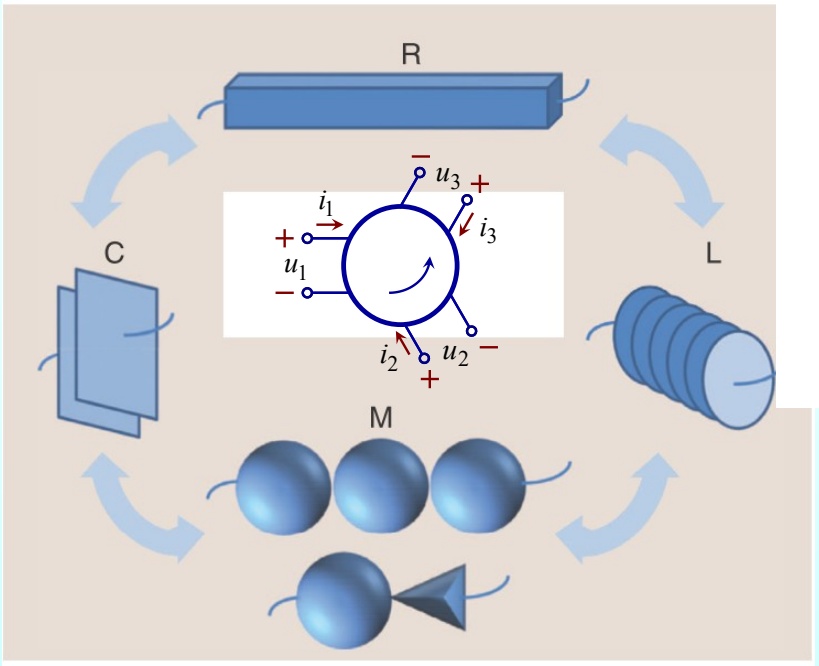
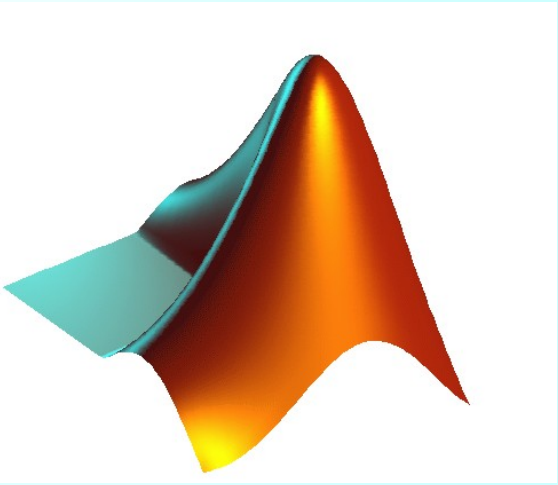
Практикум из рачунарске анализе кола

1. Увод





Милка Потребић Иваниш

Никола Баста

Практикум из рачунарске анализе кола 13Е072ПРК3, 19Е072ПРК

- **Настава се изводи према текућем студијском програму који је Електротехнички факултет акредитовао** http://www.etf.bg.ac.rs
- Студијски програм: **Електротехника и рачунарство**
- Изборно подручје (модул): **Физичка електроника, Сигнали и системи, Телекомуникације и информационе технологије**
- **Сарджај**: Рачунарске симулације електричних кола помоћу програма MATLAB/Simulink или Scilab/Xcos, LTspice и QucsStudio. Симболичко решавање кола програмима MATLAB/Symbolic Math или Symbolic Python и Maxima.

- **Наставници.** Др **Милка Потребић Иваниш**, редовни професор, соба 64 (milka_potrebic@etf.rs);
  Др **Никола Баста**, доцент, соба 80 (nbasta@etf.rs).
- **Сарадници:** мс. инж. **Ања Ковачевић**, асистент, дипл. инж. **Анђела Нинковић**, сарадник у настави*.

- Белешке са предавања и лаб. вежби, примери испитних задатака и питања, **http://tek.etf.rs**
- **Додатни материјали и консултације биће организовани преко канала на MS Teams платформи** (линкови за канале су на адреси **http://tek.etf.rs**)
- **Начин одржавања наставе.** Рачунски центар.
- **Пратити** званично представљање предмета и објављивање општих службених порука на адреси **http://tek.etf. rs/**

Начин полагања

- **ПРЕДИСПИТНЕ ОБАВЕЗЕ (70%) + ИСПИТ (30%) = УКУПНО (100%)**
- **Предиспитне обавезе (MATLAB)** реализују се полагањем колоквијума у РЦ, израдом домаћег задатка са усменом одбраном, или поенима освојеним кроз активности на часу – израда једноставнијих задатака у вредности од 5 до 10 испитних поена.
- Добровољна израда **MATLAB Onramp** теста (самостални пролазак кроз основни MATLAB курс у слободно време) носи додатних 5 бонус поена.
- Поени са свих активности се сабирају и максимално се може освојити 70%.

- **Испит** подразумева израду пројекта и његову одбрану, укупно 30%.
- Одбрана пројекта може се реализовати у свим испитним роковима. Израда пројеката се може реализовати у наведеним алтернативним софтверским алатима.
- За заинтересоване студенте може се обезбедити предрок.
- Поред класичних тема за пројекте од ове године нове теме су: Симулатори кола на Интернету. Могућност примене вештачке интелигенције у решавању електричних кола.
- *Напомена: Домаћи задаци и пројекти се могу радити у тиму од два студента, али се бране индивидуално.*

Рачунарски (софтверски) алати

- *Mathematica*, WolframAlpha, MuPAD, **Maxima**, **SymPy**, **SymPy Live**, **SymPy Gamma**
- MATLAB, **Scilab**, **Octave**, **FreeMat**, **Julia**
- LTspice, **QucsStudio**, **ngspice**, **Xyce**, **XCircuit**
- **Python**, MathCAD, MAPLE, GeoGebra, Sage
- Symbolab, SpeQ Mathematics, meta-calculator, Desmos, …
- Android apk: NCalc+, CalcES, SymJa CAS, Scientific Calculator Pro, CYZSoft Scientific Calculator Plus

Free/Libre Open Source Software (FLOSS)

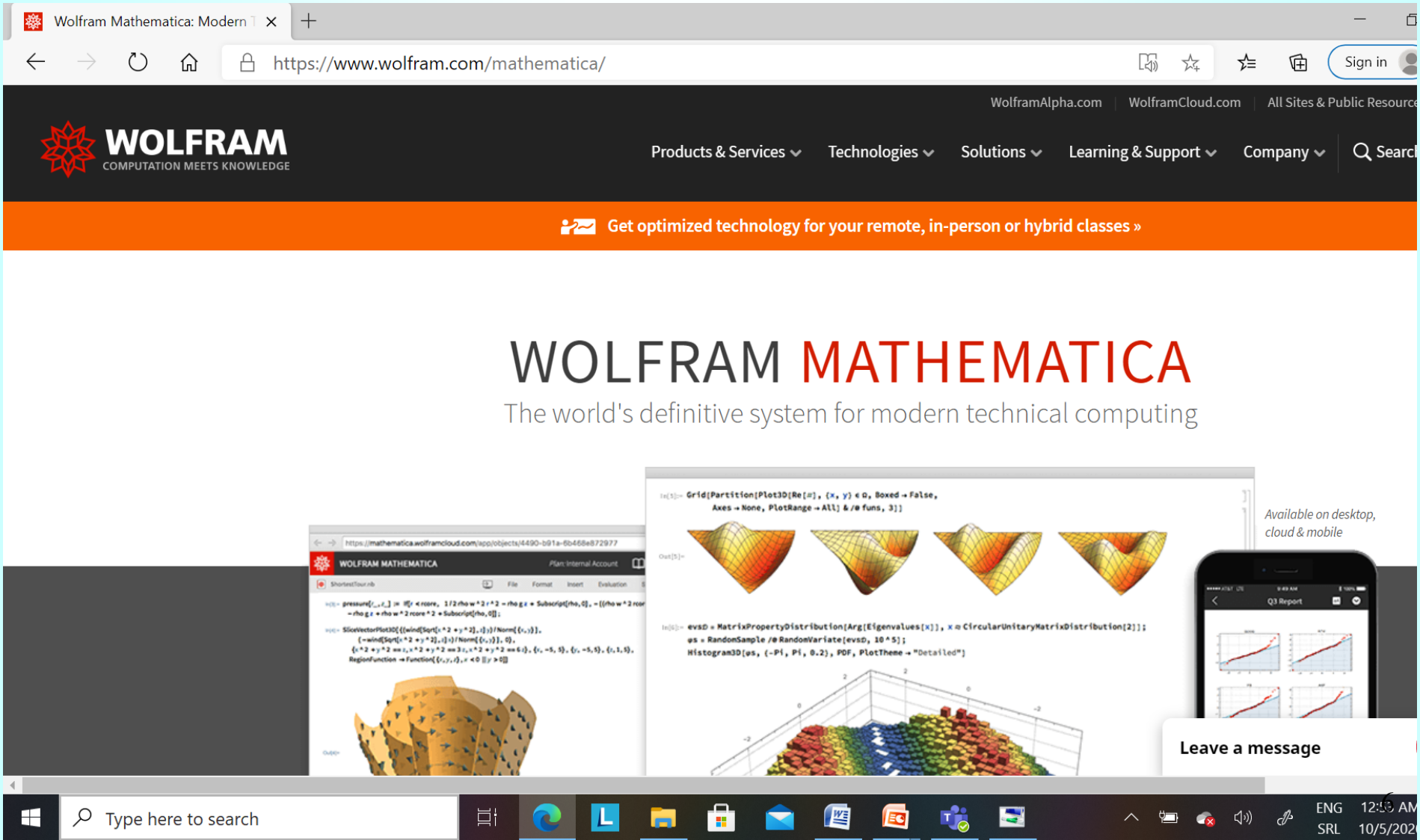
Моделовање електричних кола

- PSpice, ngSpice, TINA-TI, Micro-Cap, Plexim, 5SpiceXcos, SciLab, …
- **Online Circuit Simulator**: DoCircuits, Circuit Lab, PartSim, EasyEda, Multisim, PartQuest (SystemVision Cloud)...
- **Android circuit simulator:**
  Electric Circuit Studio,
  Circuit Safari SPICE Simulator...

др Милка Потребић Иваниш, ред. проф., др Никола Баста, доцент, 13E072PRK3, 19E072PRK, Универзитет у Београду – Електротехнички факултет, 2025.

Mathematica

https://www.wolfram.com/mathematica/



Mathematica free on every Raspberry Pi!

https://www.wolfram.com/raspberry-pi/

www.wolframalpha.com

Give your brain a quick workout with Wolfram Problem Generator.

WolframAlpha computational... knowledge engine

y' +1000 * y = 10000, y(0)=-10

Examples Random

Input:

$\{y'(x) + 1000\, y(x) = 10\,000,\ y(0) = -10\}$

ODE classification:

first-order linear ordinary differential equation

Alternate forms:

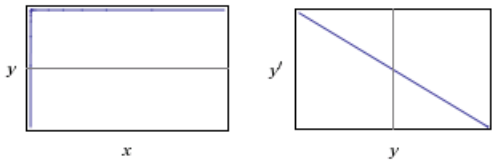
$\{y'(x) = 10\,000 - 1000\, y(x),\ y(0) = -10\}$

$\{y'(x) + 1000\,(y(x) - 10) = 0,\ y(0) = -10\}$

Differential equation solution: Approximate form | Step-by-step solution

$y(x) = 10 - 20\, e^{-1000\, x}$

Plots of the solution:



New to Wolfram|Alpha?

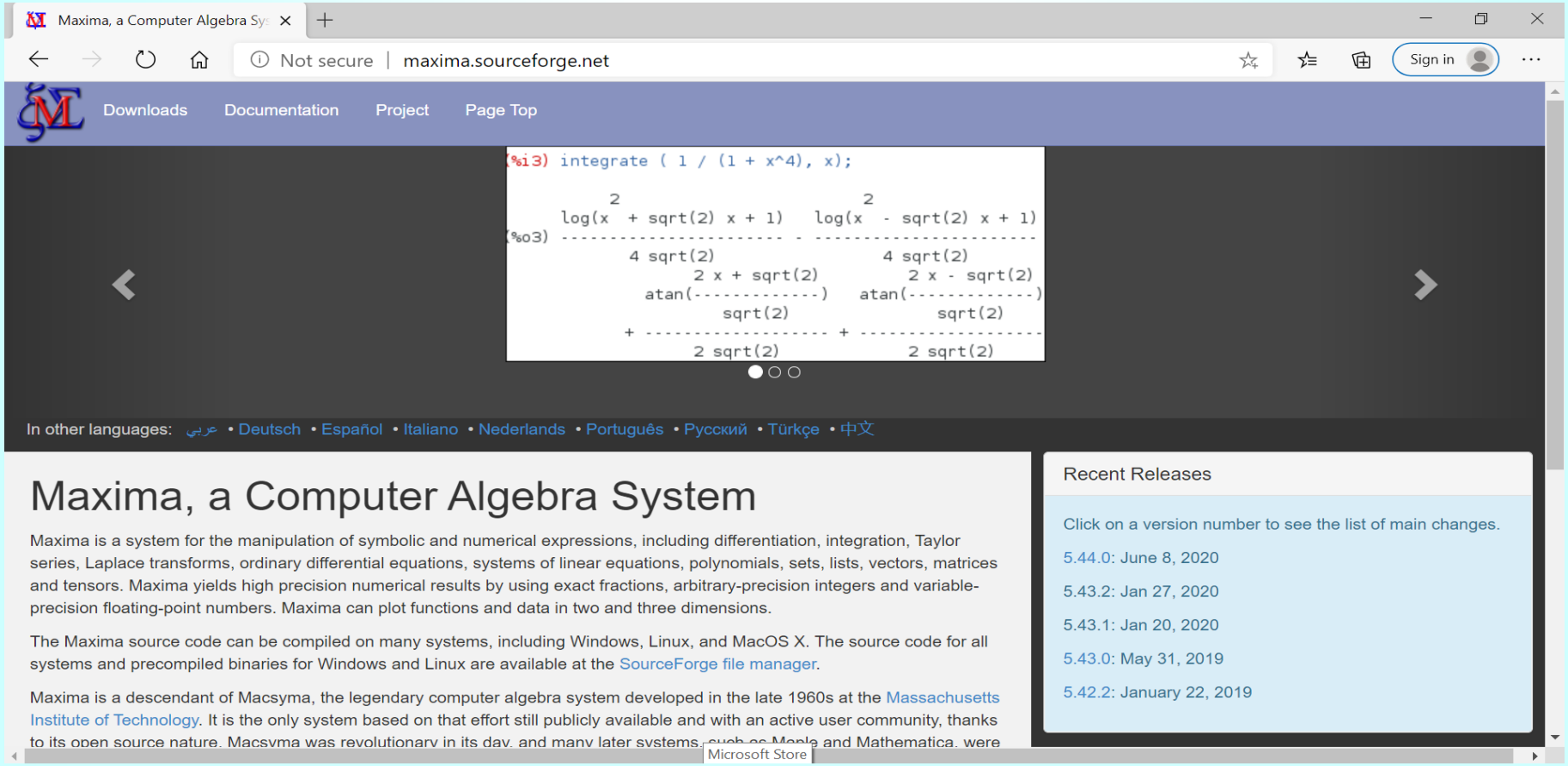
Take the Tour >>

New! Wolfram Problem Generator

Need a hint?

