathrm{heaviside}(t)\,\left(\mathrm{e}^{\frac{R\,t}{L}} - 1\right)}{R}$$

MATLAB: Symbolic Math Toolbox

Гринова функција и конволуциони интеграл

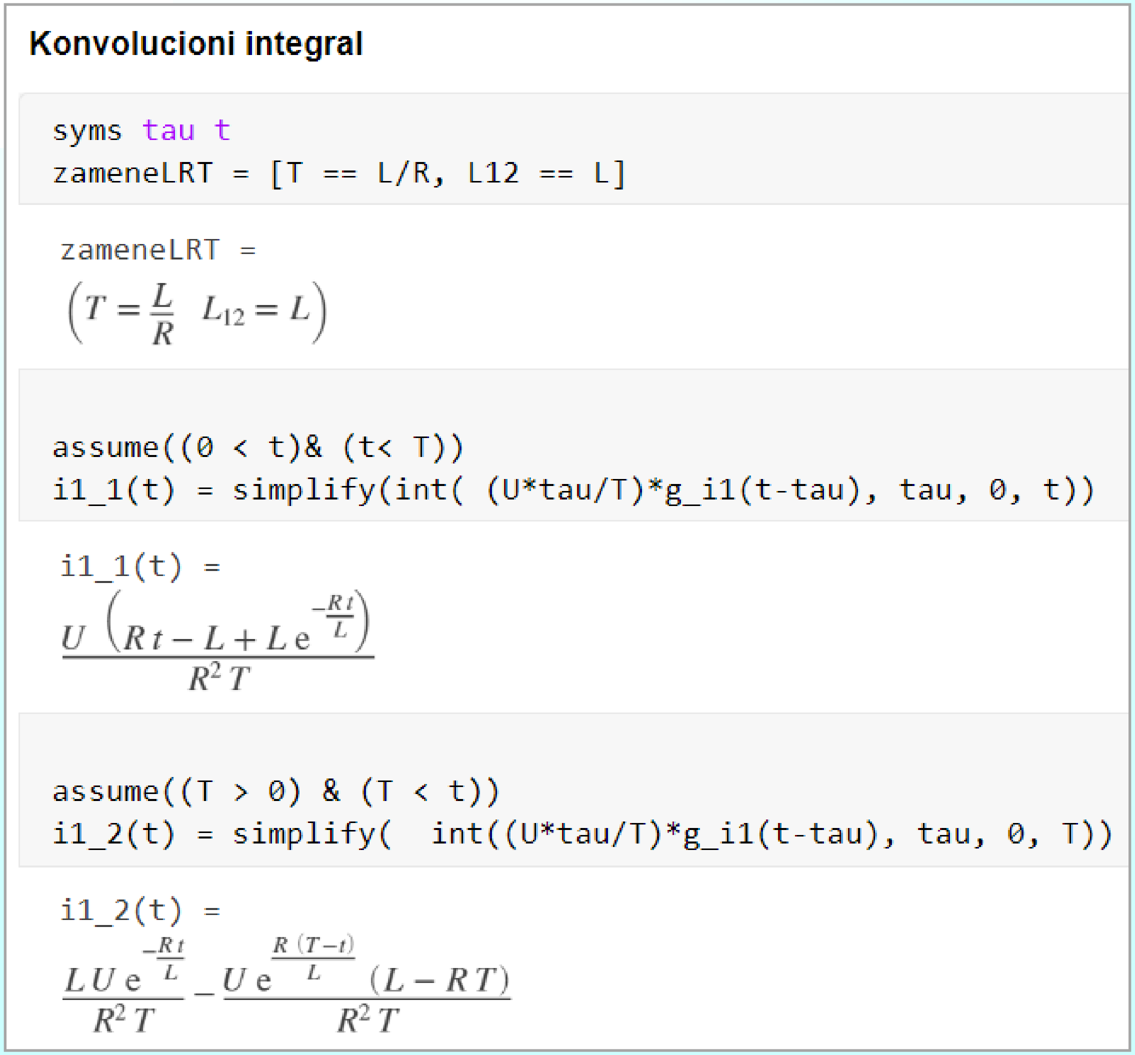


Grinova funkcija

```
g_i1(t) = simplify(diff(resenjeDiff))
```

g_i1(t) =

$$\frac{e^{-\frac{R\,t}{L}}\,\mathrm{heaviside}(t)}{L}$$



Konvolucioni integral

```
syms tau t
zameneLRT = [T == L/R, L12 == L]
```

zameneLRT =

$$\left(T = \frac{L}{R}\quad L_{12} = L\right)$$

```
assume((0 < t)& (t< T))
i1_1(t) = simplify(int( (U*tau/T)*g_i1(t-tau), tau, 0, t))
```

i1_1(t) =

$$\frac{U\left(R\,t - L + L\,e^{-\frac{R\,t}{L}}\right)}{R^2\,T}$$

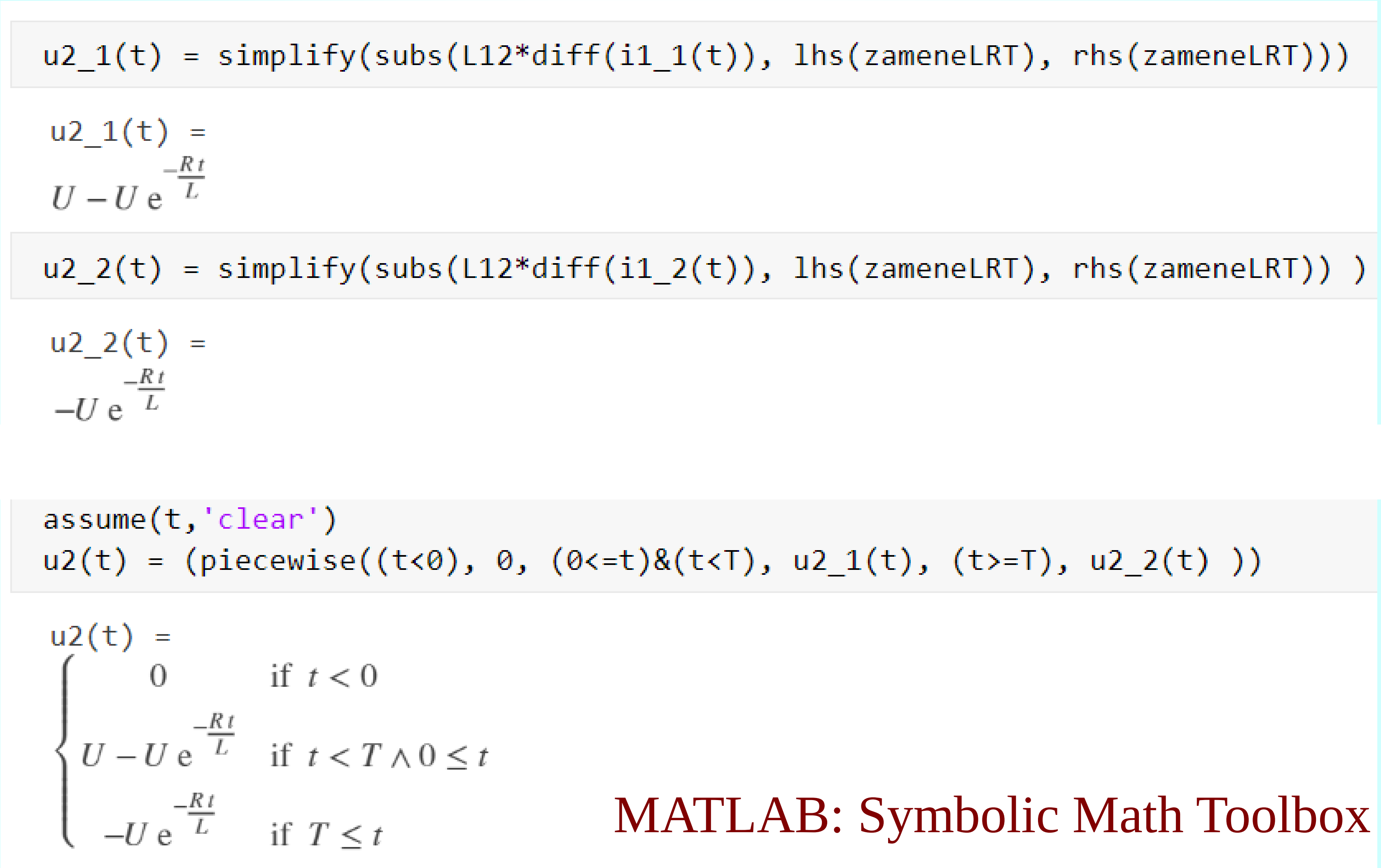
```
assume((T > 0) & (T < t))
i1_2(t) = simplify(  int((U*tau/T)*g_i1(t-tau), tau, 0, T))
```

i1_2(t) =

$$\frac{L\,U\,e^{-\frac{R\,t}{L}}}{R^2\,T} - \frac{U\,e^{\frac{R\,(T-t)}{L}}\,(L - R\,T)}{R^2\,T}$$

MATLAB: Symbolic Math Toolbox

Конволуциони интеграл (наставак)



```
u2_1(t) = simplify(subs(L12*diff(i1_1(t)), lhs(zameneLRT), rhs(zameneLRT)))
```

u2_1(t) =

$$U - U\,\mathrm{e}^{-\frac{R\,t}{L}}$$

```
u2_2(t) = simplify(subs(L12*diff(i1_2(t)), lhs(zameneLRT), rhs(zameneLRT)) )
```

u2_2(t) =

$$-U\,\mathrm{e}^{-\frac{R\,t}{L}}$$

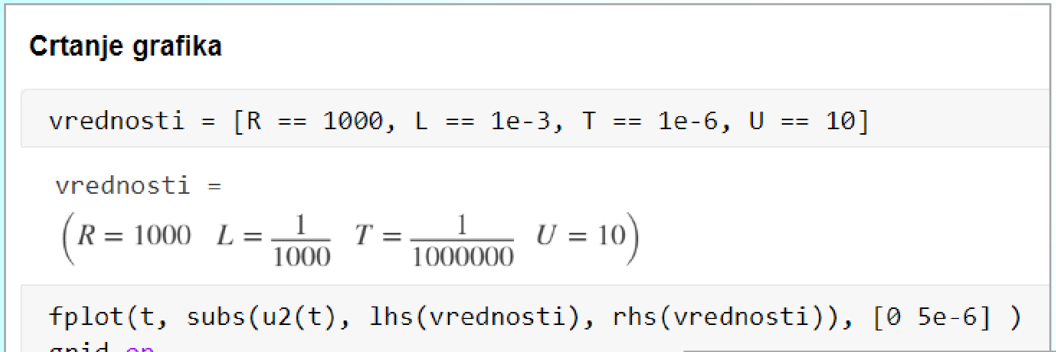
```
assume(t,'clear')
u2(t) = (piecewise((t<0), 0, (0<=t)&(t<T), u2_1(t), (t>=T), u2_2(t) ))
```

u2(t) =

$$\begin{cases} 0 & \text{if } t < 0 \\ U - U\,\mathrm{e}^{-\frac{R\,t}{L}} & \text{if } t < T \wedge 0 \le t \\ -U\,\mathrm{e}^{-\frac{R\,t}{L}} & \text{if } T \le t \end{cases}$$