Maxima

http://maxima.sourceforge.net/



Maxima on line

http://maxima.cesga.es/

Servicio proporcionado por CESGA

Maxima on line

Help: Español, English Galego

```
expand((x-2)^3*(x+1/3)^2);

solve(x^2-x+2=0);

invert(matrix([2,3,1], [a,0,0], [1,4,8]));

integrate(x * sin(x), x);

draw3d(explicit(x^2+y^2,x,-1,1,y,-1,1));
```

Clic Clear

AVISO LEGAL

(%i1) `expand((x-2)^3*(x+1/3)^2);`

(%o1) $x^5-\frac{16\,x^4}{3}+\frac{73\,x^3}{9}-\frac{2\,x^2}{3}-4\,x-\frac{8}{9}$

(%i2) `solve(x^2-x+2=0);`

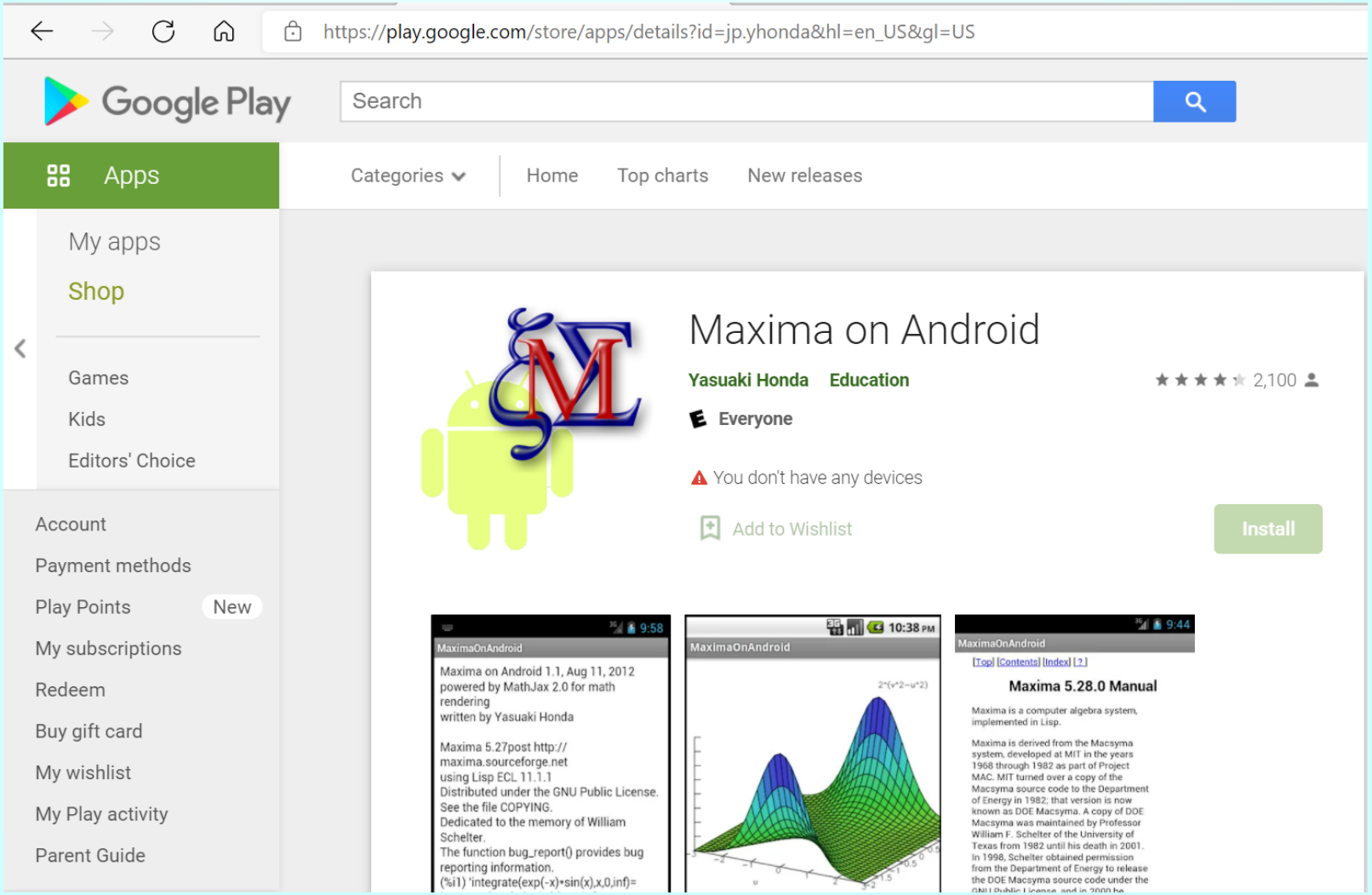
(%o2) $\left[x=-\frac{\sqrt{7}\,i-1}{2},x=\frac{\sqrt{7}\,i+1}{2}\right]$

(%i3) `invert(matrix([2,3,1], [a,0,0], [1,4,8]));`

(%o3) $\begin{pmatrix} 0 & \frac{1}{a} & 0 \\ \frac{2}{5} & -\frac{3}{4\,a} & -\frac{1}{20} \\ -\frac{1}{5} & \frac{1}{4\,a} & \frac{3}{20} \end{pmatrix}$