MATLAB: Symbolic Math Toolbox

График функције одзива



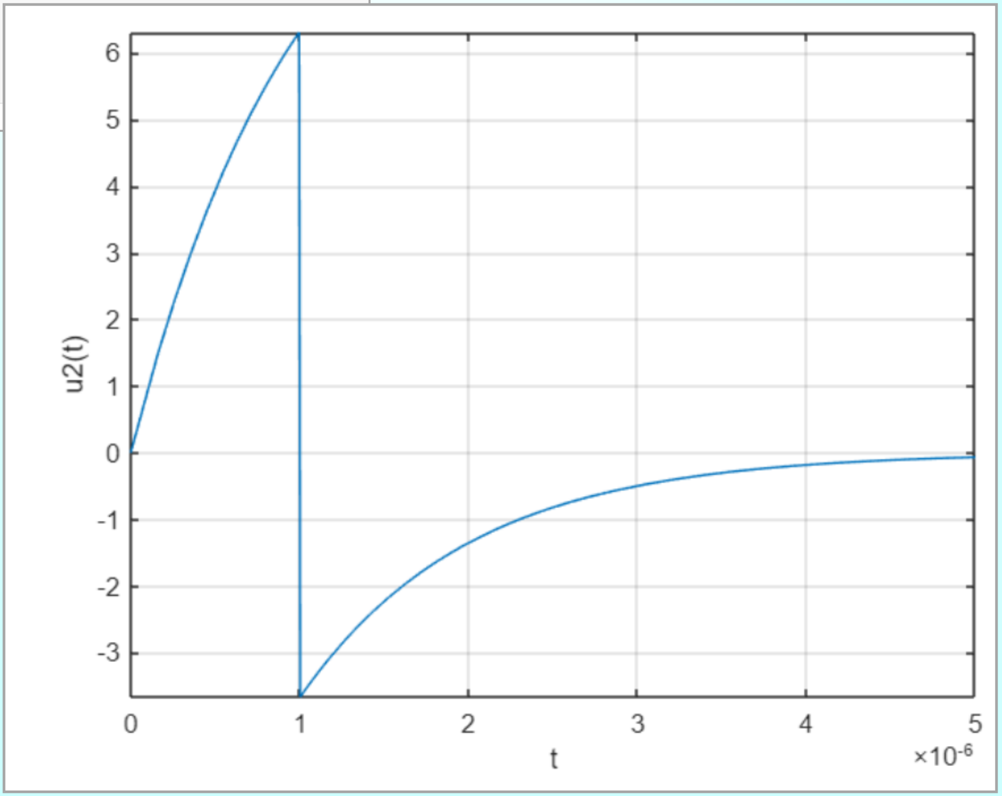
Crtanje grafika

```
vrednosti = [R == 1000, L == 1e-3, T == 1e-6, U == 10]
```

vrednosti =

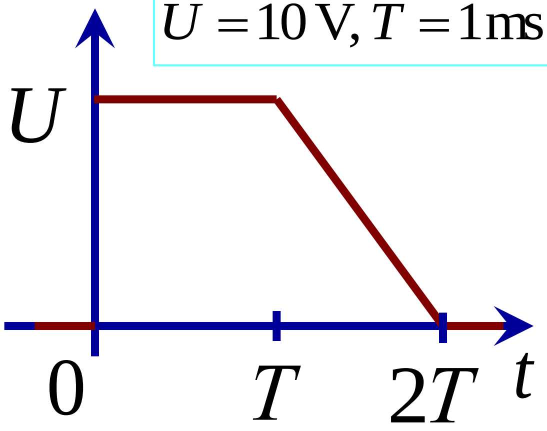
$$\left(R = 1000 \quad L = \frac{1}{1000} \quad T = \frac{1}{1000000} \quad U = 10 \right)$$

```
fplot(t, subs(u2(t), lhs(vrednosti), rhs(vrednosti)), [0 5e-6] )
grid on
xlabel('t')
ylabel('u2(t)')
```