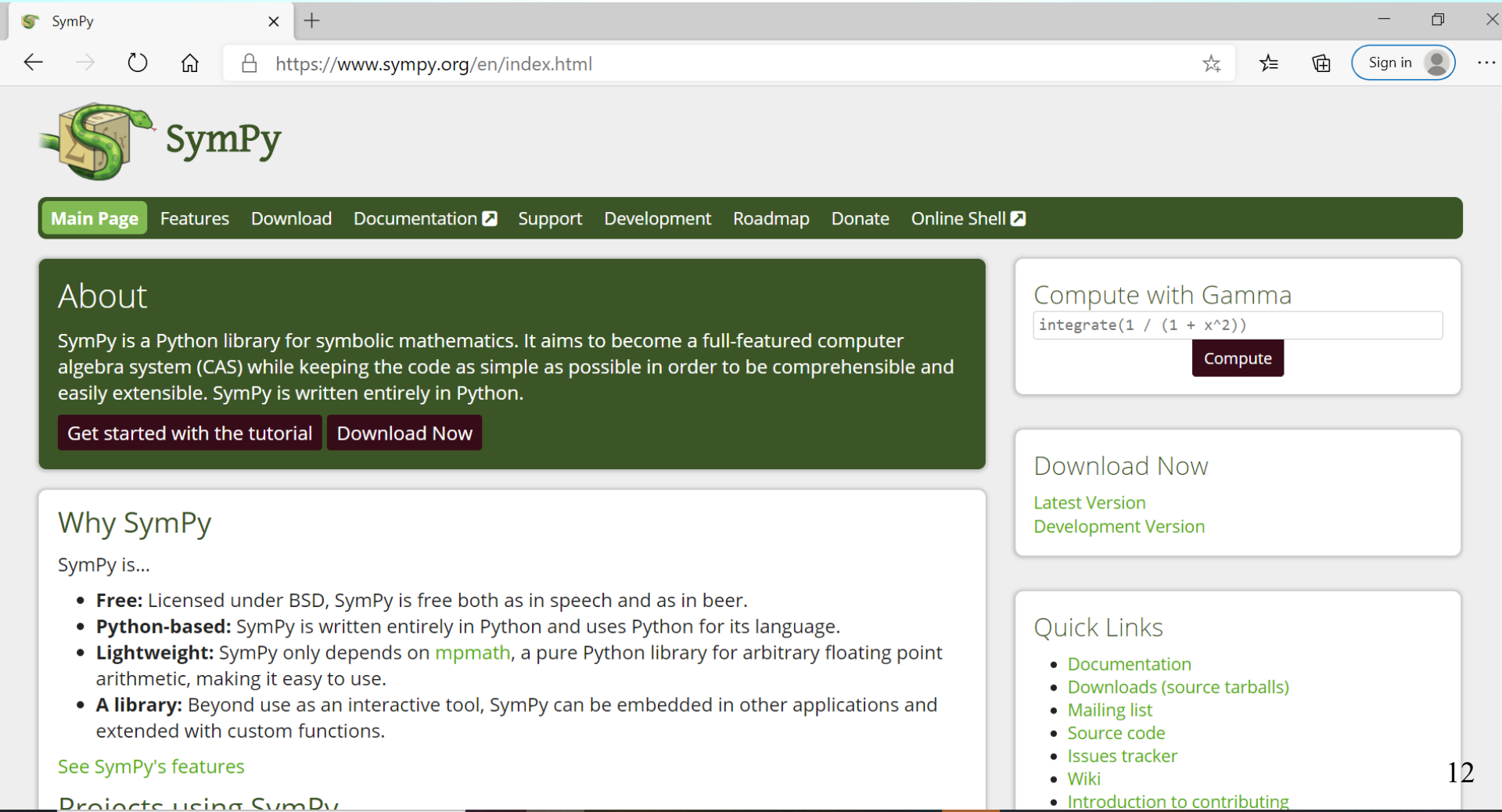
Maxima on Android

https://play.google.com/store/apps/details?id=jp.yhonda&hl=en_US&gl=US



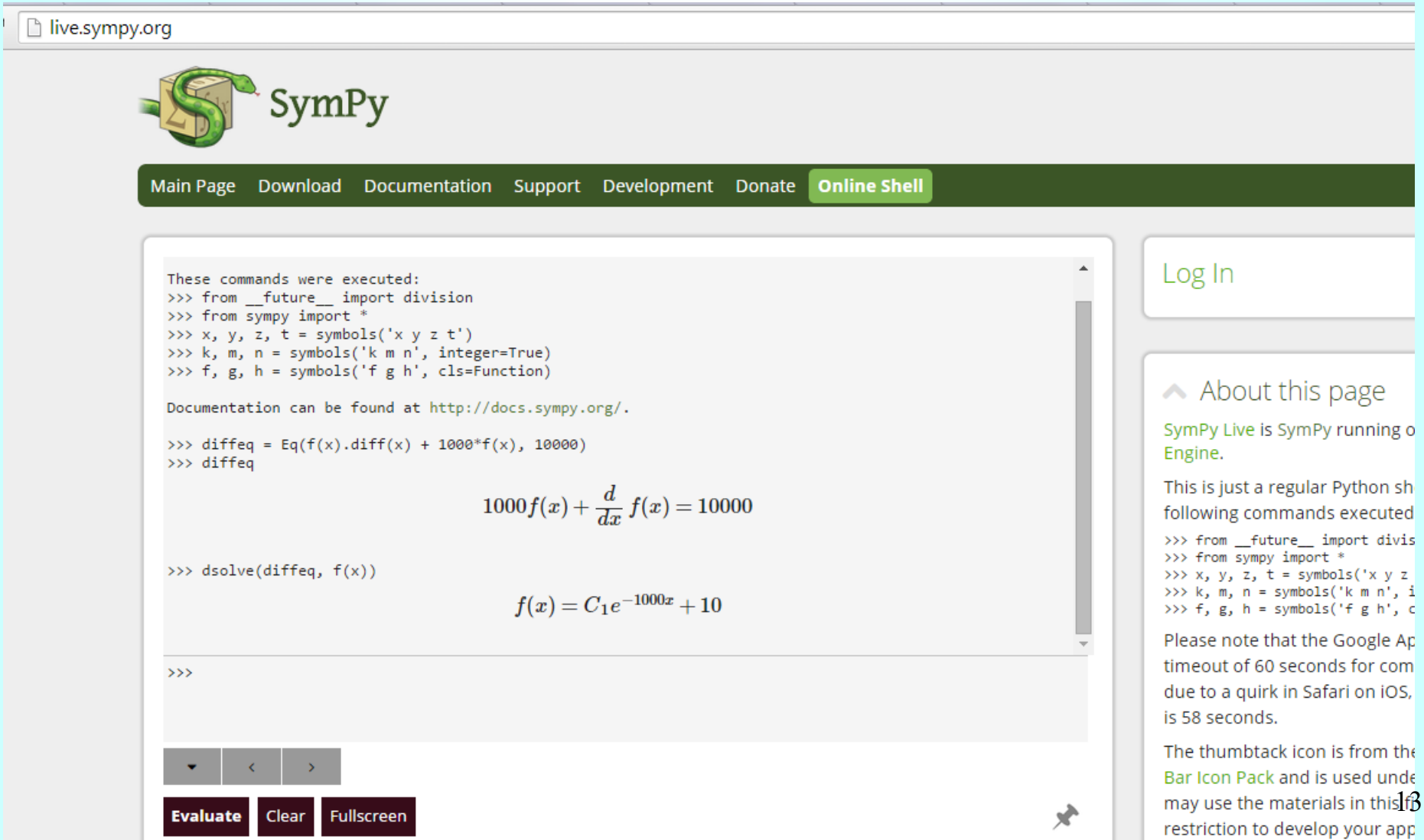
SymPy

www.sympy.org/



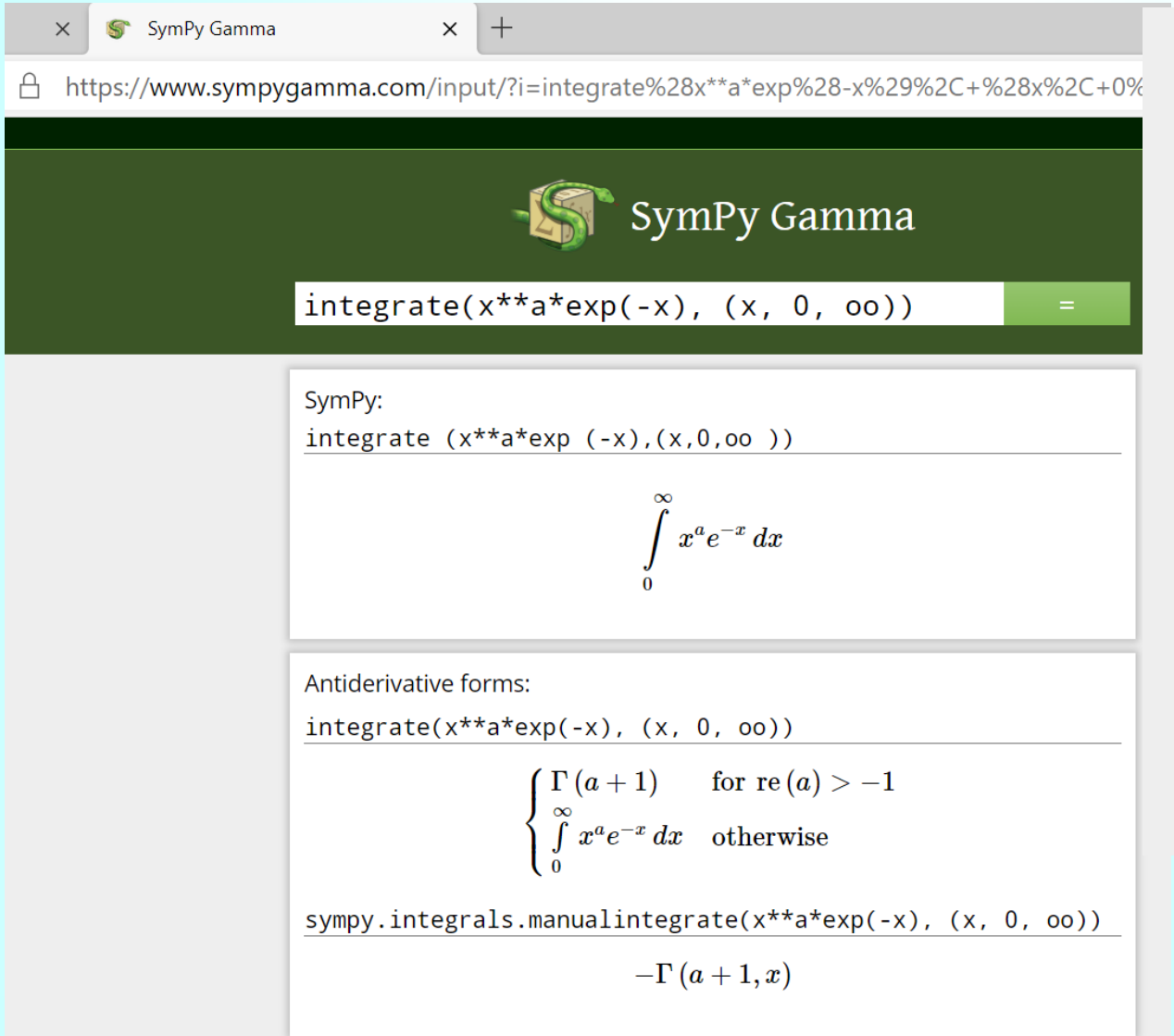
SymPy live

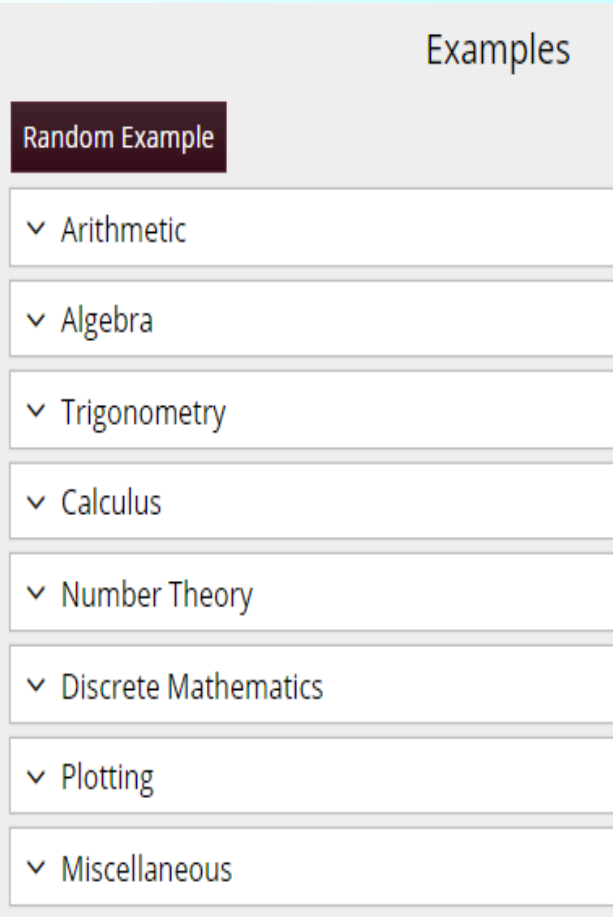
live.sympy.org

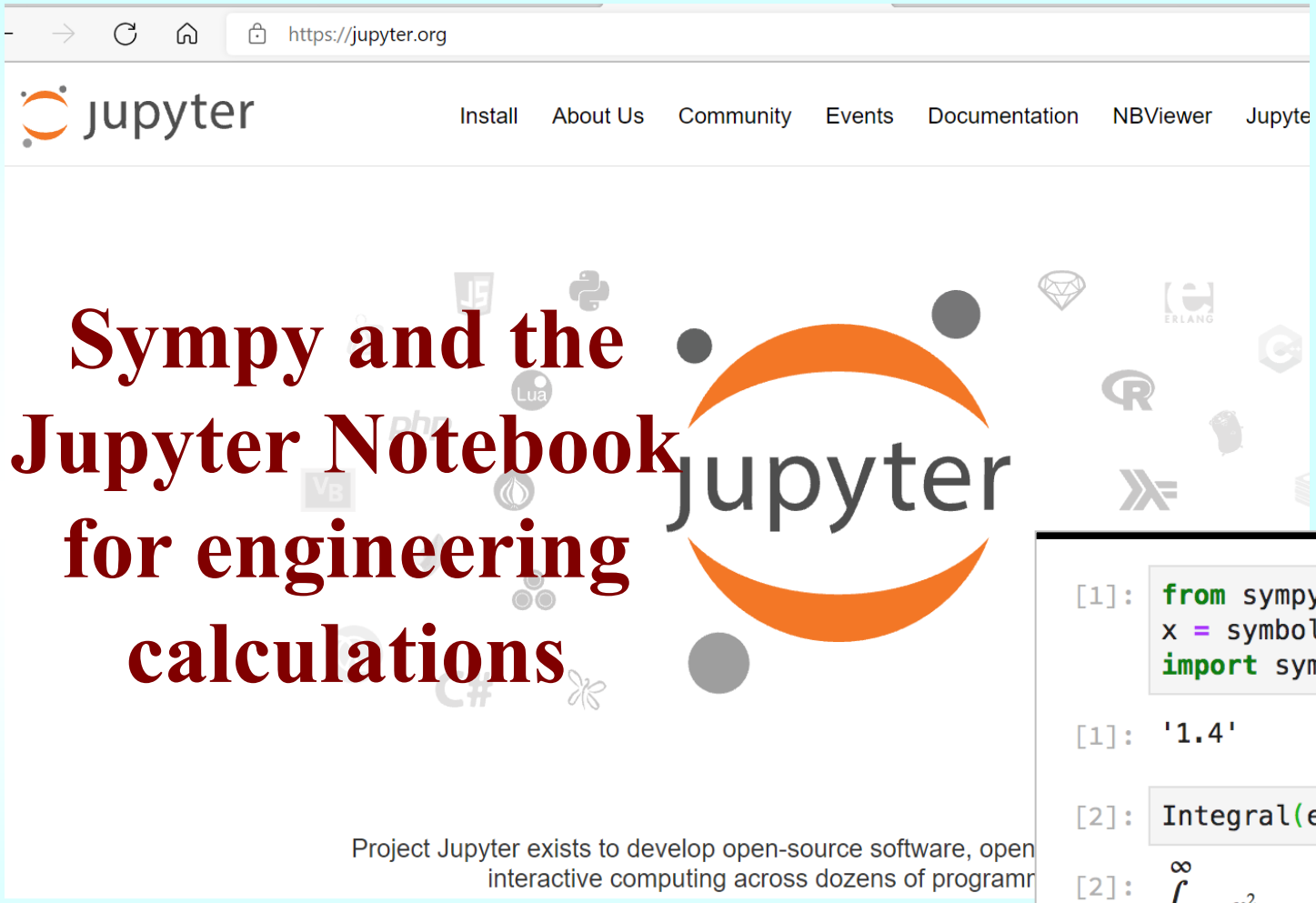


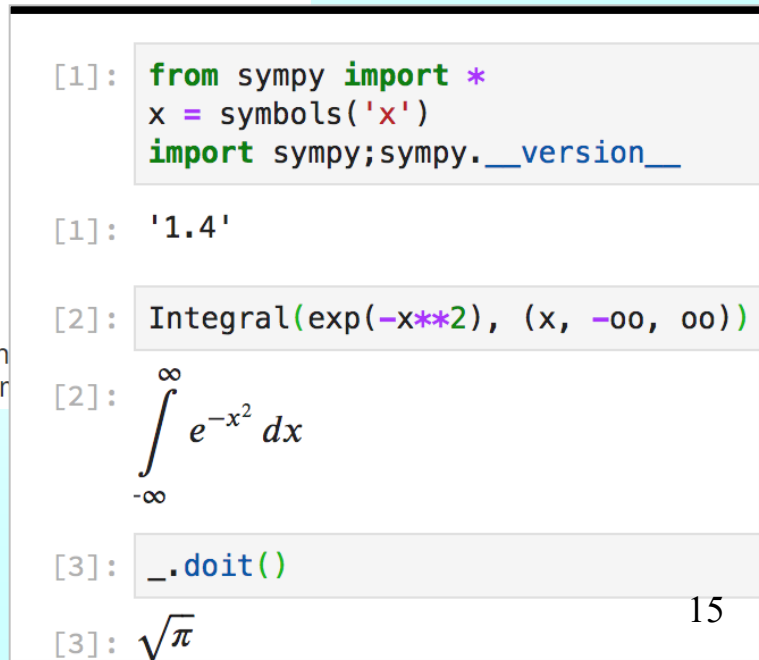
SymPy Gamma

https://www.sympygamma.com/

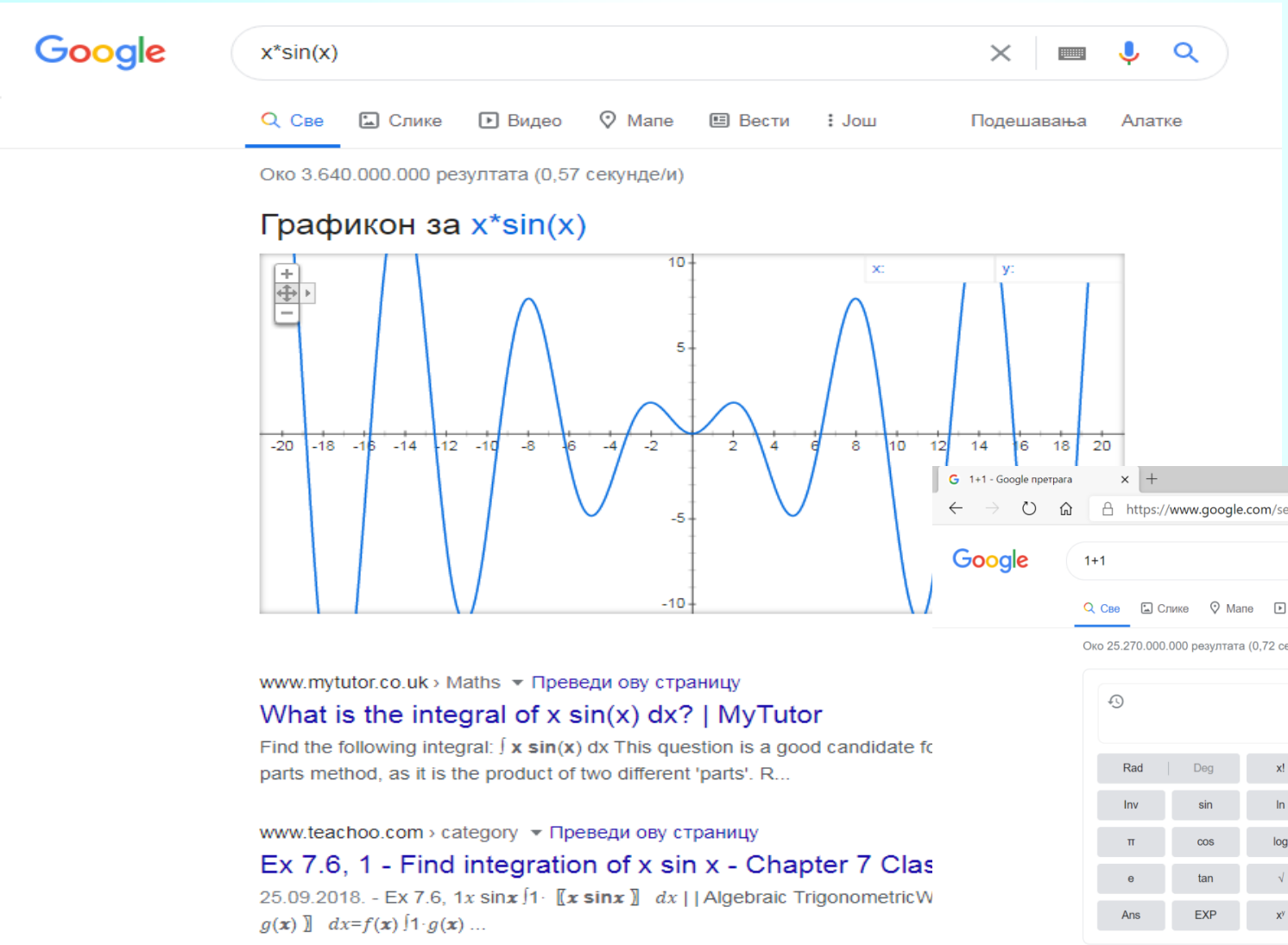


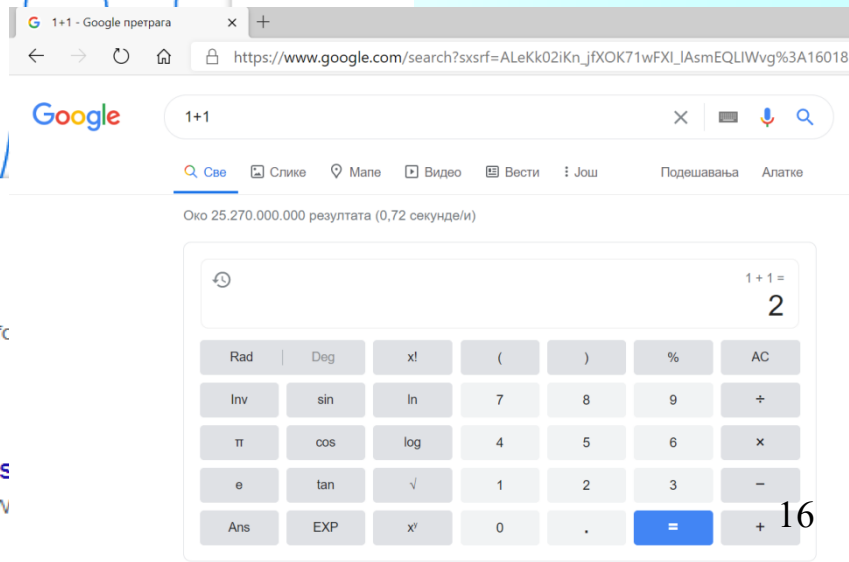



https://jupyter.org
jupyter
Install
About Us
Community
Events
Documentation
NBViewer
Jupyte
Sympy and the Jupyter Notebook for engineering calculations
jupyter
Project Jupyter exists to develop open-source software, open
interactive computing across dozens of programr


[1]:
from sympy import *
x = symbols('x')
import sympy;sympy.__version__
[1]:
'1.4'
[2]:
Integral(exp(-x**2), (x, -oo, oo))
[2]:
$\int_{-\infty}^{\infty} e^{-x^2}\,dx$
[3]:
_.doit()
[3]:
$\sqrt{\pi}$