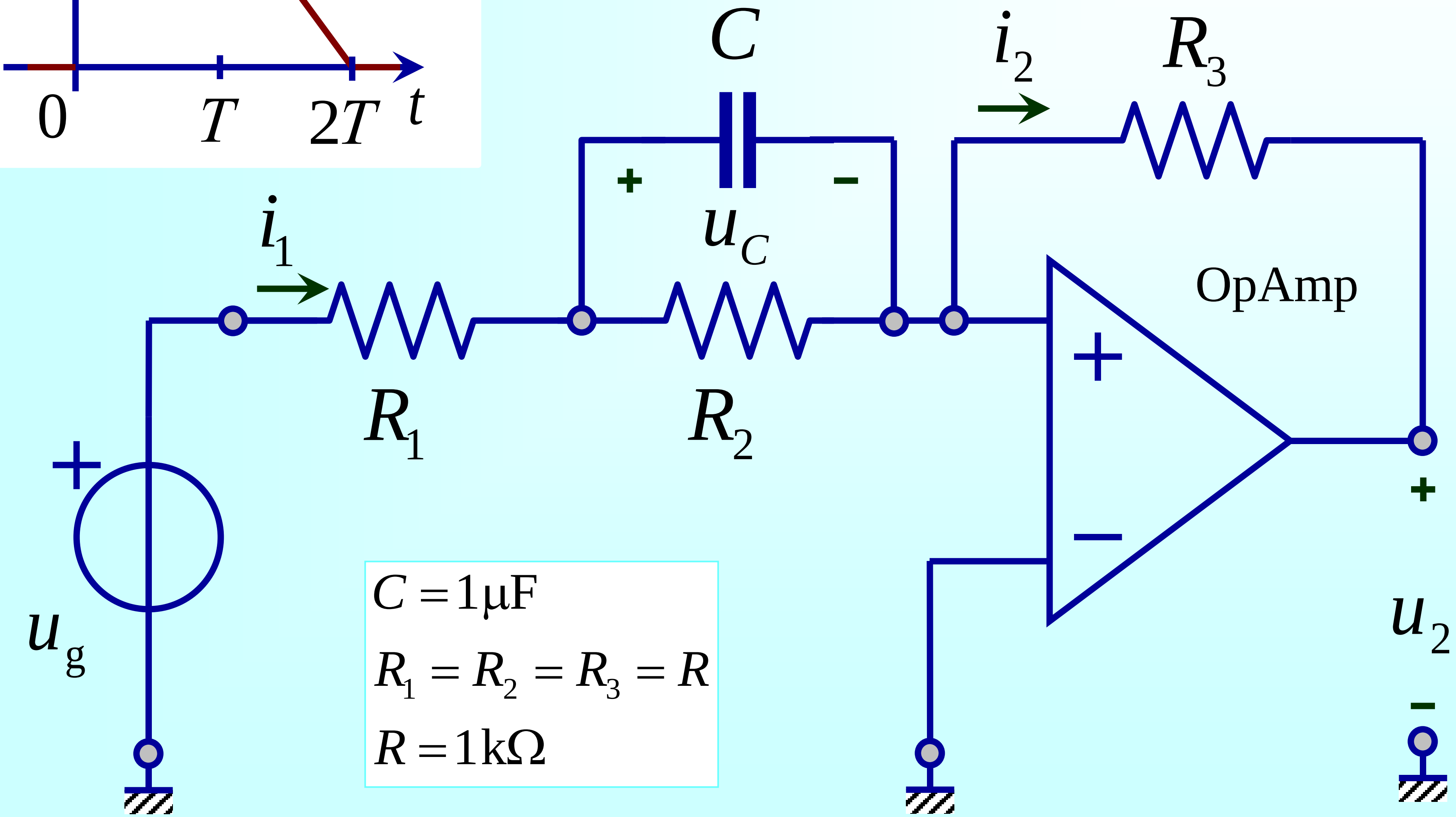


MATLAB: Symbolic Math Toolbox

Одзив на сложену побуду



$U = 10\,\text{V},\ T = 1\,\text{ms}$



$C = 1\,\mu\text{F}$

$R_1 = R_2 = R_3 = R$

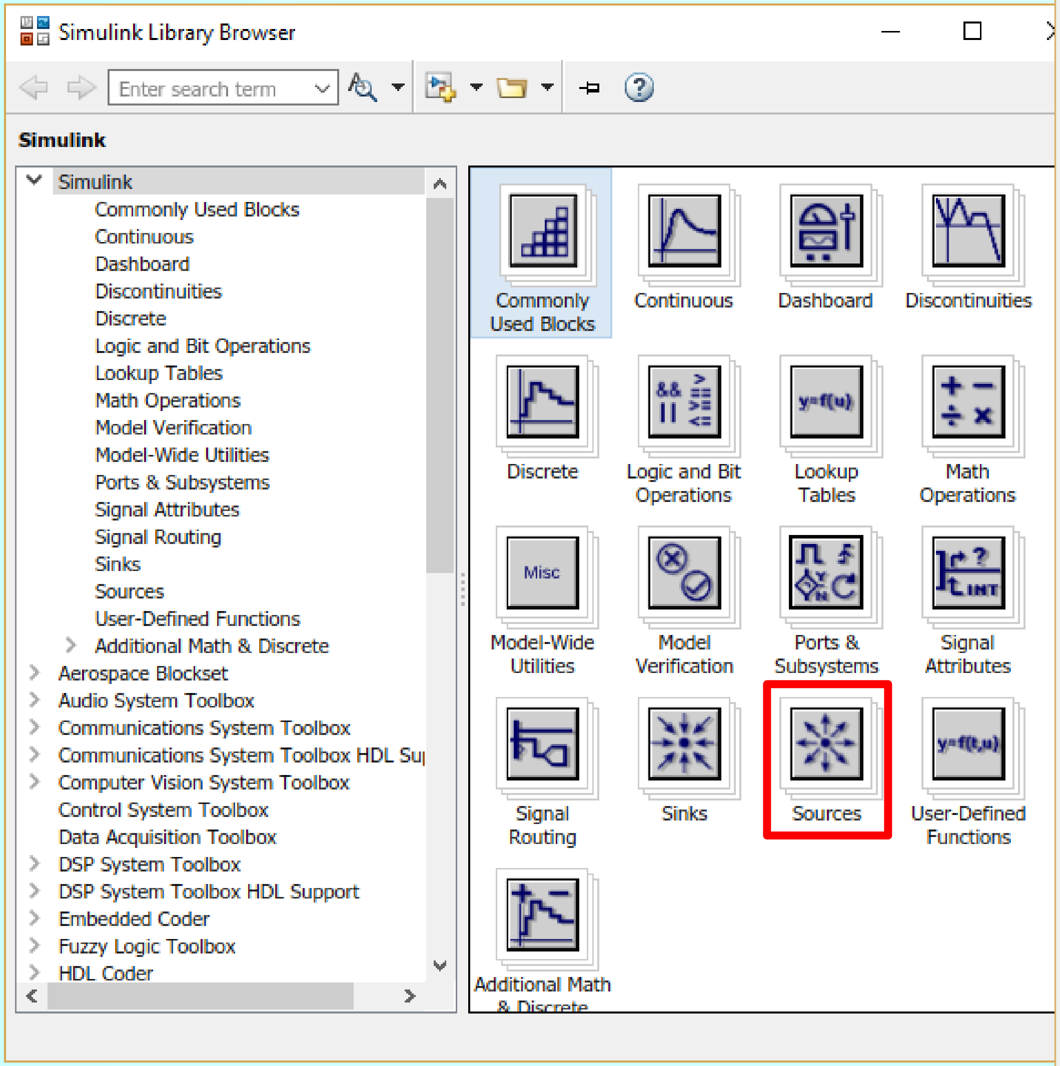
$R = 1\,\text{k}\Omega$

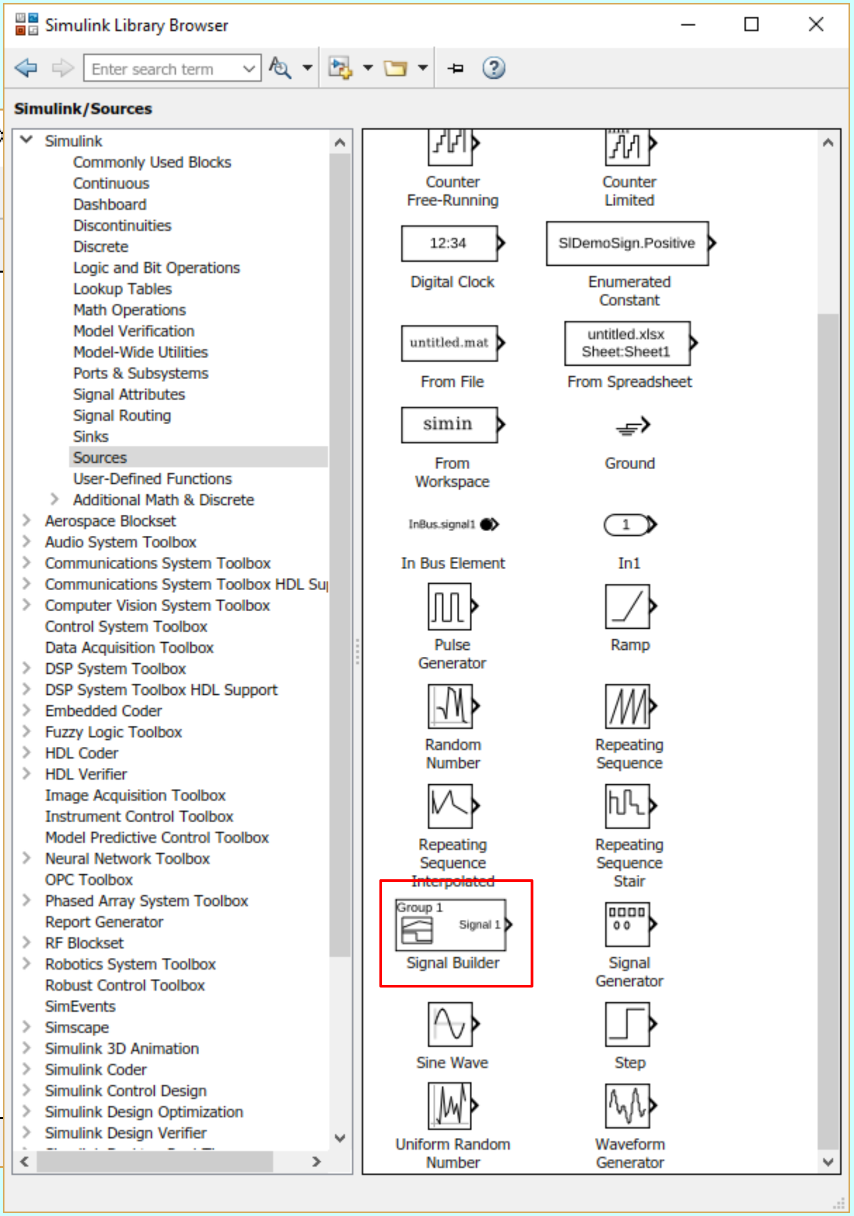
Задатак - сложена побуда

- Параметри кола са слике су познати: $R_1 = R_2 = R_3 = R$. Нема сакупљене енергије.
- Применом конволуционог интеграла одредити напон кондензатора $u_C(t)$, а затим и излазни напон $u_2(t)$ и нацртати његов график.
- Напон генератора се мења са временом као на диаграму, при чему је $T = RC$.