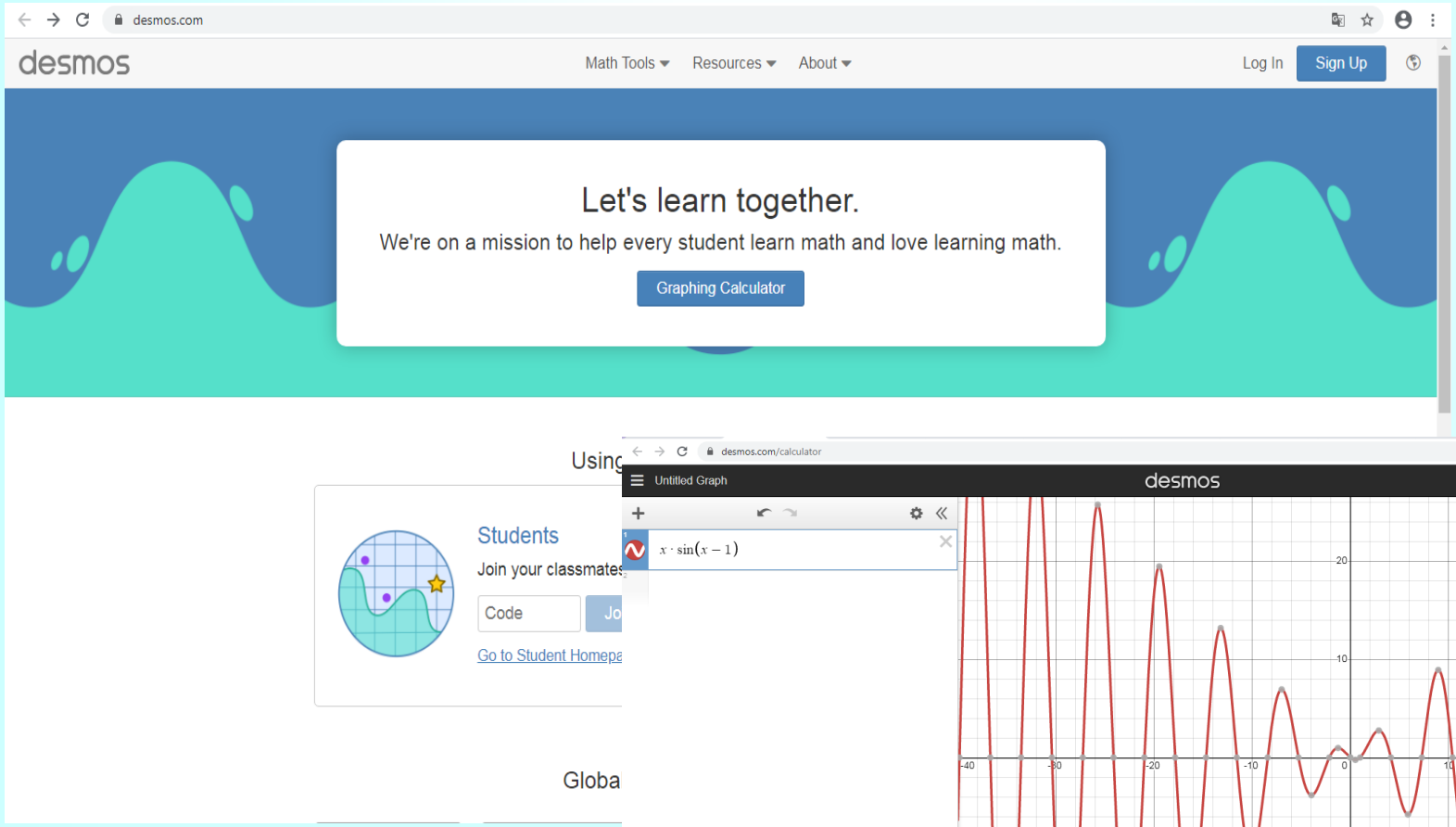
Google калкулатор

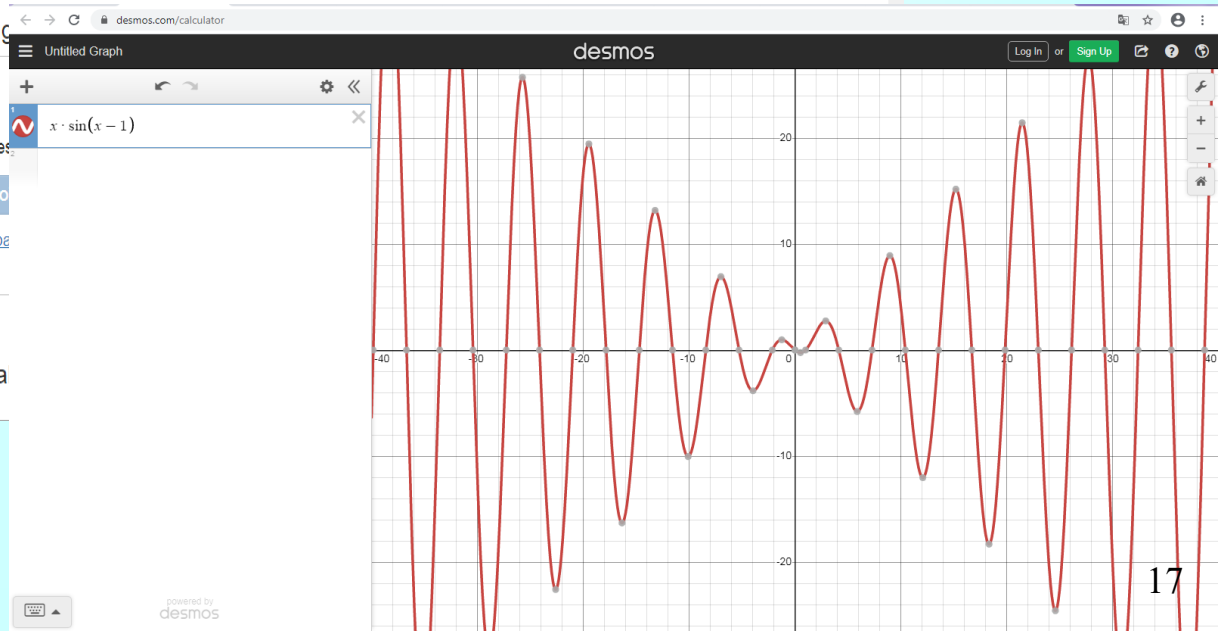




Graphing Calculator – Desmos

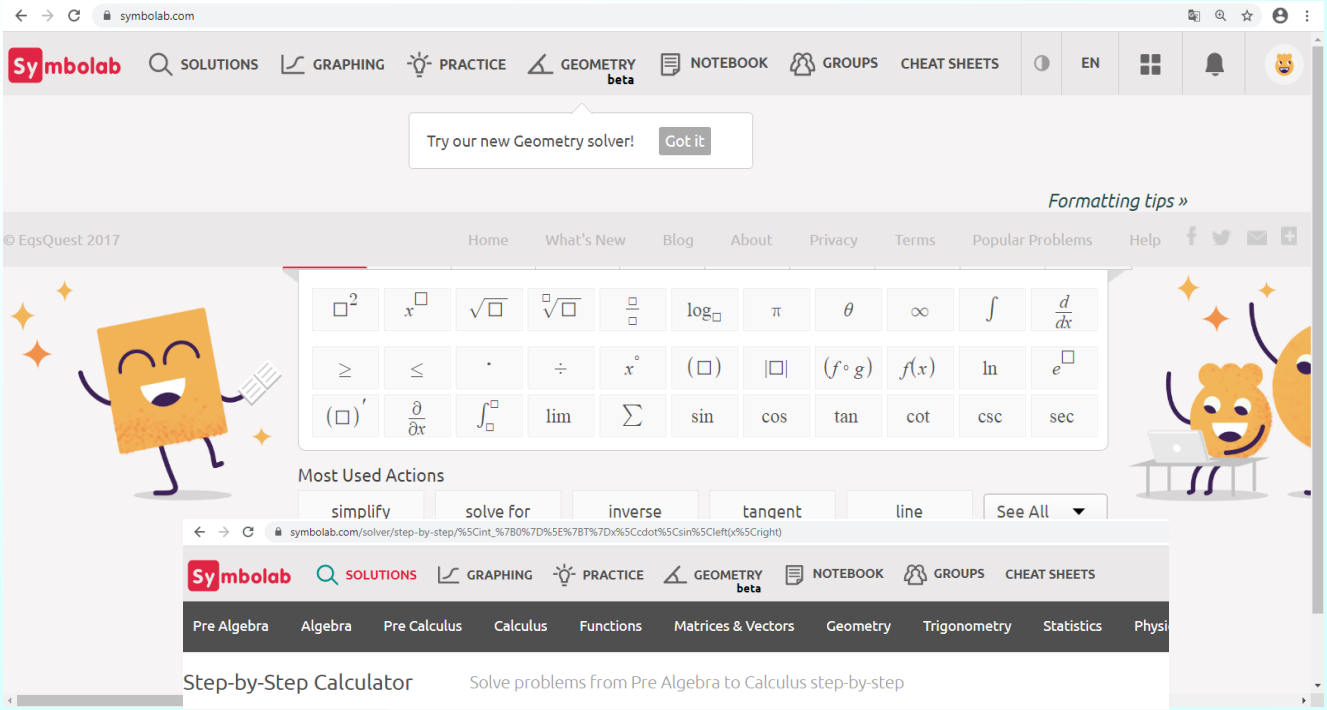
https://www.desmos.com/calculator

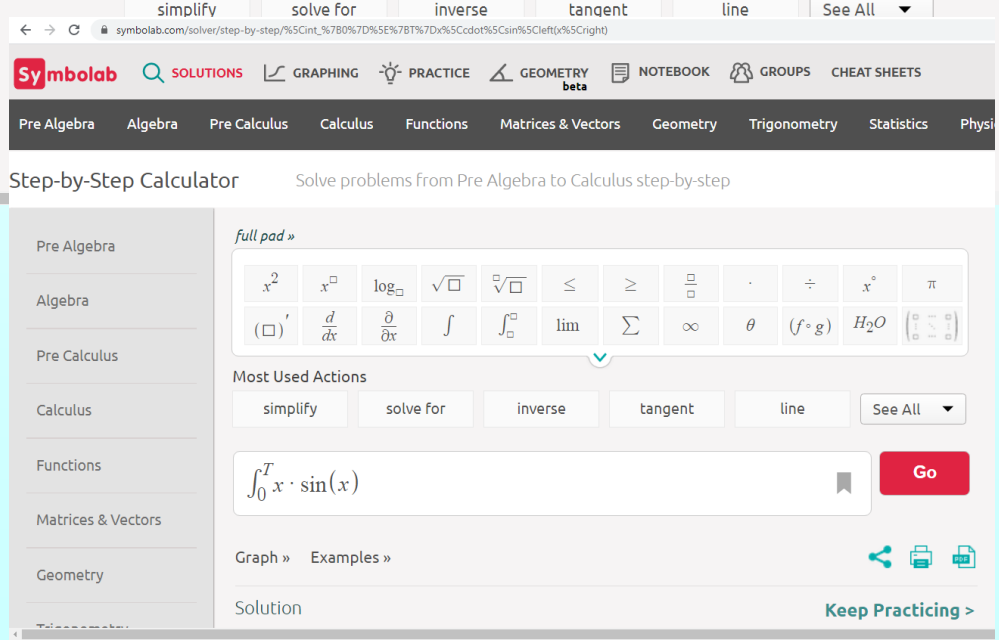


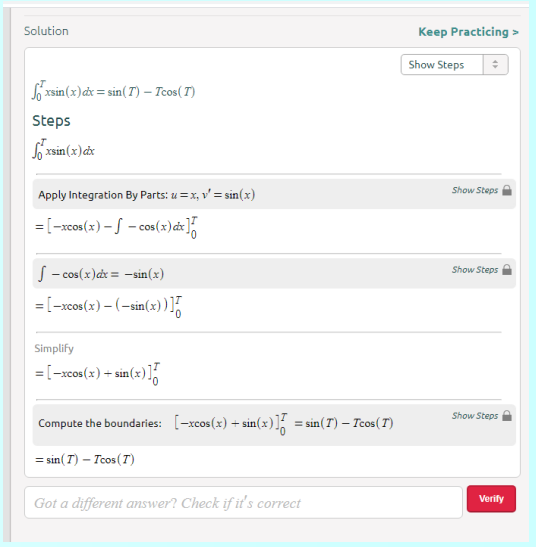


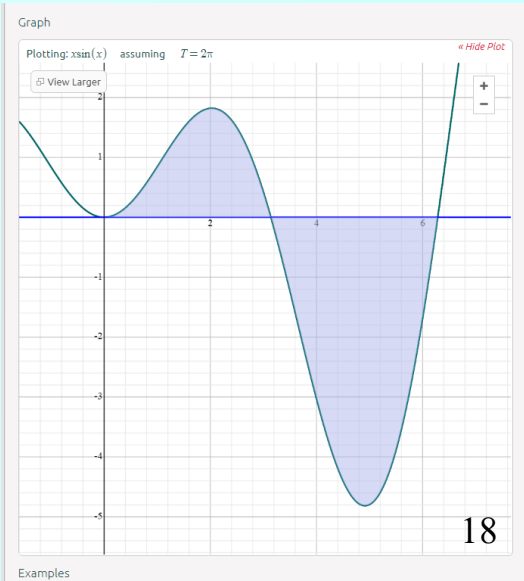
Symbolab Math Solver - Step by Step calculator