MATLAB: Simulink

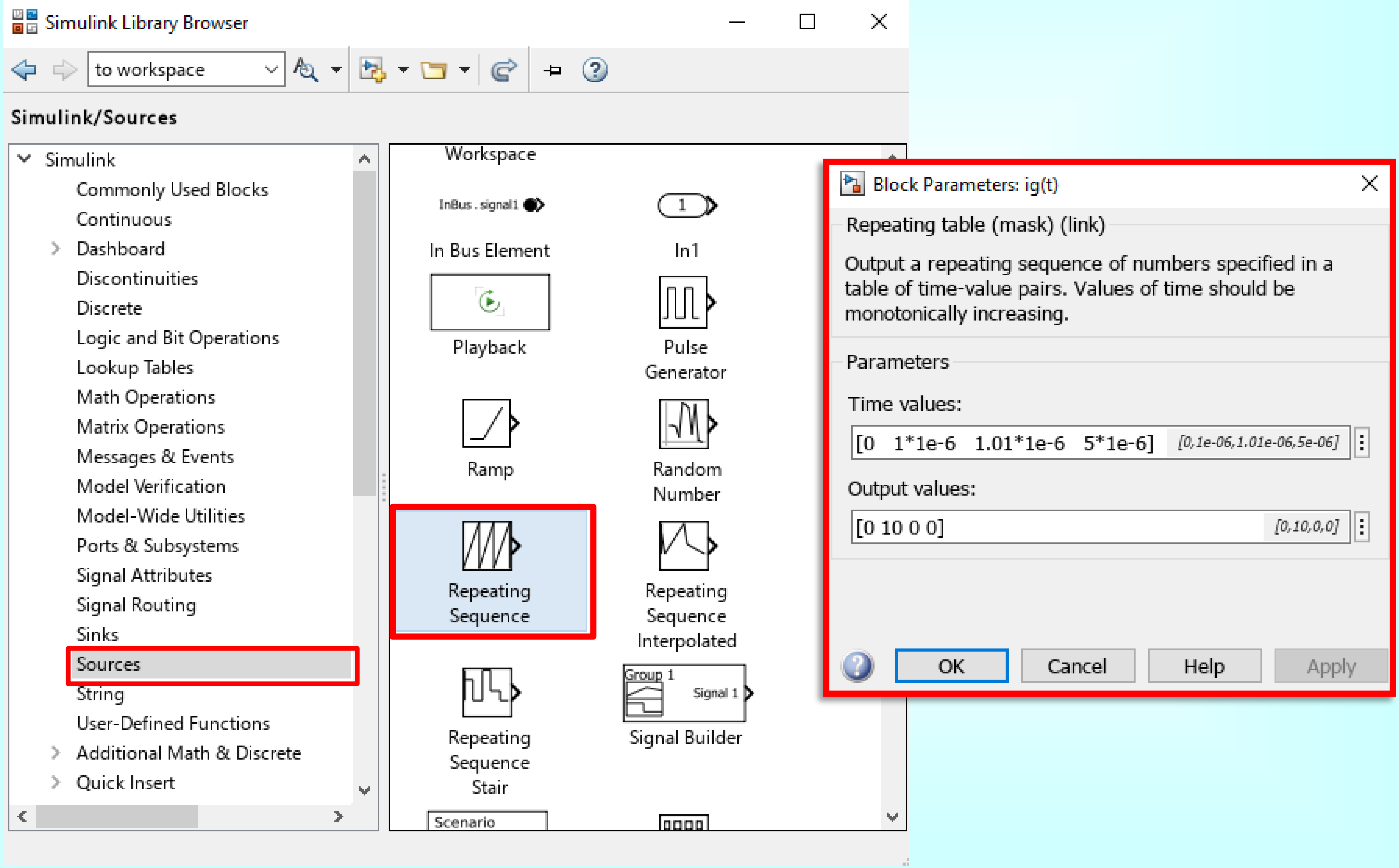
Sources





MATLAB: Simulink

Sources



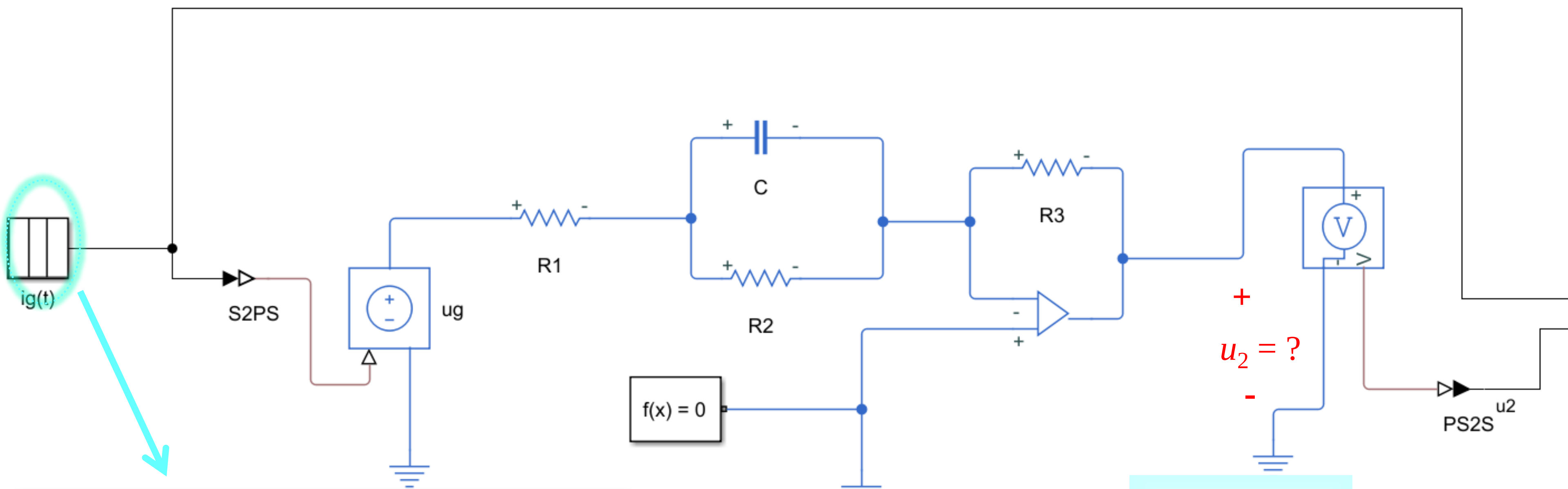
Repeating table (mask) (link)

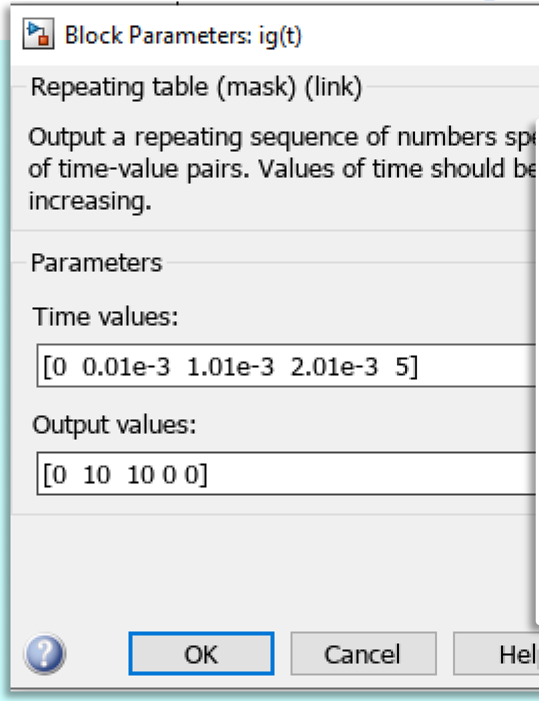
Output a repeating sequence of numbers specified in a table of time-value pairs. Values of time should be monotonically increasing.

Parameters

Time values: [0 1*1e-6 1.01*1e-6 5*1e-6] [0,1e-06,1.01e-06,5e-06]

Output values: [0 10 0 0] [0,10,0,0]





Repeating table (mask) (link)

Output a repeating sequence of numbers spe
of time-value pairs. Values of time should be
increasing.

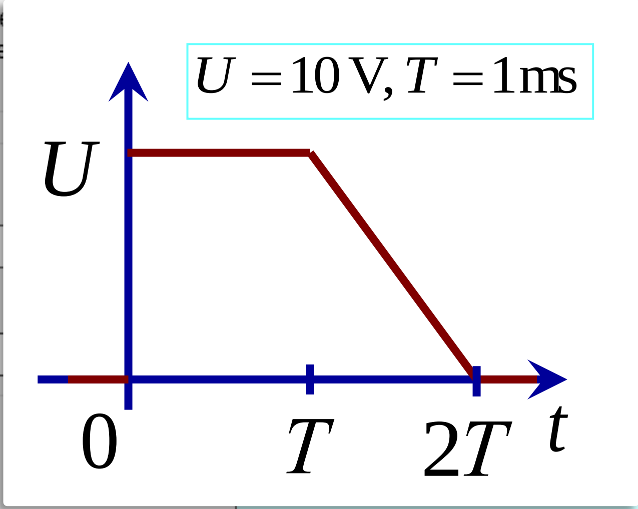
Parameters

Time values:

[0 0.01e-3 1.01e-3 2.01e-3 5]

Output values:

[0 10 10 0 0]



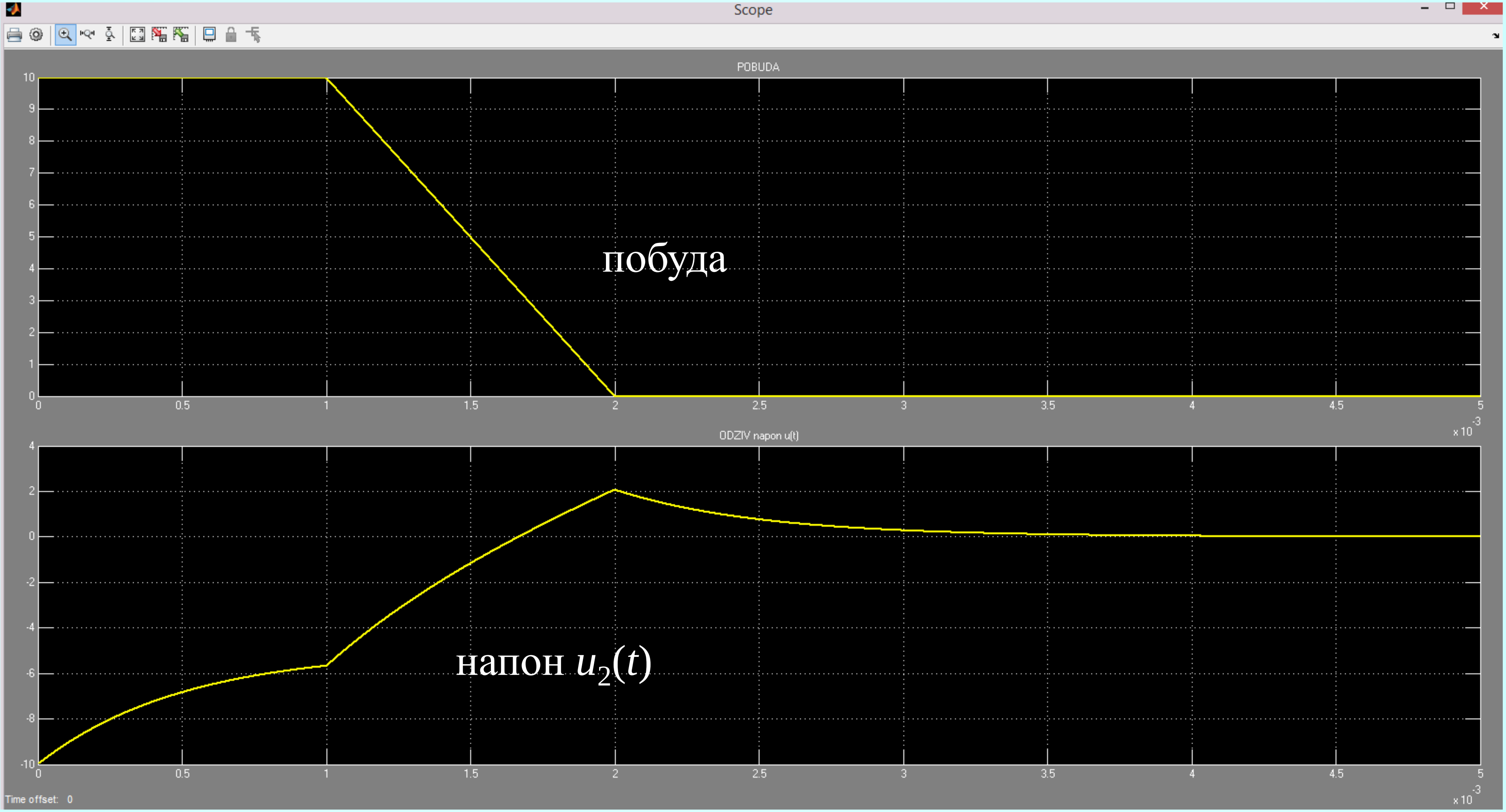
$U = 10\,\mathrm{V}, T = 1\,\mathrm{ms}$

$$u_2 = ?$$

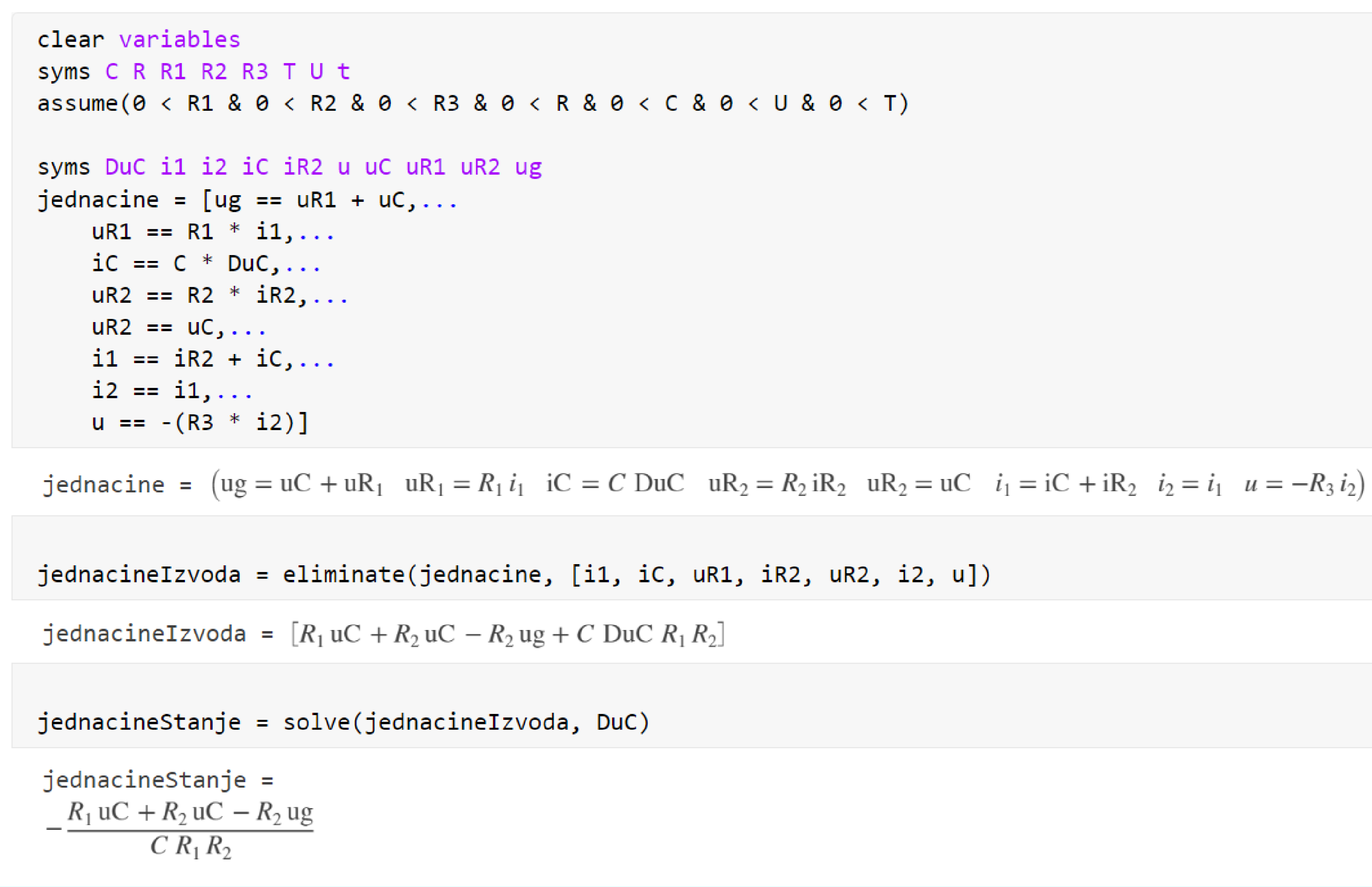
$$R_1 = R_2 = R_3 = 1\,\mathrm{k\Omega}$$

$$C_1 = 1\,\mu\mathrm{F}$$

MATLAB: Simscape
Foundation Library
Utilities
Simulink


Scope
POBUDA
10
9
8
7
6
5
4
3
2
1
0
0.5
1.5
2.5
3.5
4.5
x 10^-3
побуда
ODZIV napon u(t)
-2
-4
-6
-8
-10
напон $u_2(t)$
Time offset: 0

Једначине



```
clear variables
syms C R R1 R2 R3 T U t
assume(0 < R1 & 0 < R2 & 0 < R3 & 0 < R & 0 < C & 0 < U & 0 < T)

syms DuC i1 i2 iC iR2 u uC uR1 uR2 ug
jednacine = [ug == uR1 + uC,...
    uR1 == R1 * i1,...
    iC == C * DuC,...
    uR2 == R2 * iR2,...
    uR2 == uC,...
    i1 == iR2 + iC,...
    i2 == i1,...
    u == -(R3 * i2)]
```

jednacine = $\left(\mathrm{ug} = \mathrm{uC} + \mathrm{uR}_1 \quad \mathrm{uR}_1 = R_1\, i_1 \quad \mathrm{iC} = C\,\mathrm{DuC} \quad \mathrm{uR}_2 = R_2\,\mathrm{iR}_2 \quad \mathrm{uR}_2 = \mathrm{uC} \quad i_1 = \mathrm{iC} + \mathrm{iR}_2 \quad i_2 = i_1 \quad u = -R_3\, i_2\right)$

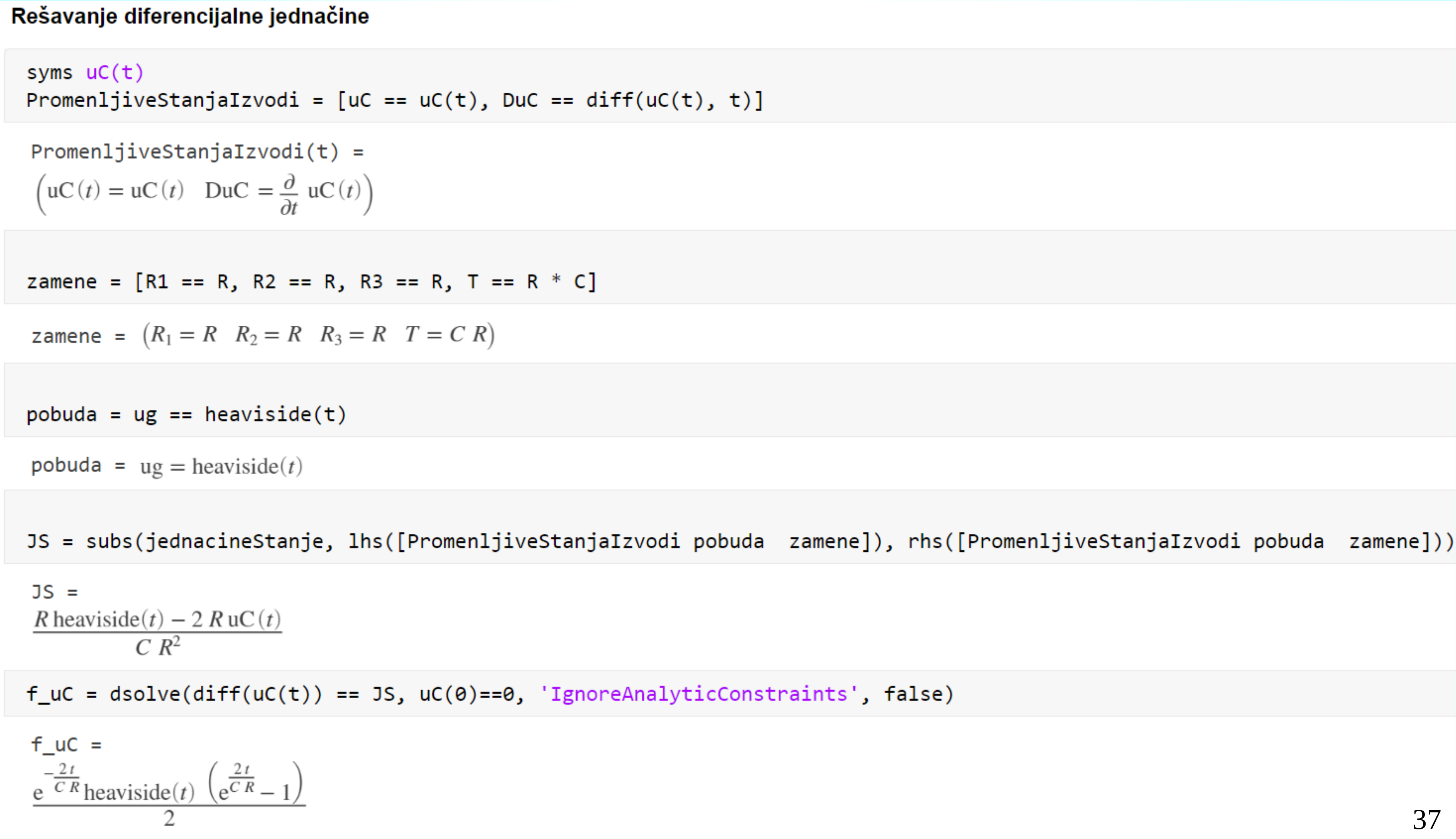
```
jednacineIzvoda = eliminate(jednacine, [i1, iC, uR1, iR2, uR2, i2, u])
```

jednacineIzvoda = $\left[R_1\,\mathrm{uC} + R_2\,\mathrm{uC} - R_2\,\mathrm{ug} + C\,\mathrm{DuC}\,R_1\,R_2\right]$

```
jednacineStanje = solve(jednacineIzvoda, DuC)
```

jednacineStanje =
$$-\frac{R_1\,\mathrm{uC} + R_2\,\mathrm{uC} - R_2\,\mathrm{ug}}{C\,R_1\,R_2}$$

Решавање диференцијалне једначине



Rešavanje diferencijalne jednačine

```
syms uC(t)
PromenljiveStanjaIzvodi = [uC == uC(t), DuC == diff(uC(t), t)]
```

PromenljiveStanjaIzvodi(t) =

$$\left(\mathrm{uC}(t) = \mathrm{uC}(t) \quad \mathrm{DuC} = \frac{\partial}{\partial t}\,\mathrm{uC}(t)\right)$$

```
zamene = [R1 == R, R2 == R, R3 == R, T == R * C]
```

zamene = $\left(R_1 = R \quad R_2 = R \quad R_3 = R \quad T = C\,R\right)$

```
pobuda = ug == heaviside(t)
```

pobuda = $\mathrm{ug} = \mathrm{heaviside}(t)$

```
JS = subs(jednacineStanje, lhs([PromenljiveStanjaIzvodi pobuda  zamene]), rhs([PromenljiveStanjaIzvodi pobuda  zamene]))
```