https://www.symbolab.com/



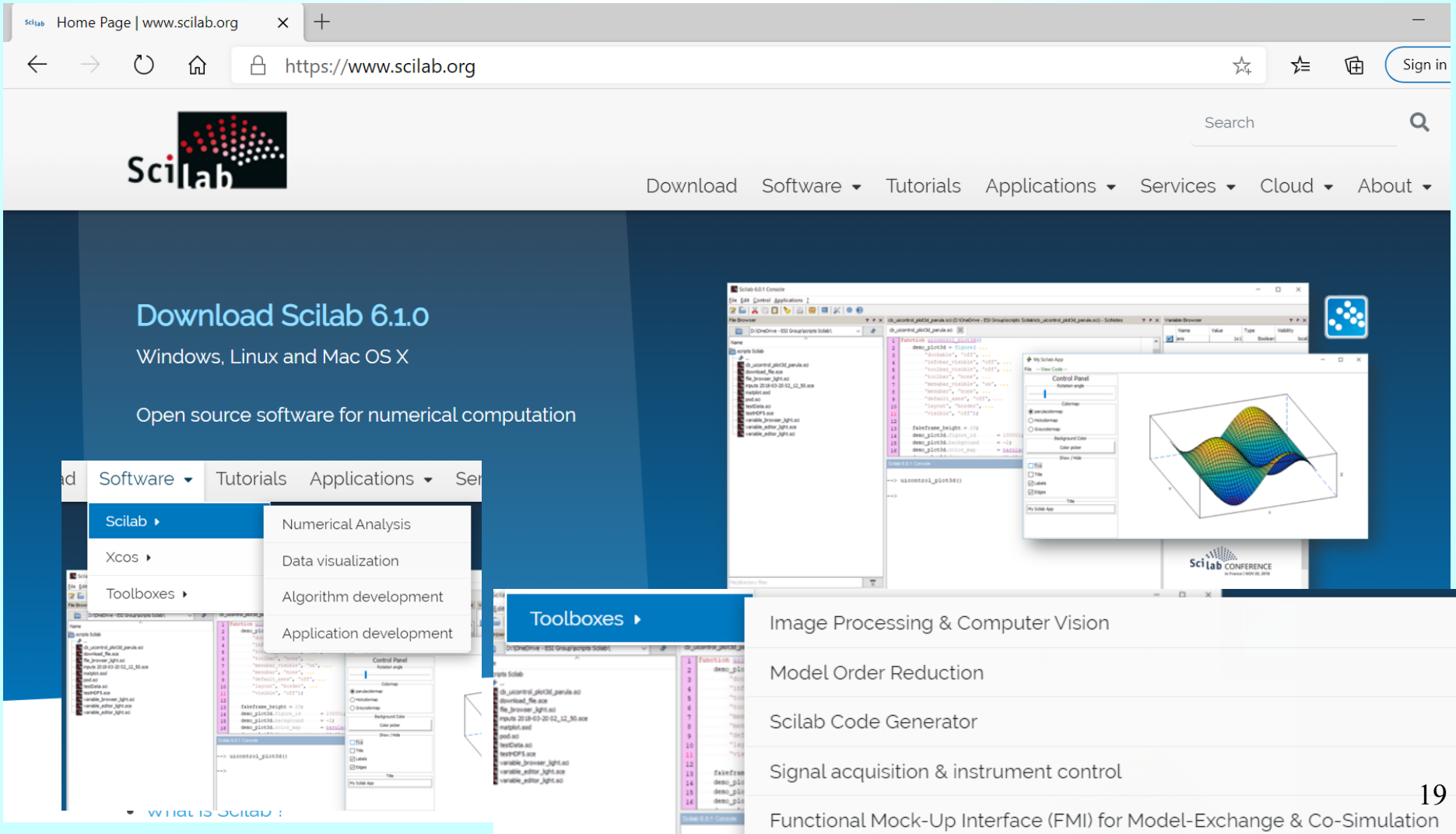






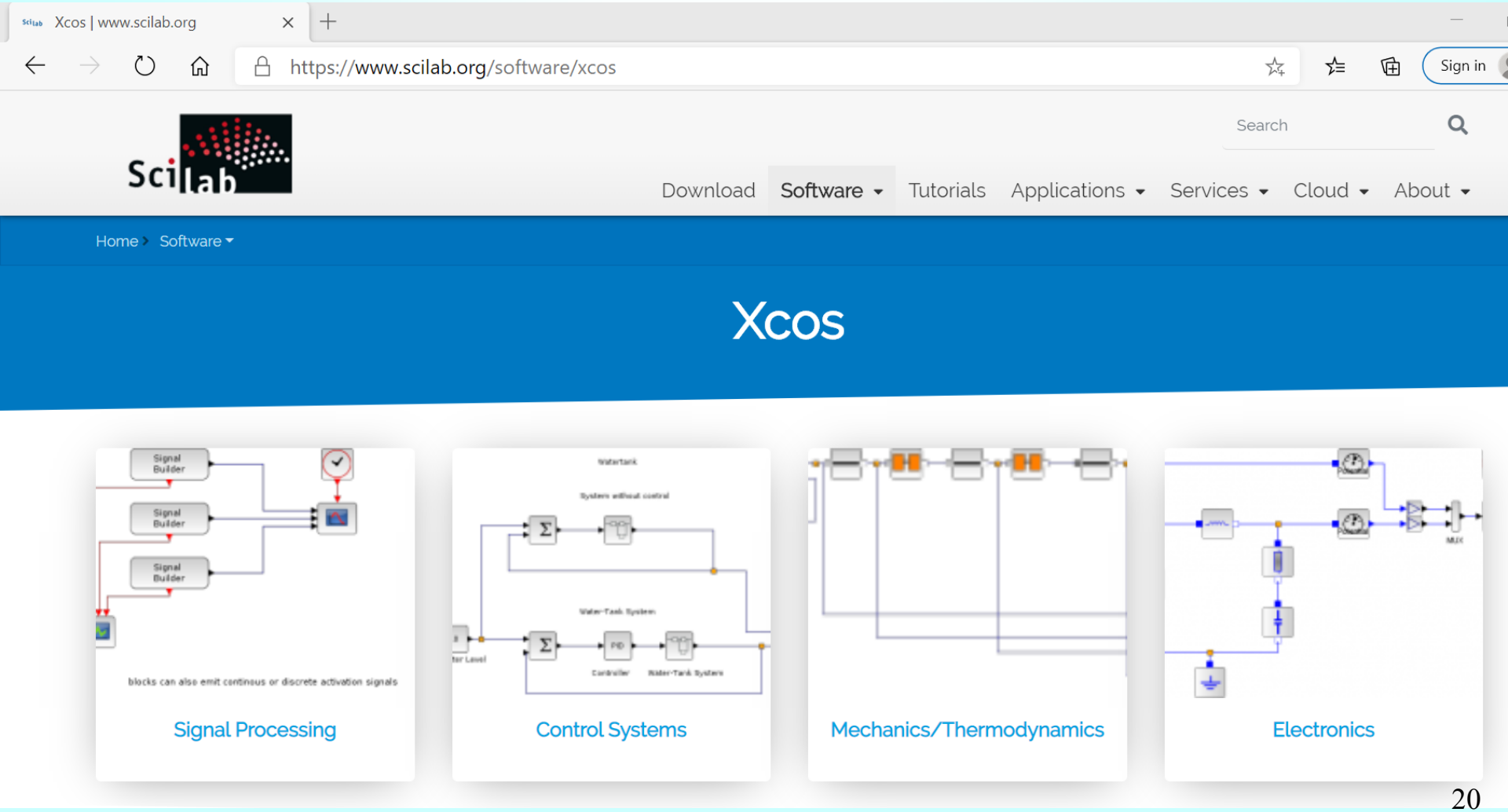
SciLab

https://www.scilab.org/



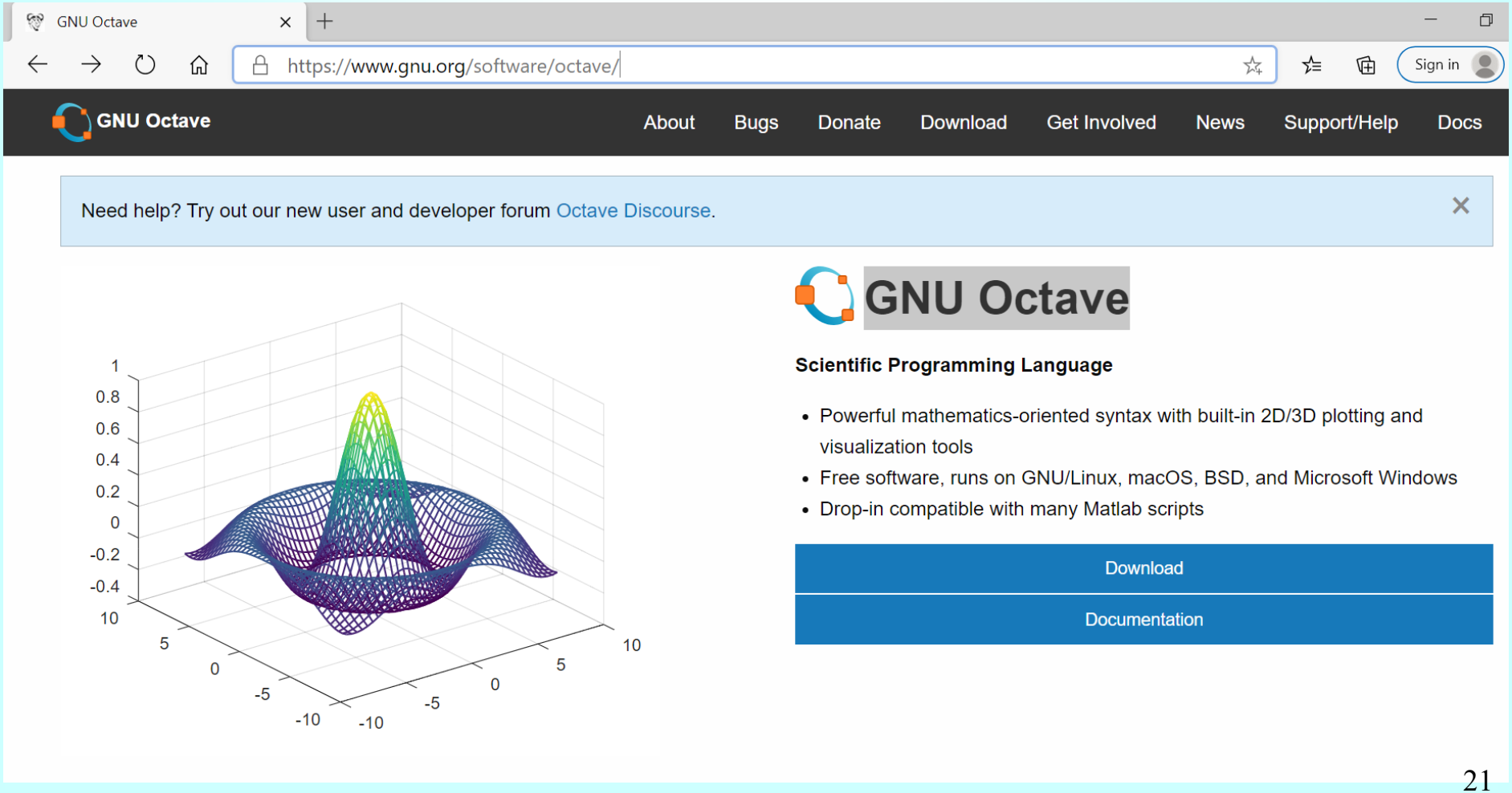
Xcos

https://www.scilab.org/software/xcos



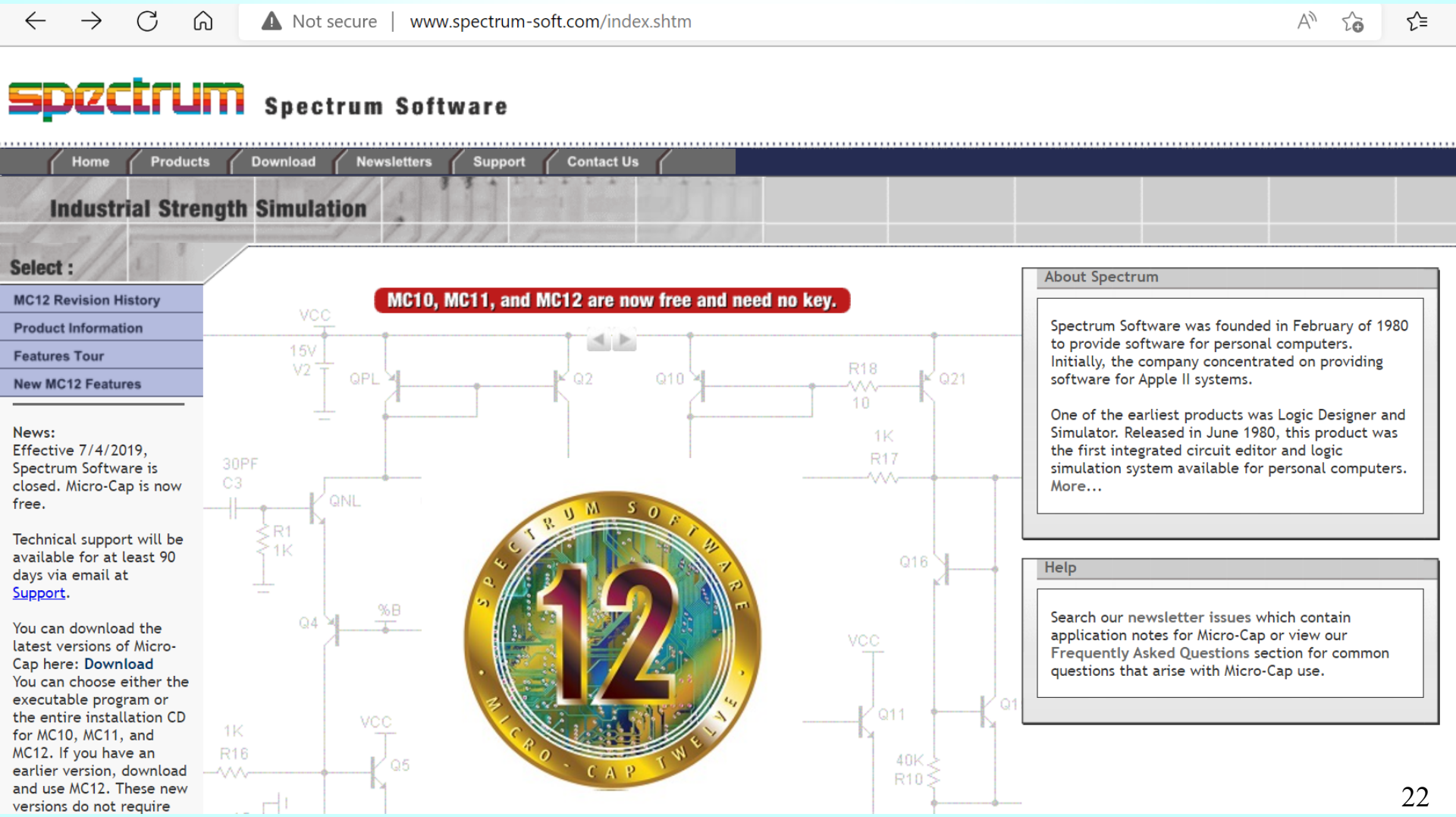
GNU Octave

https://www.gnu.org/software/octave/



Micro-Cap

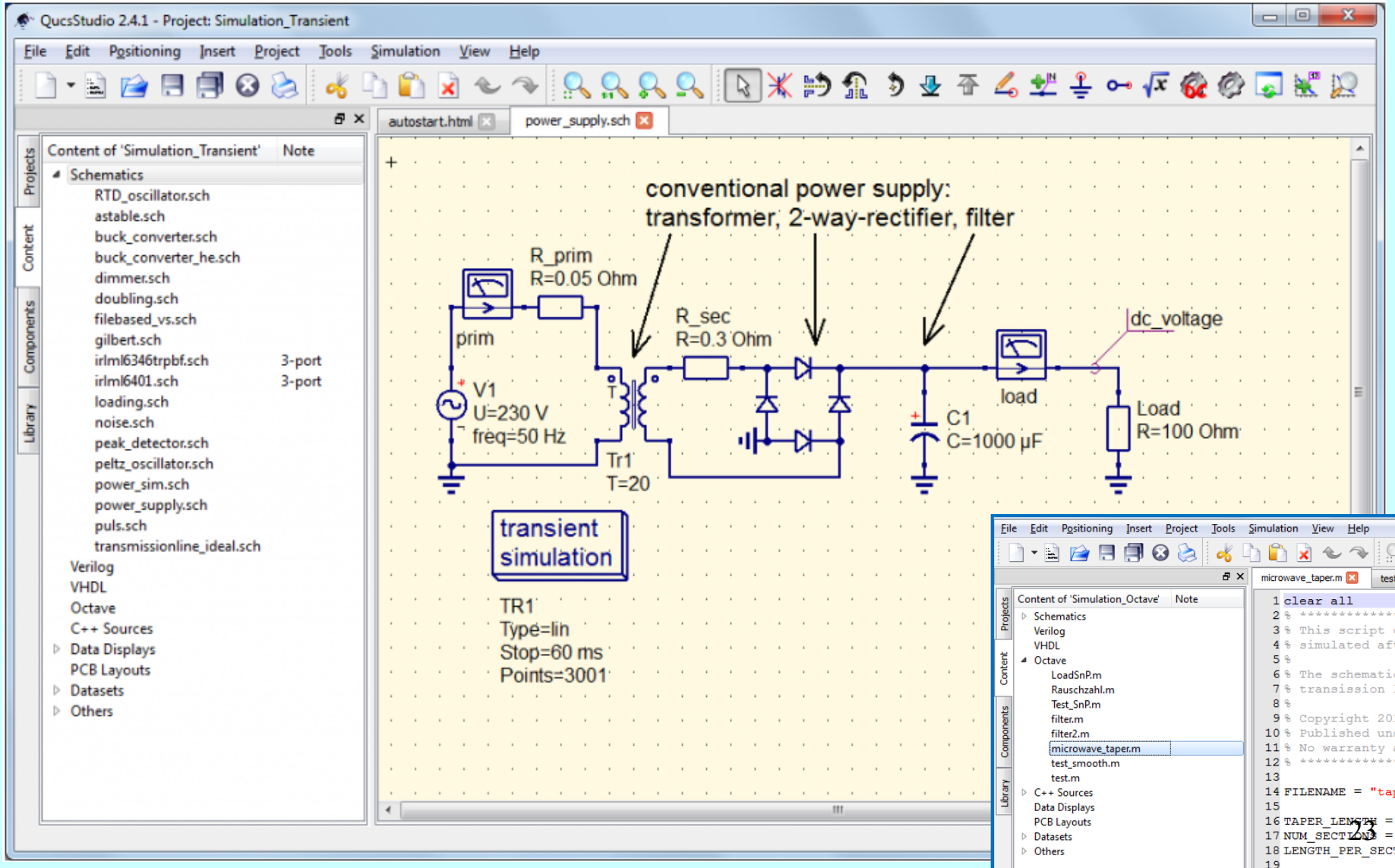
http://www.spectrum-soft.com/index.shtm



QucsStudio

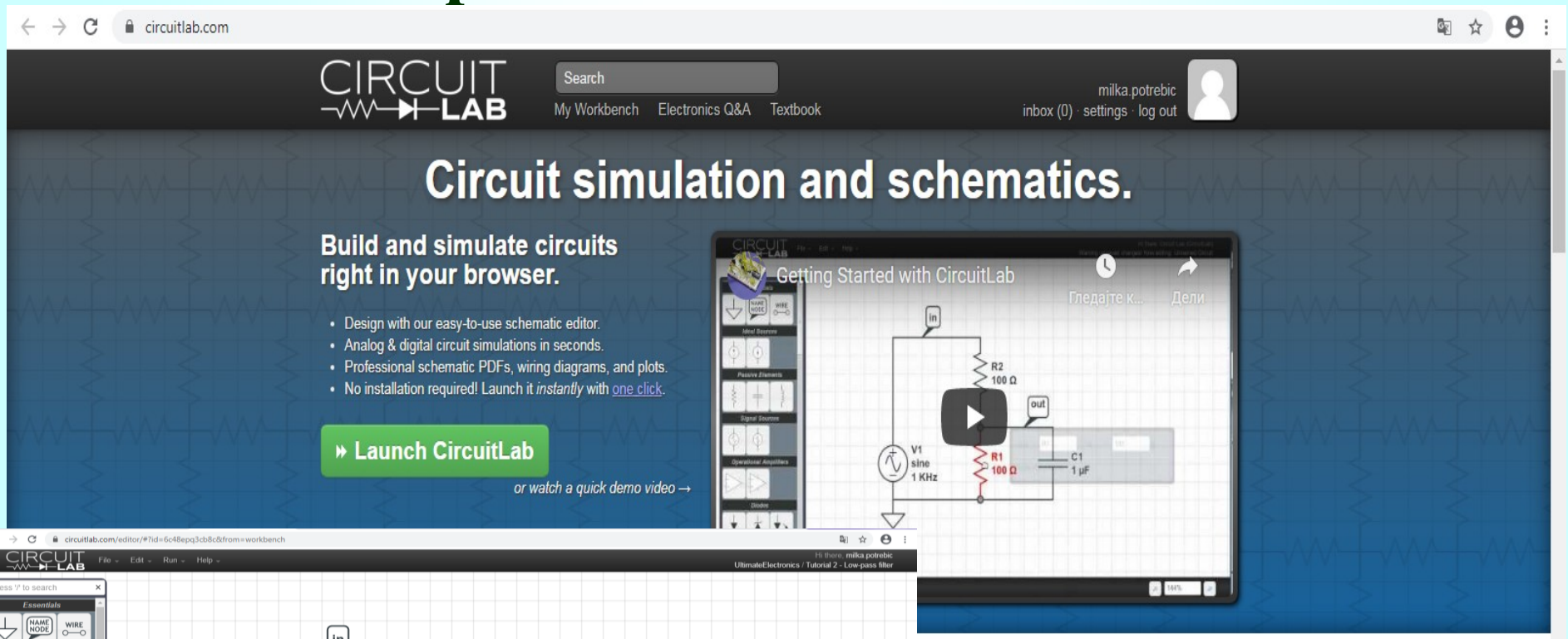
http://qucsstudio.de

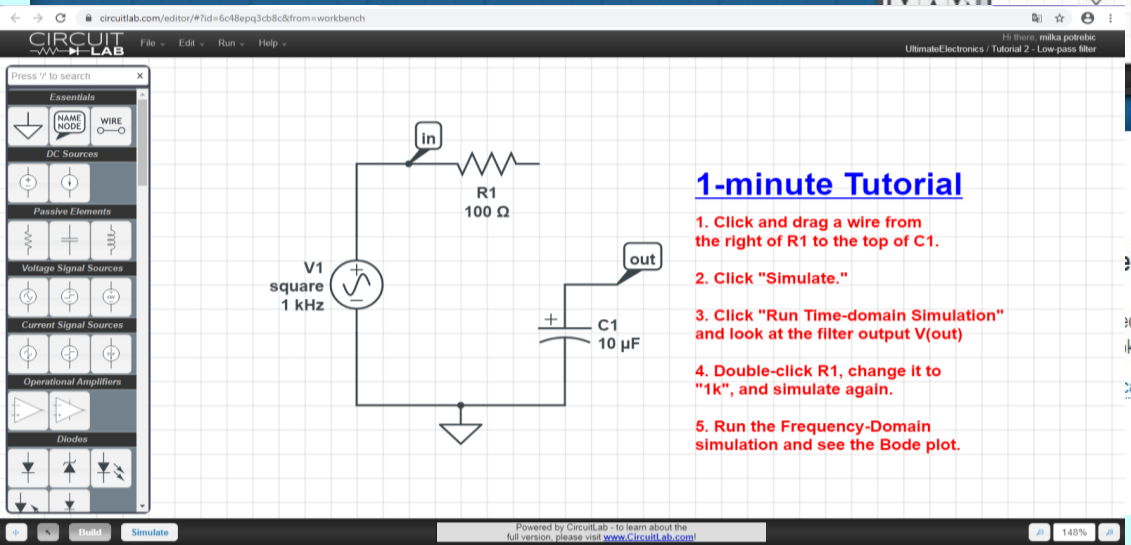
Octave engine inside!



CircuitLab

https://www.circuitlab.com/





DoCircuits Circuit Simulator

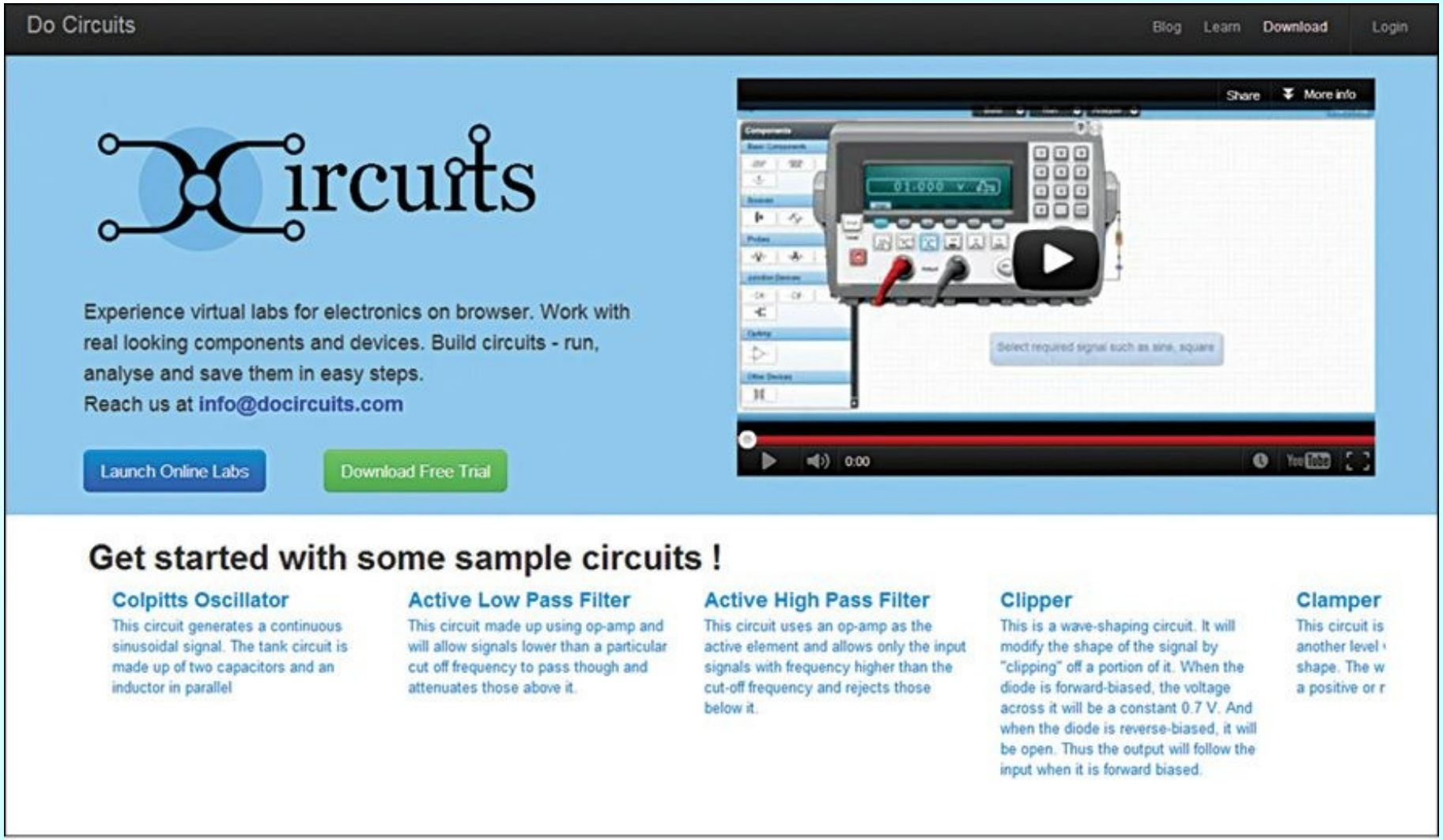


Fig. 1: DoCircuits - Home

ngSpice

http://ngspice.sourceforge.net/

Not secure | ngspice.sourceforge.net

NGSPICE

MIXED MODE - MIXED LEVEL CIRCUIT SIMULATOR

BASED ON BERKELEY'S SPICE3F5

NGSPICE SUMMARY

Home | News | Screenshots | Download | Documentation | Tutorials | Extras/Options | Applications | Development | Simulation Environments | Recipes

Ngspice Home

- Home
- What is ngspice ?
- Features, Extras & Options
- F.A.Q.
- Tutorials

Sourceforge Developer Pages

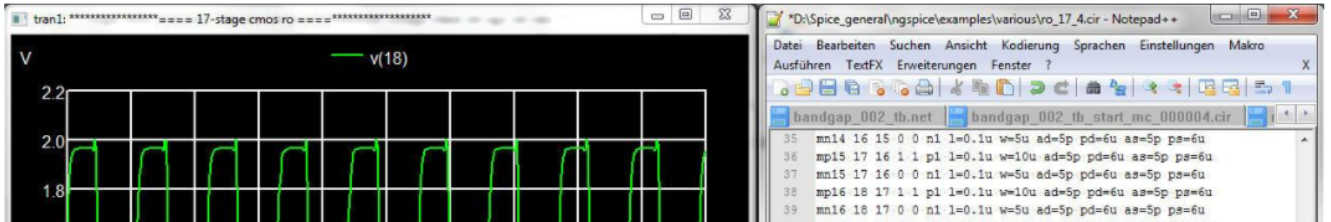
ngspice - open source spice simulator

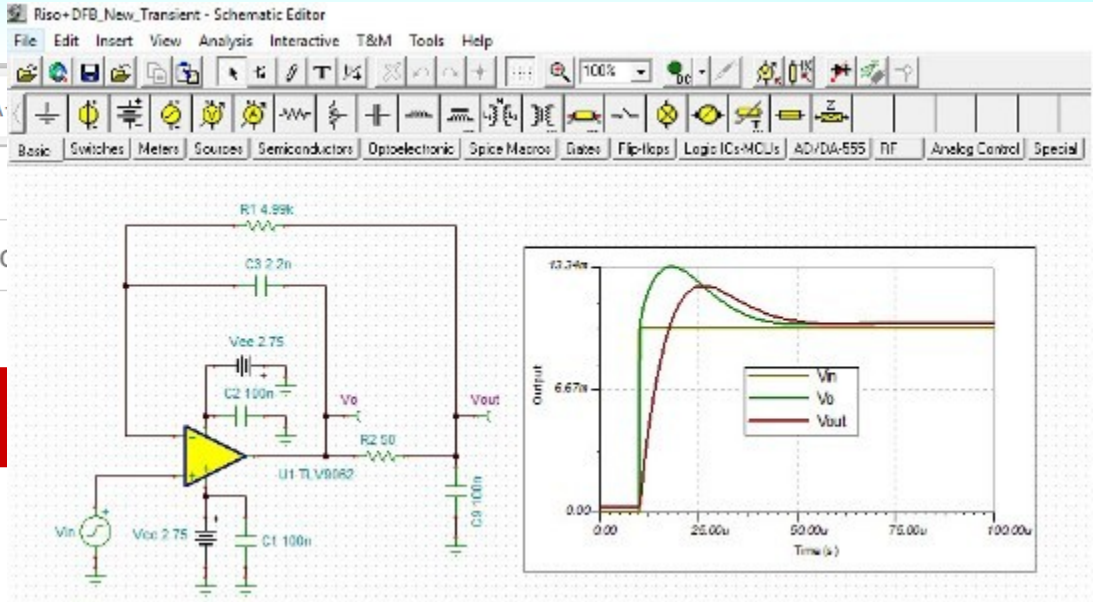
ngspice is the open source spice simulator for electric and electronic circuits.

Such a circuit may comprise of JFETs, bipolar and MOS transistors, passive elements like R, L, or C, diodes, transmission lines and other devices, all interconnected in a netlist. Digital circuits are simulated as well, event driven and fast, from single gates to complex circuits. And you may enter the combination of both analog and digital as a mixed-signal circuit.

ngspice offers a wealth of device models for active, passive, analog, and digital elements. Model parameters are provided by our collections, by the semiconductor device manufacturers, or from semiconductor foundries. The user adds her circuits as a netlist, and the output is one or more graphs of currents, voltages and other electrical quantities or is saved in a data file.

ngspice does not provide schematic entry. Its input is command line or file based. There are however third party interfaces available.





TINA-TI

SPICE-based analog simulation program

TINA-TI

https://www.ti.com/tool/TINA-TI

Overview

TINA-TI provides all the conventional DC, transient and frequency domain analysis of SPICE and much more. TINA has extensive post-processing capability that allows you to format results the way you want them. Virtual instruments allow you to

Electric Circuit Studio

https://play.google.com/store/apps/details?id=com.ecstudiosystems.electriccircuitstudio&hl=en_US&gl=US

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Parent Guide

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ECStudio Systems Education

2,709

Everyone

Contains Ads

You don't have any devices

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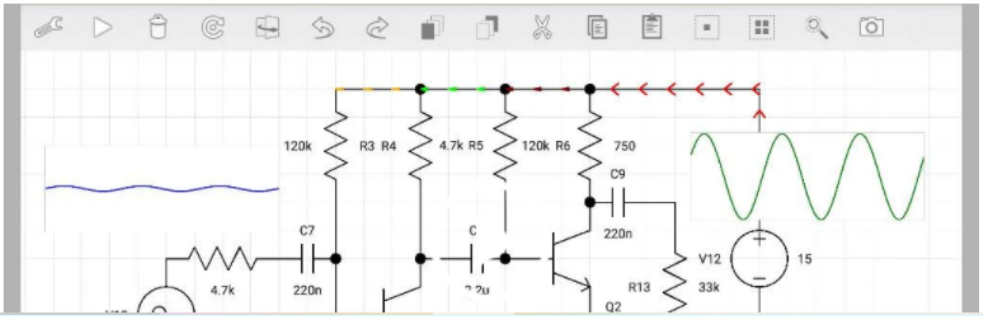
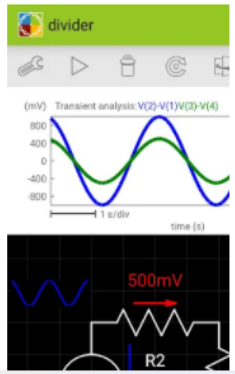
Install





CircuitSafari SPICE Simulator

https://play.google.com/store/apps/details?id=com.logipipe.circuitsafari&hl=en_US&gl=US

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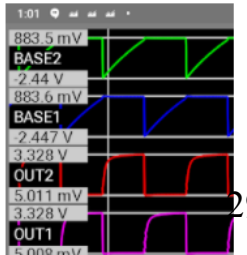

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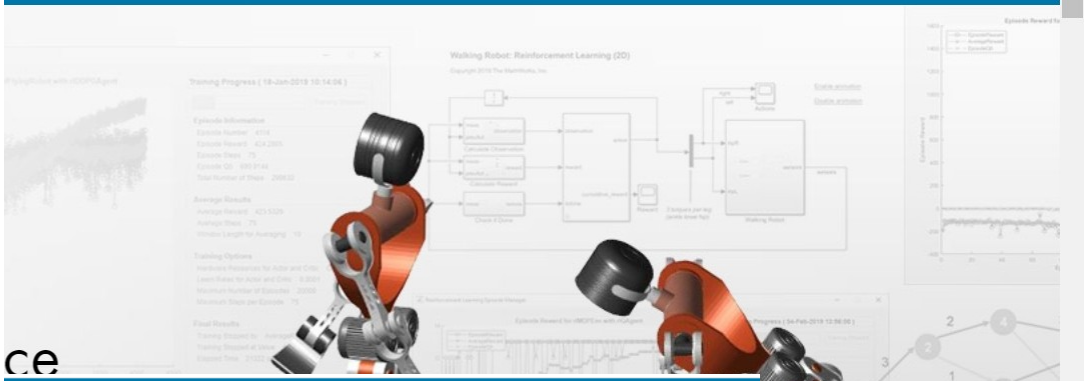
MATLAB® combines a desktop environment tuned for iterative analysis and design processes with a programming language that expresses matrix and array mathematics directly. It includes the Live Editor for creating scripts that combine code, output, and formatted text in an executable notebook.



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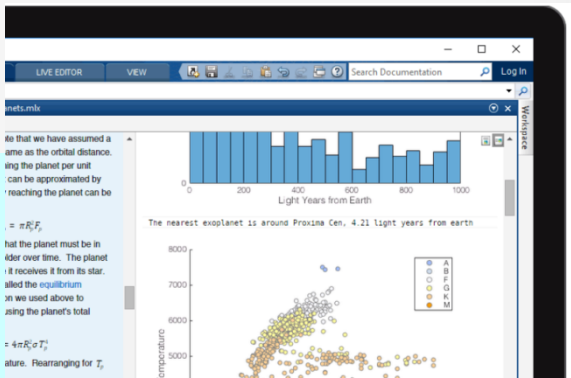

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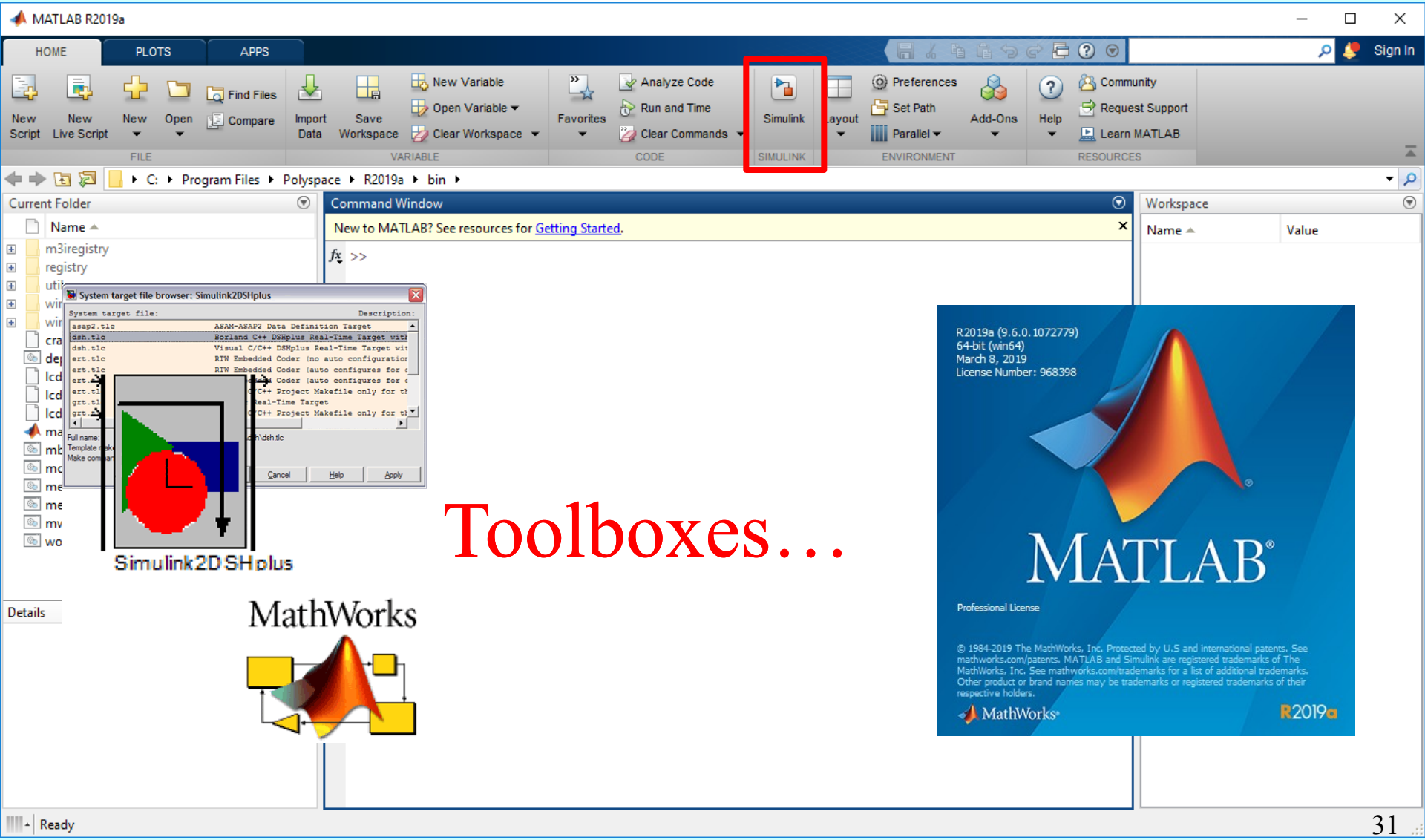
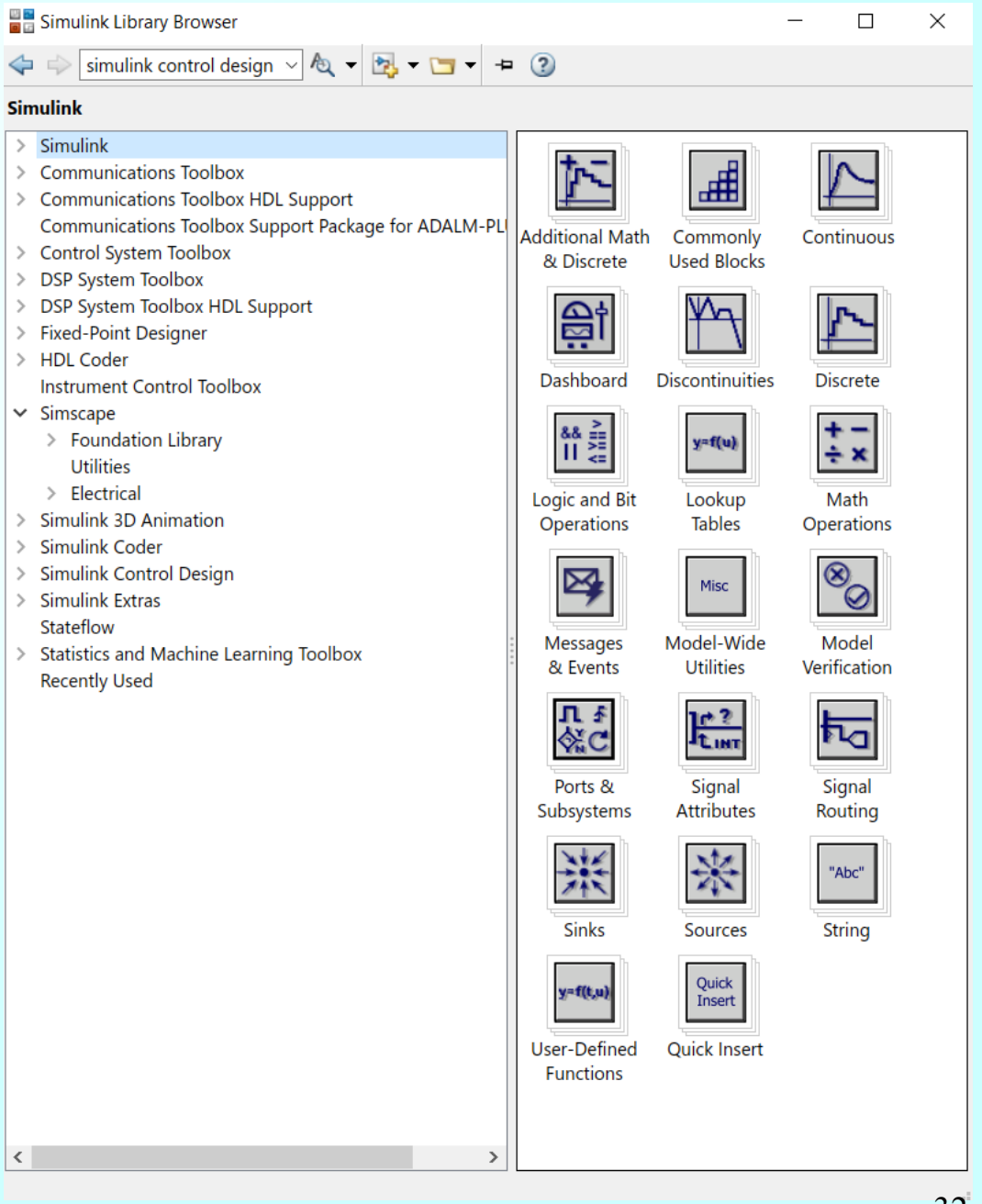


Toolboxes…

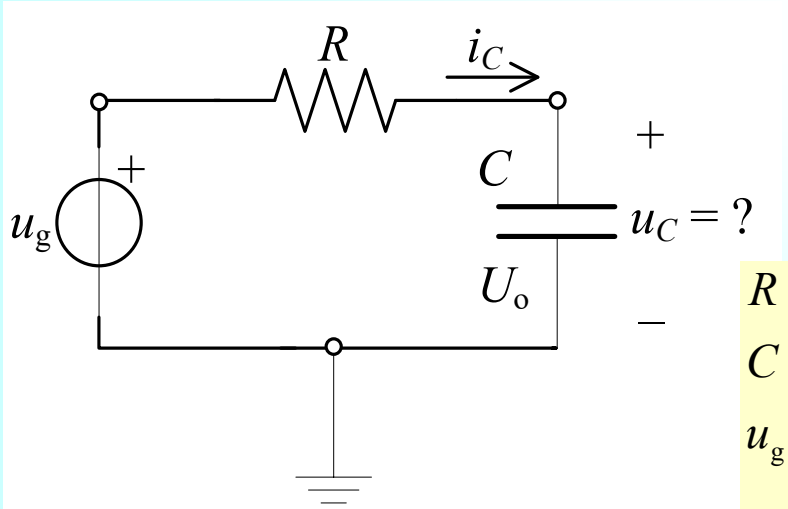
MATLAB Simulink

- Алатка за нумеричку анализу математичких модела
- Систем се дефинише графички, уметањем блокова и њиховим повезивањем у мрежу