JS =

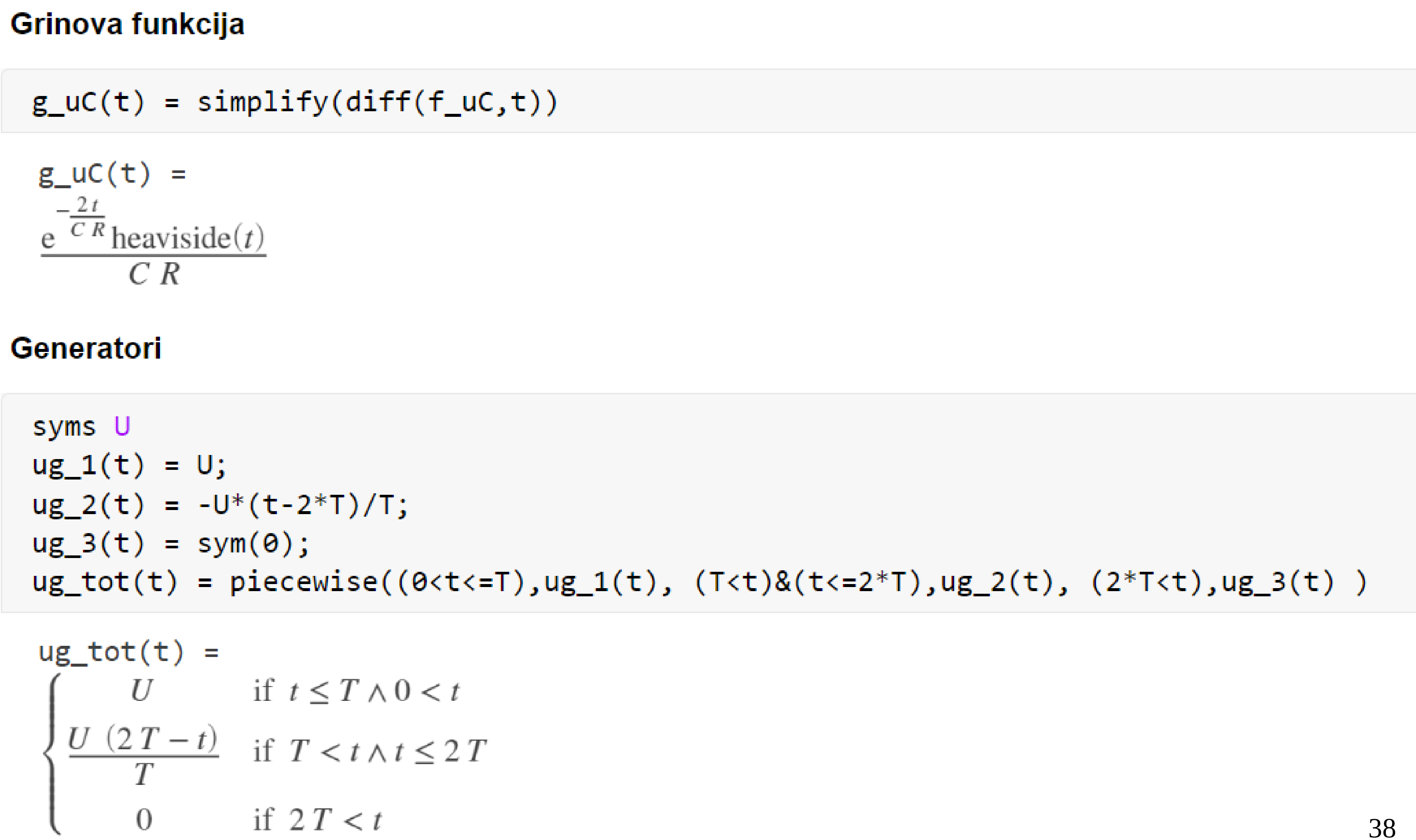
$$\frac{R\,\mathrm{heaviside}(t) - 2\,R\,\mathrm{uC}(t)}{C\,R^2}$$

```
f_uC = dsolve(diff(uC(t)) == JS, uC(0)==0, 'IgnoreAnalyticConstraints', false)
```

f_uC =

$$\frac{e^{-\frac{2\,t}{C\,R}}\,\mathrm{heaviside}(t)\,\left(e^{\frac{2\,t}{C\,R}} - 1\right)}{2}$$

Гринова функција и побуда



Grinova funkcija

```
g_uC(t) = simplify(diff(f_uC,t))
```

g_uC(t) =

$$\frac{e^{-\frac{2\,t}{C\,R}}\,\mathrm{heaviside}(t)}{C\,R}$$

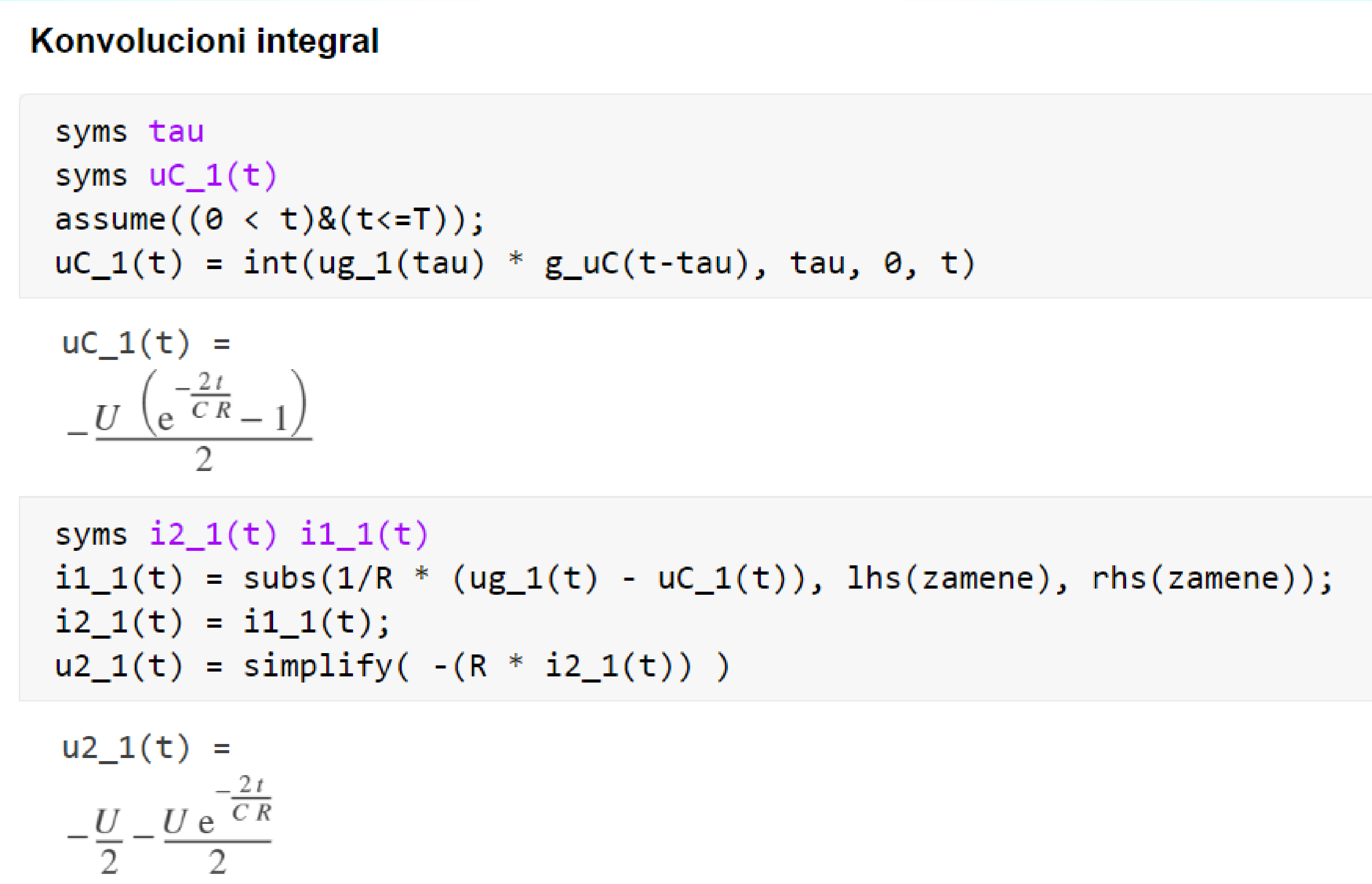
Generatori

```
syms U
ug_1(t) = U;
ug_2(t) = -U*(t-2*T)/T;
ug_3(t) = sym(0);
ug_tot(t) = piecewise((0<t<=T),ug_1(t), (T<t)&(t<=2*T),ug_2(t), (2*T<t),ug_3(t) )
```

ug_tot(t) =

$$\begin{cases} U & \text{if } t \le T \wedge 0 < t \\ \frac{U\,(2\,T-t)}{T} & \text{if } T < t \wedge t \le 2\,T \\ 0 & \text{if } 2\,T < t \end{cases}$$

Конволуциони интеграл (1)



Konvolucioni integral

```
syms tau
syms uC_1(t)
assume((0 < t)&(t<=T));
uC_1(t) = int(ug_1(tau) * g_uC(t-tau), tau, 0, t)
```

uC_1(t) =

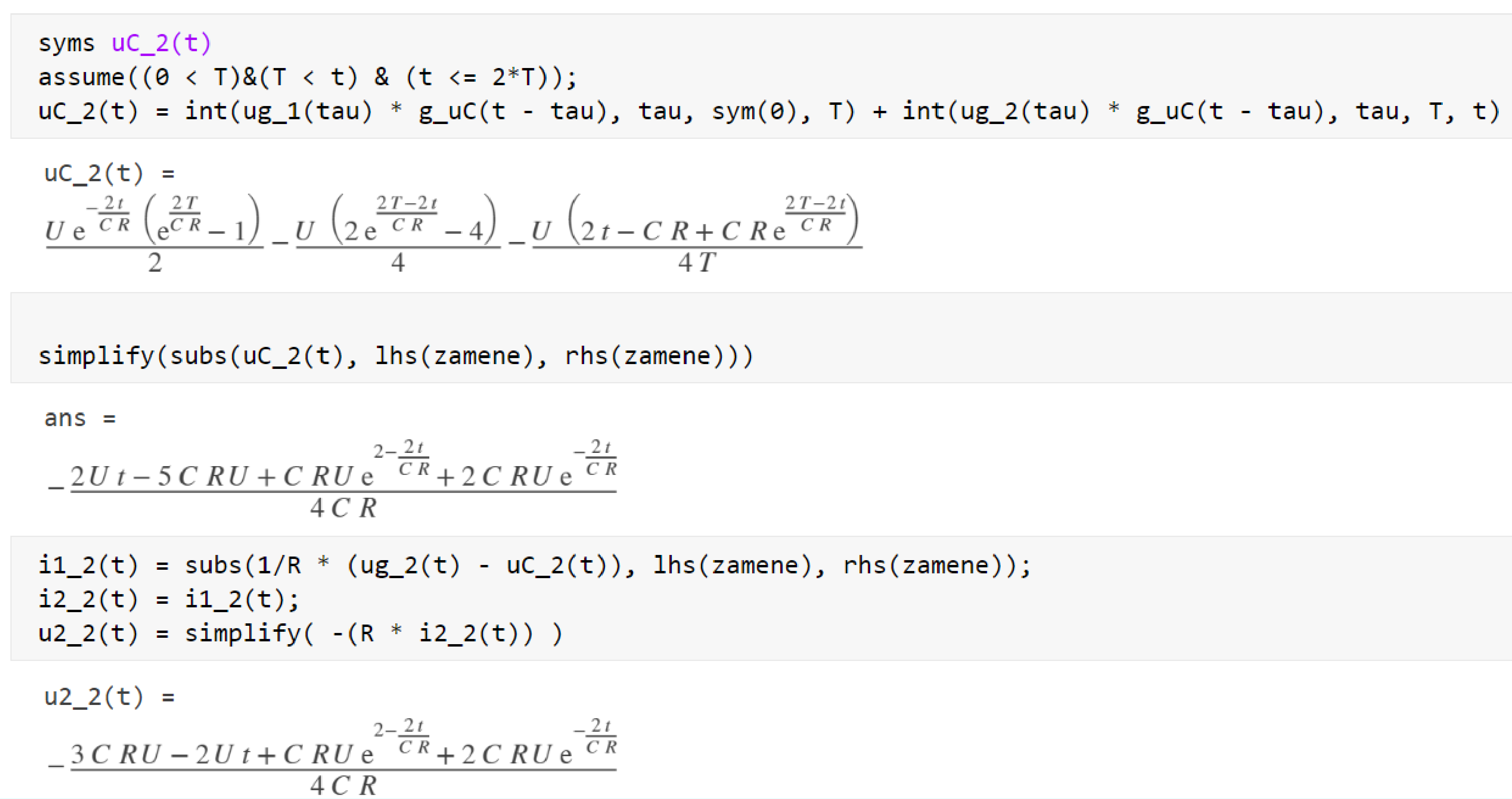
$$-\frac{U\left(e^{-\frac{2t}{CR}}-1\right)}{2}$$

```
syms i2_1(t) i1_1(t)
i1_1(t) = subs(1/R * (ug_1(t) - uC_1(t)), lhs(zamene), rhs(zamene));
i2_1(t) = i1_1(t);
u2_1(t) = simplify( -(R * i2_1(t)) )
```