Simulink > Simscape

- Библиотека за симулацију и нумеричку анализу мултидоменских проблема (механика, електродинамика, оптика)
- Пример једноставног електричног кола:

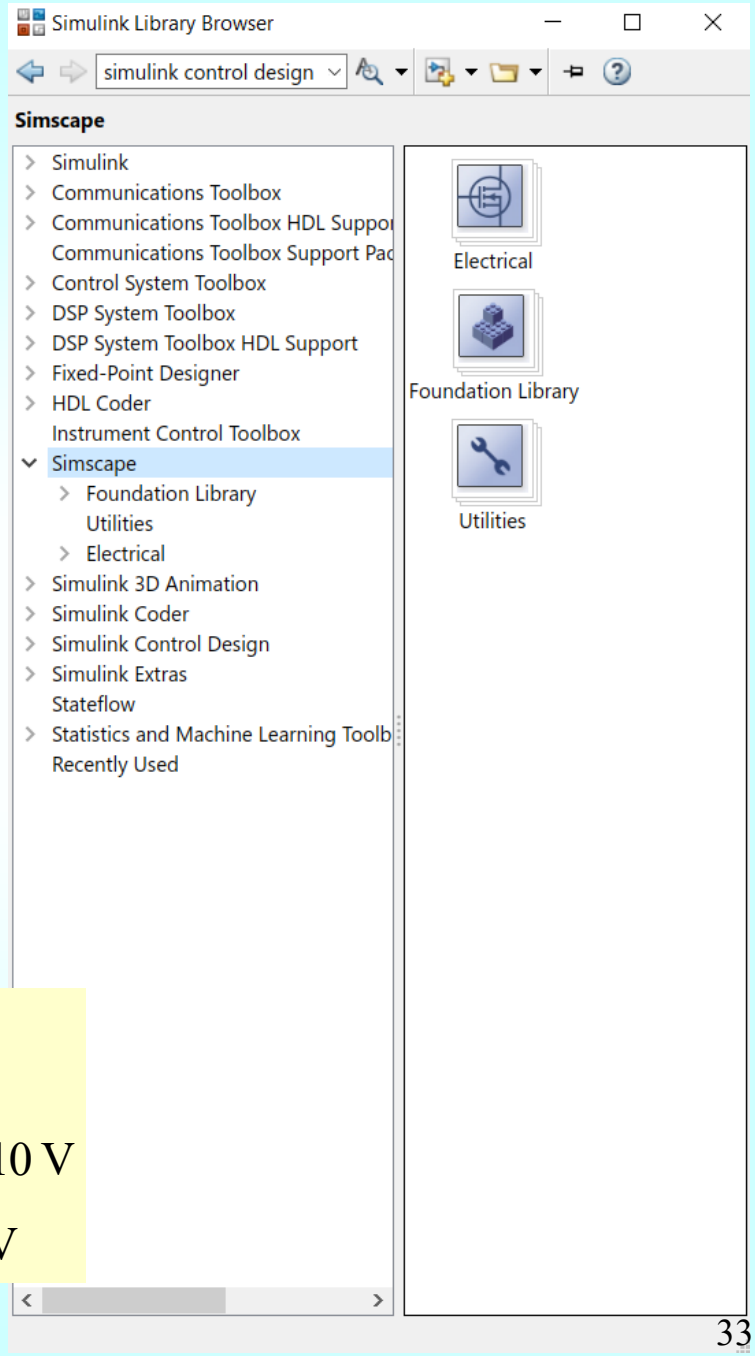


$R = 1\,\mathrm{k\Omega}$

$C = 1\,\mu\mathrm{F}$

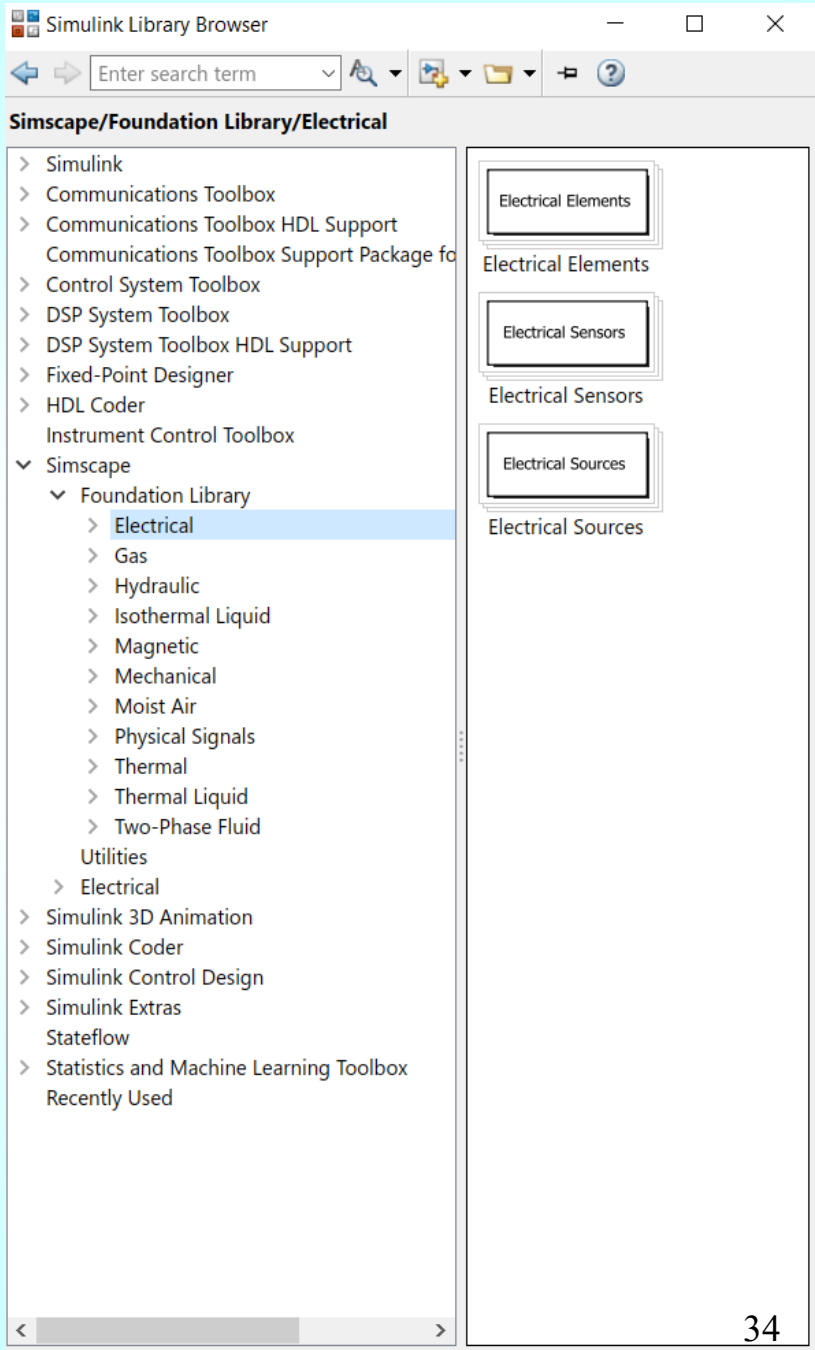
$u_g(t) = U\,h(t), U = 10\,\mathrm{V}$

$u_C(t_0^-) = U_0 = -10\,\mathrm{V}$



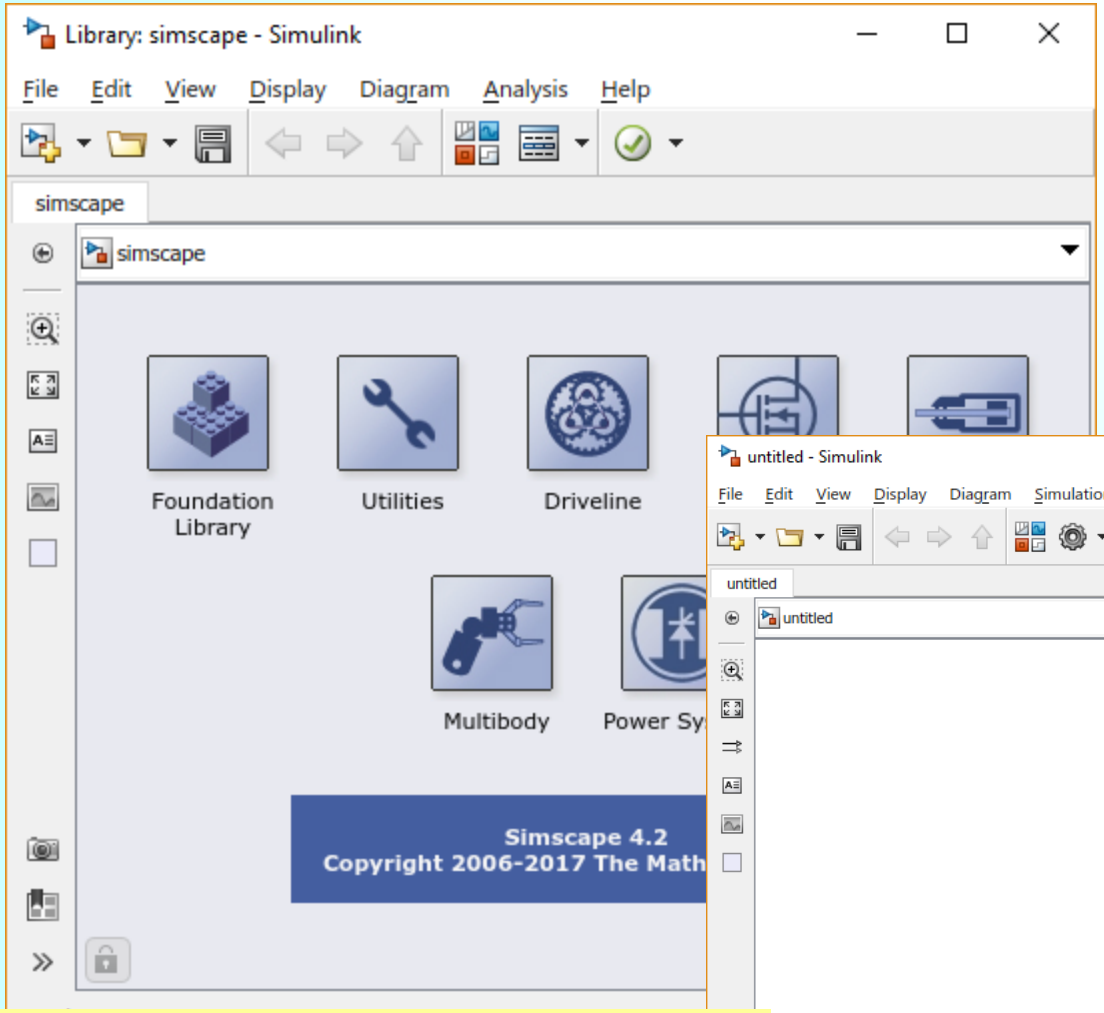
Simulink > Simscape > Foundation Library > Electrical

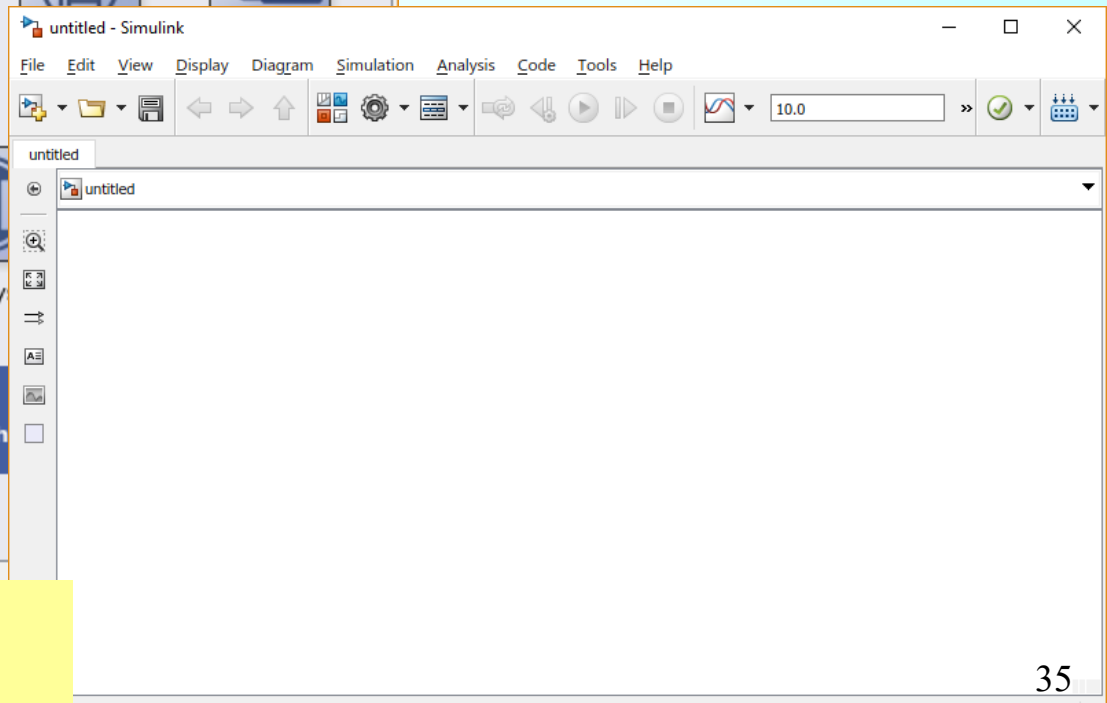
- Симулација линеарних и нелинеатних електричних кола



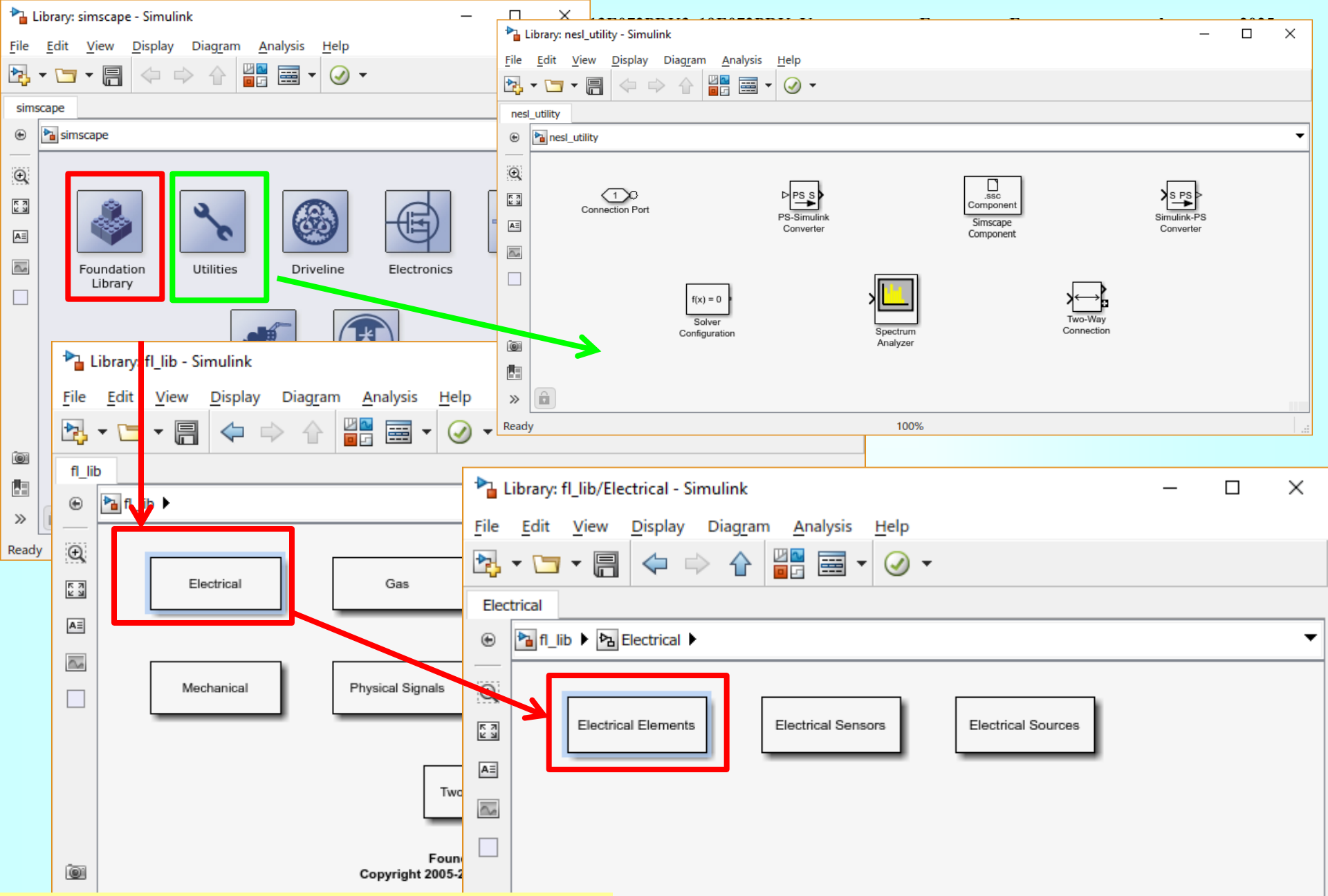
MATLAB > Simscape > Foundation Library, Utilities