u2_1(t) =

$$-\frac{U}{2}-\frac{U\,e^{-\frac{2t}{CR}}}{2}$$

Конволуциони интеграл (2)



```
syms uC_2(t)
assume((0 < T)&(T < t) & (t <= 2*T));
uC_2(t) = int(ug_1(tau) * g_uC(t - tau), tau, sym(0), T) + int(ug_2(tau) * g_uC(t - tau), tau, T, t)
```

uC_2(t) =

$$\frac{U\,e^{-\frac{2t}{CR}}\left(e^{\frac{2T}{CR}}-1\right)}{2}-\frac{U\left(2\,e^{\frac{2T-2t}{CR}}-4\right)}{4}-\frac{U\left(2t-C\,R+C\,R\,e^{\frac{2T-2t}{CR}}\right)}{4T}$$

```
simplify(subs(uC_2(t), lhs(zamene), rhs(zamene)))
```

ans =

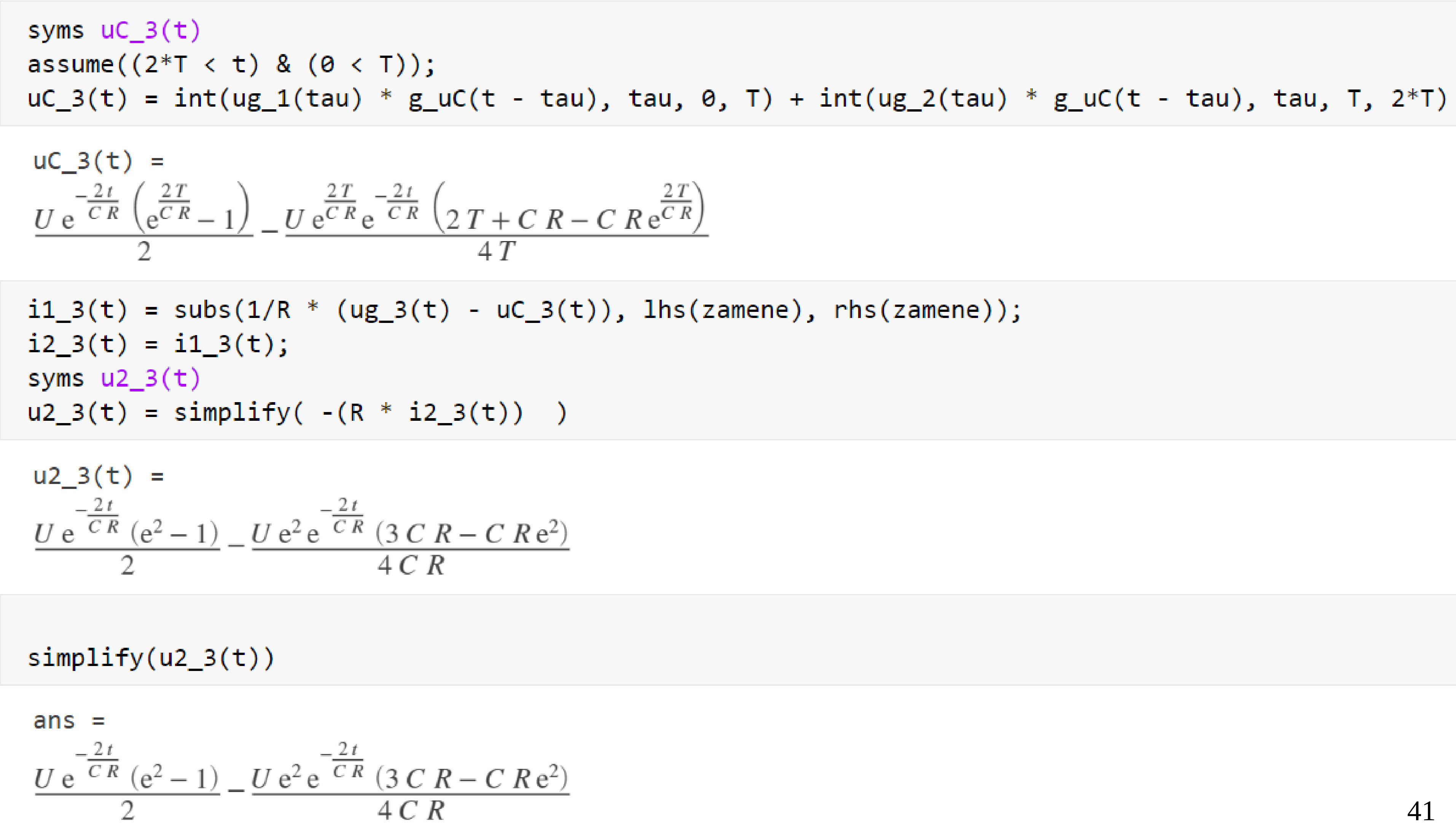
$$-\frac{2Ut-5\,C\,RU+C\,RU\,e^{2-\frac{2t}{CR}}+2\,C\,RU\,e^{-\frac{2t}{CR}}}{4\,C\,R}$$

```
i1_2(t) = subs(1/R * (ug_2(t) - uC_2(t)), lhs(zamene), rhs(zamene));
i2_2(t) = i1_2(t);
u2_2(t) = simplify( -(R * i2_2(t)) )
```

u2_2(t) =

$$-\frac{3\,C\,RU-2Ut+C\,RU\,e^{2-\frac{2t}{CR}}+2\,C\,RU\,e^{-\frac{2t}{CR}}}{4\,C\,R}$$

Конволуциони интеграл (3)



```
syms uC_3(t)
assume((2*T < t) & (0 < T));
uC_3(t) = int(ug_1(tau) * g_uC(t - tau), tau, 0, T) + int(ug_2(tau) * g_uC(t - tau), tau, T, 2*T)
```

uC_3(t) =

$$\frac{U\,e^{-\frac{2\,t}{C\,R}}\left(e^{\frac{2\,T}{C\,R}}-1\right)}{2}-\frac{U\,e^{\frac{2\,T}{C\,R}}\,e^{-\frac{2\,t}{C\,R}}\left(2\,T+C\,R-C\,R\,e^{\frac{2\,T}{C\,R}}\right)}{4\,T}$$

```
i1_3(t) = subs(1/R * (ug_3(t) - uC_3(t)), lhs(zamene), rhs(zamene));
i2_3(t) = i1_3(t);
syms u2_3(t)
u2_3(t) = simplify( -(R * i2_3(t))  )
```

u2_3(t) =

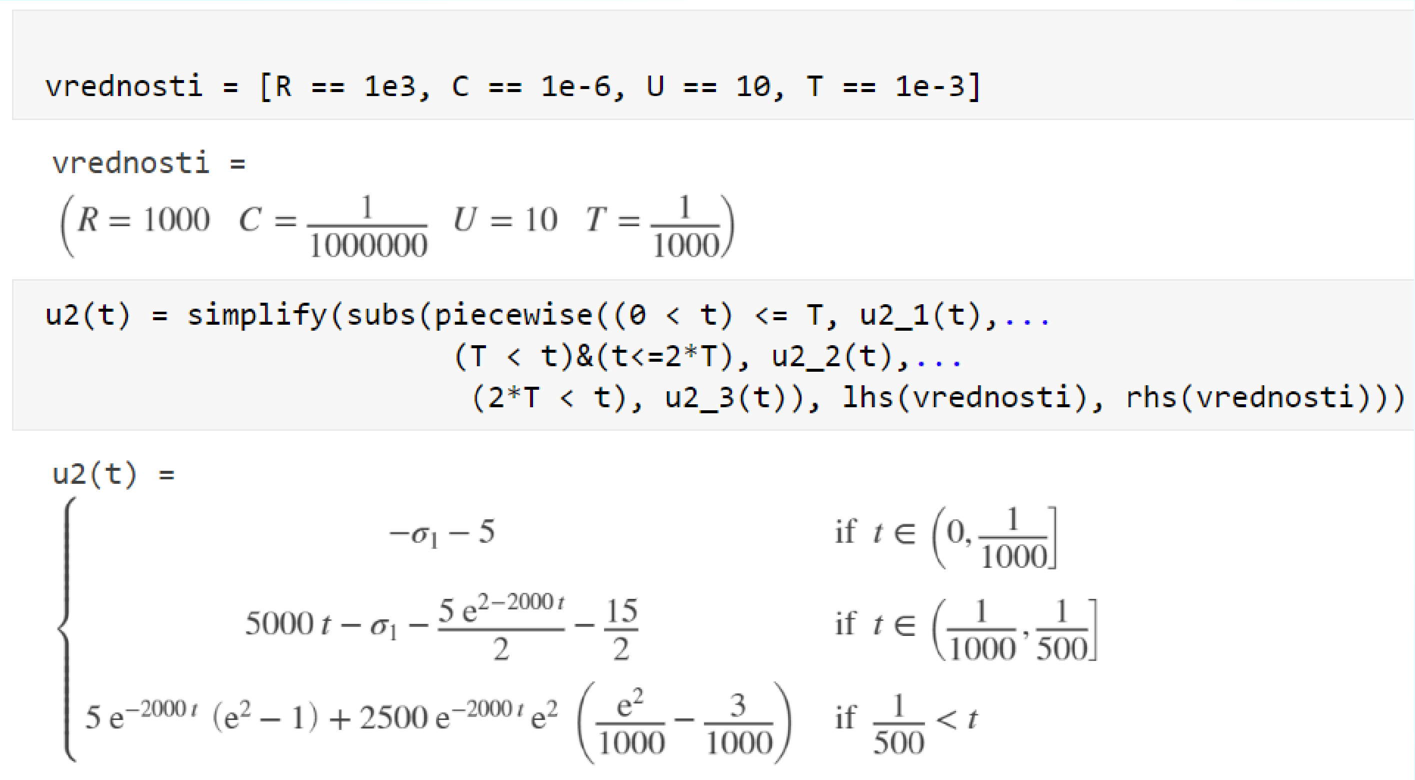
$$\frac{U\,e^{-\frac{2\,t}{C\,R}}\,(e^2-1)}{2}-\frac{U\,e^2\,e^{-\frac{2\,t}{C\,R}}\,(3\,C\,R-C\,R\,e^2)}{4\,C\,R}$$

```
simplify(u2_3(t))
```

ans =

$$\frac{U\,e^{-\frac{2\,t}{C\,R}}\,(e^2-1)}{2}-\frac{U\,e^2\,e^{-\frac{2\,t}{C\,R}}\,(3\,C\,R-C\,R\,e^2)}{4\,C\,R}$$

Вредности



```
vrednosti = [R == 1e3, C == 1e-6, U == 10, T == 1e-3]
```

vrednosti =

$$\left(R = 1000 \quad C = \frac{1}{1000000} \quad U = 10 \quad T = \frac{1}{1000}\right)$$

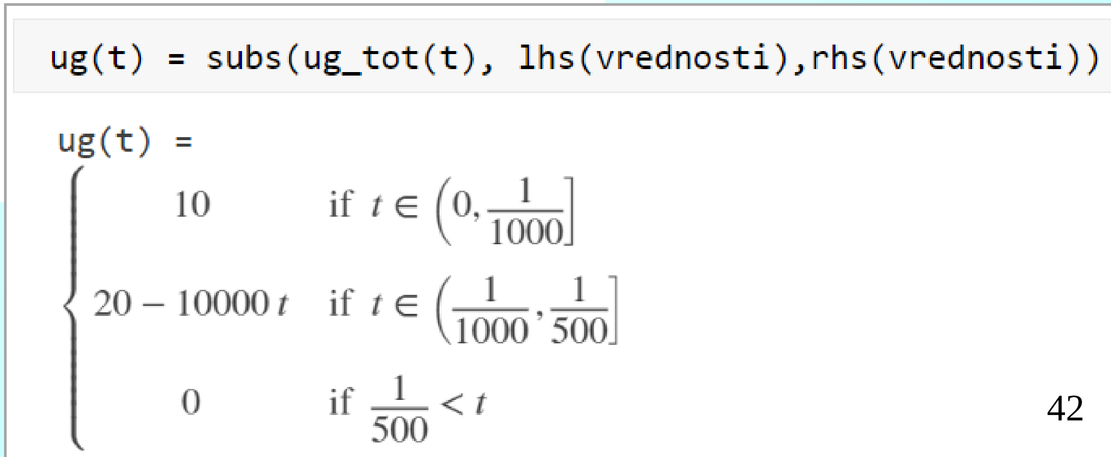
```
u2(t) = simplify(subs(piecewise((0 < t) <= T, u2_1(t),...
                      (T < t)&(t<=2*T), u2_2(t),...
                       (2*T < t), u2_3(t)), lhs(vrednosti), rhs(vrednosti)))
```

u2(t) =

$$\begin{cases} -\sigma_1 - 5 & \text{if } t \in \left(0, \frac{1}{1000}\right] \\ 5000\,t - \sigma_1 - \frac{5\,e^{2-2000\,t}}{2} - \frac{15}{2} & \text{if } t \in \left(\frac{1}{1000}, \frac{1}{500}\right] \\ 5\,e^{-2000\,t}\,(e^2 - 1) + 2500\,e^{-2000\,t}\,e^2\left(\frac{e^2}{1000} - \frac{3}{1000}\right) & \text{if } \frac{1}{500} < t \end{cases}$$

where

$$\sigma_1 = 5\,e^{-2000\,t}$$

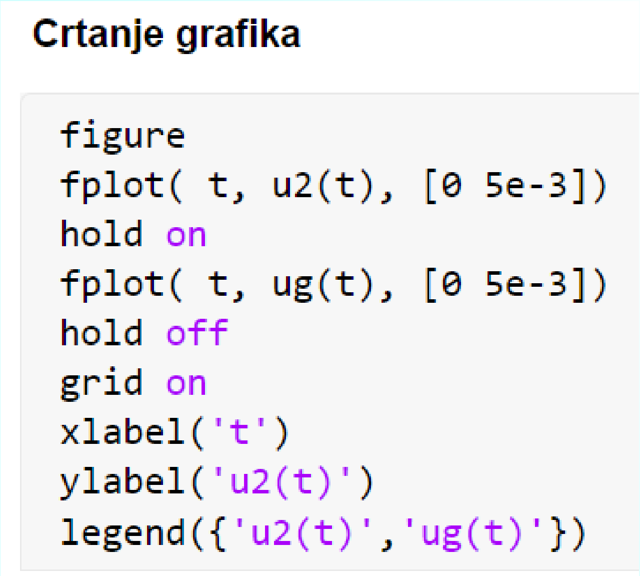


```
ug(t) = subs(ug_tot(t), lhs(vrednosti),rhs(vrednosti))
```

ug(t) =

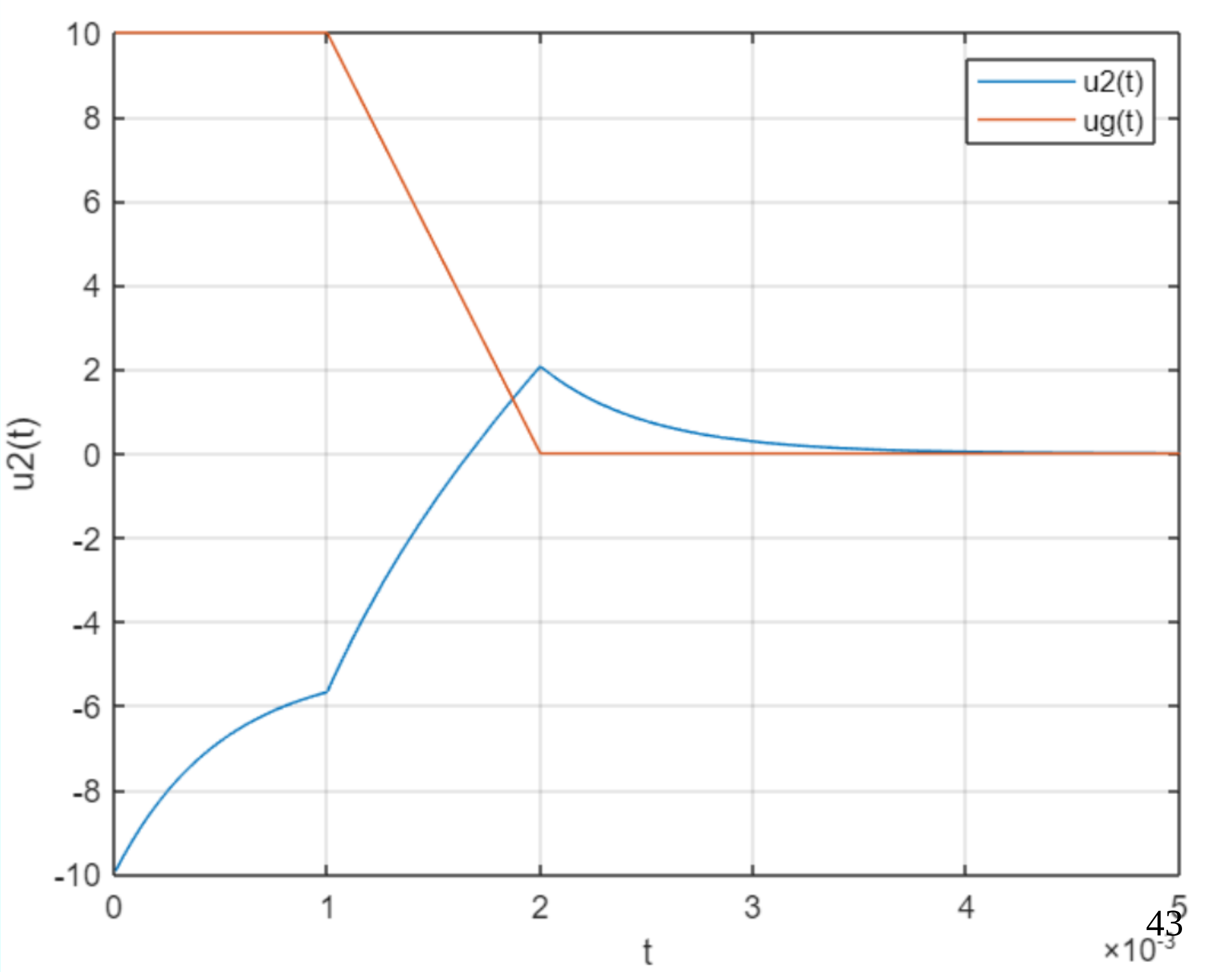
$$\begin{cases} 10 & \text{if } t \in \left(0, \frac{1}{1000}\right] \\ 20 - 10000\,t & \text{if } t \in \left(\frac{1}{1000}, \frac{1}{500}\right] \\ 0 & \text{if } \frac{1}{500} < t \end{cases}$$

Цртање графика одзива и побуде



Crtanje grafika

```
figure
fplot( t, u2(t), [0 5e-3])
hold on
fplot( t, ug(t), [0 5e-3])
hold off
grid on
xlabel('t')
ylabel('u2(t)')
legend({'u2(t)','ug(t)'})
```


u2(t)
t
×10⁻³
u2(t)
ug(t)