Симулација електричног кола коришћењем **Simscape > Foundation Library** библиотеке

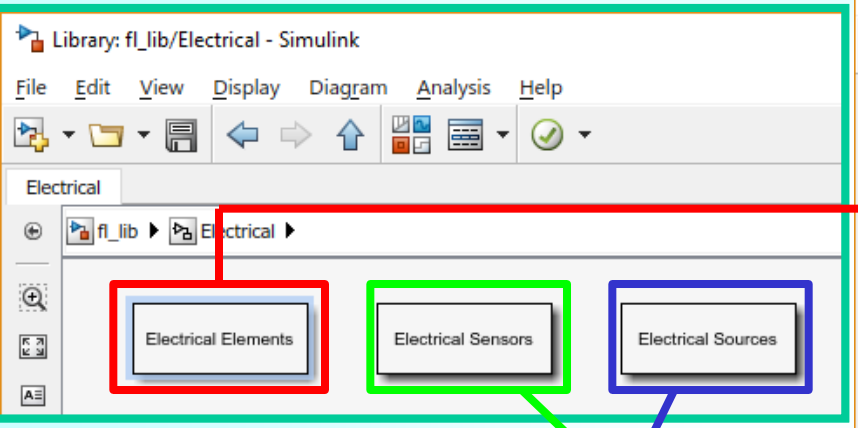




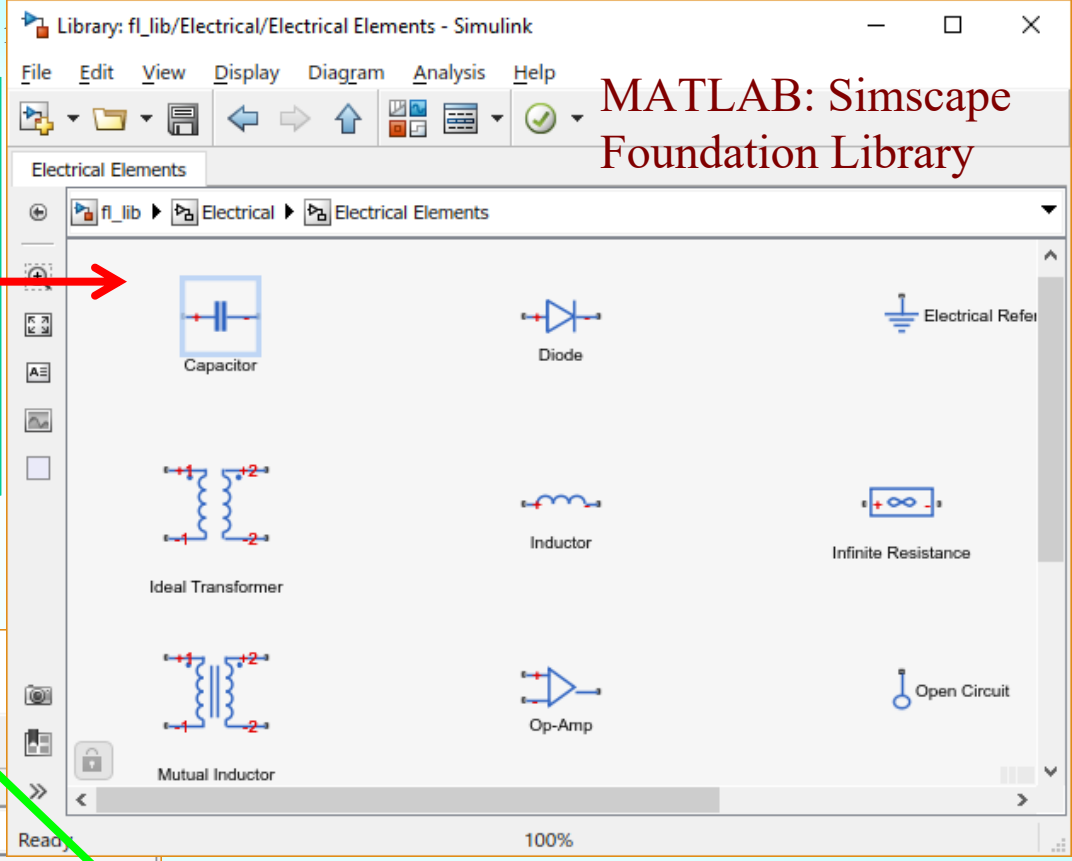
MATLAB > Simscape > Foundation Library, Utilities

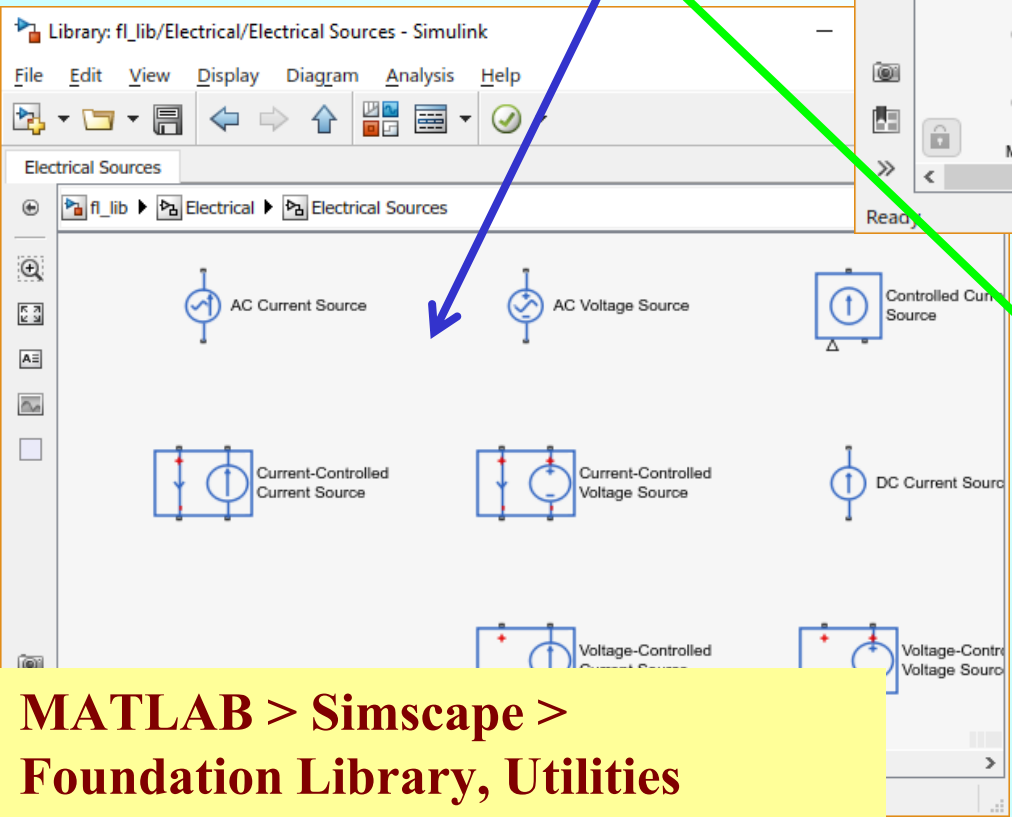


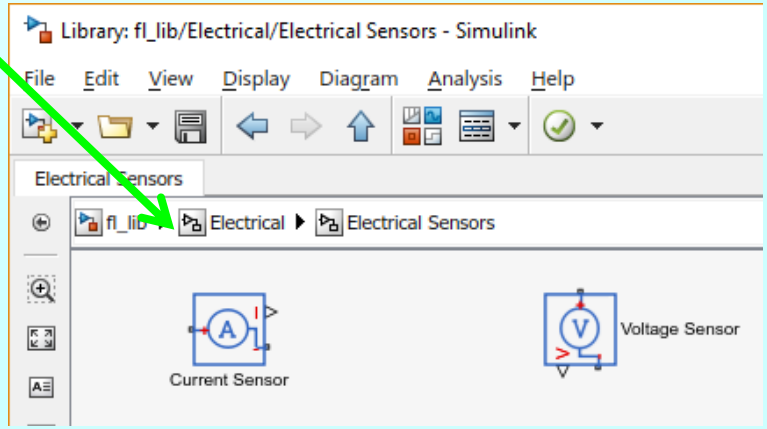
MATLAB > Simscape > Foundation Library, Utilities



MATLAB: Simscape Foundation Library

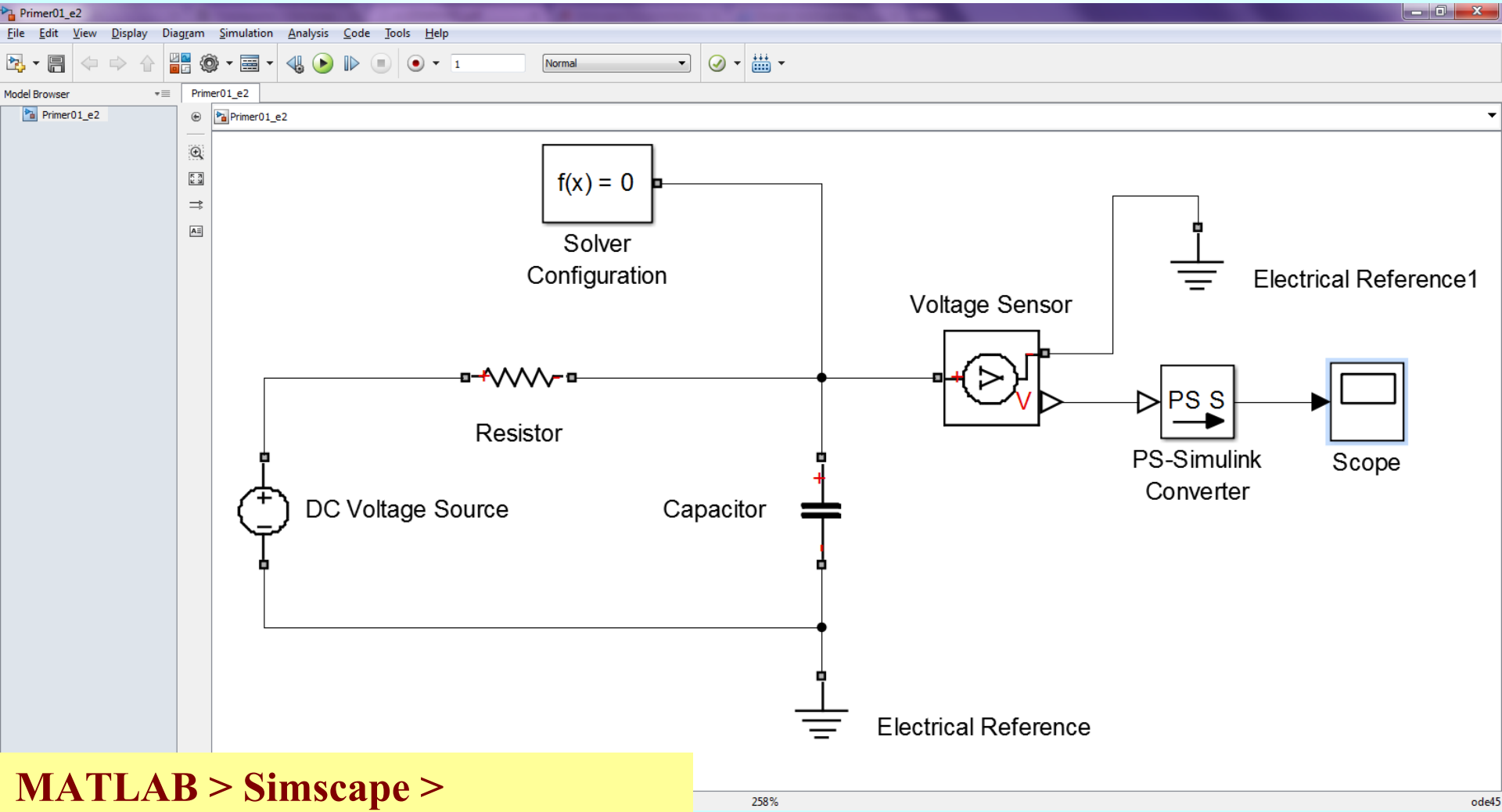




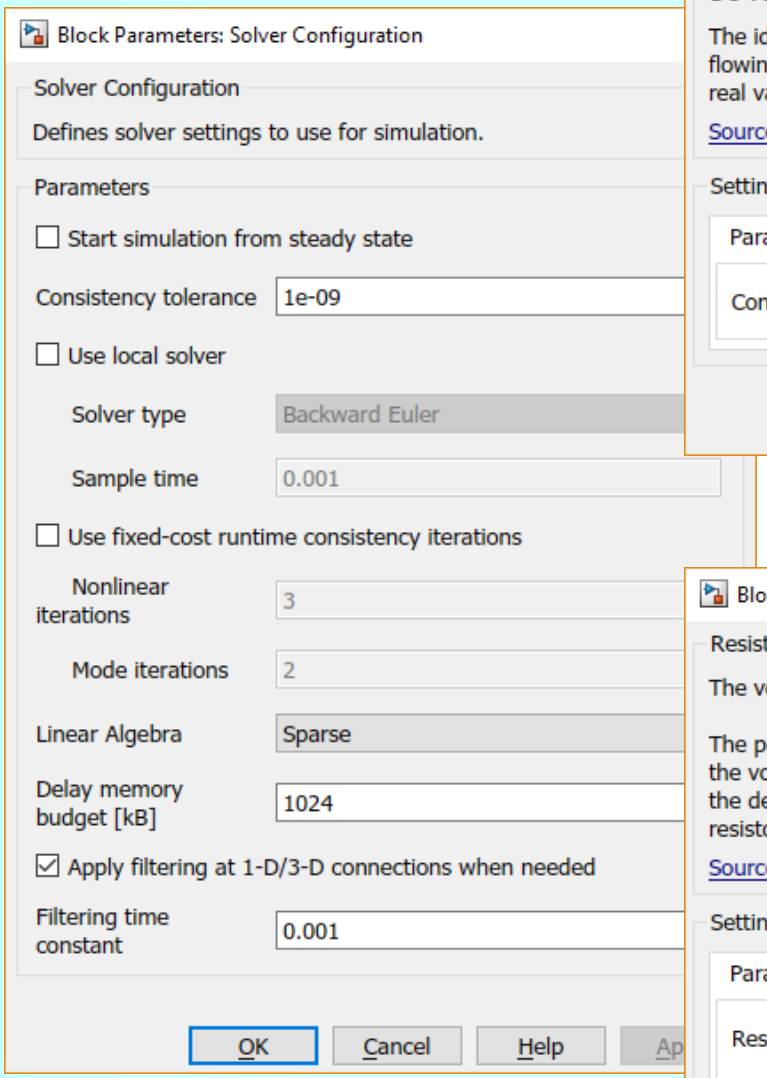


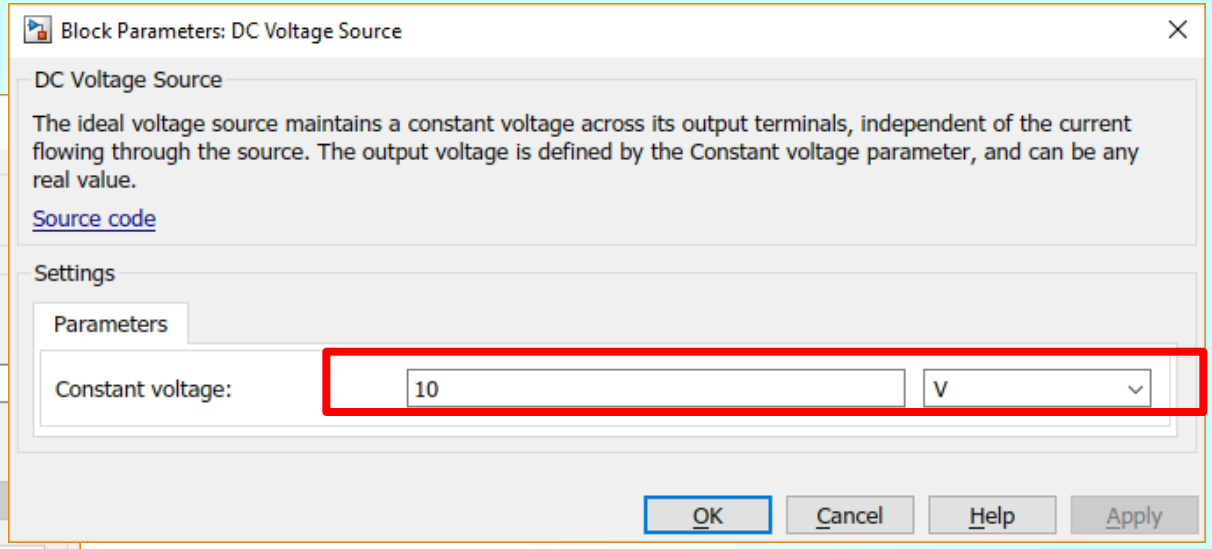
MATLAB > Simscape > Foundation Library, Utilities

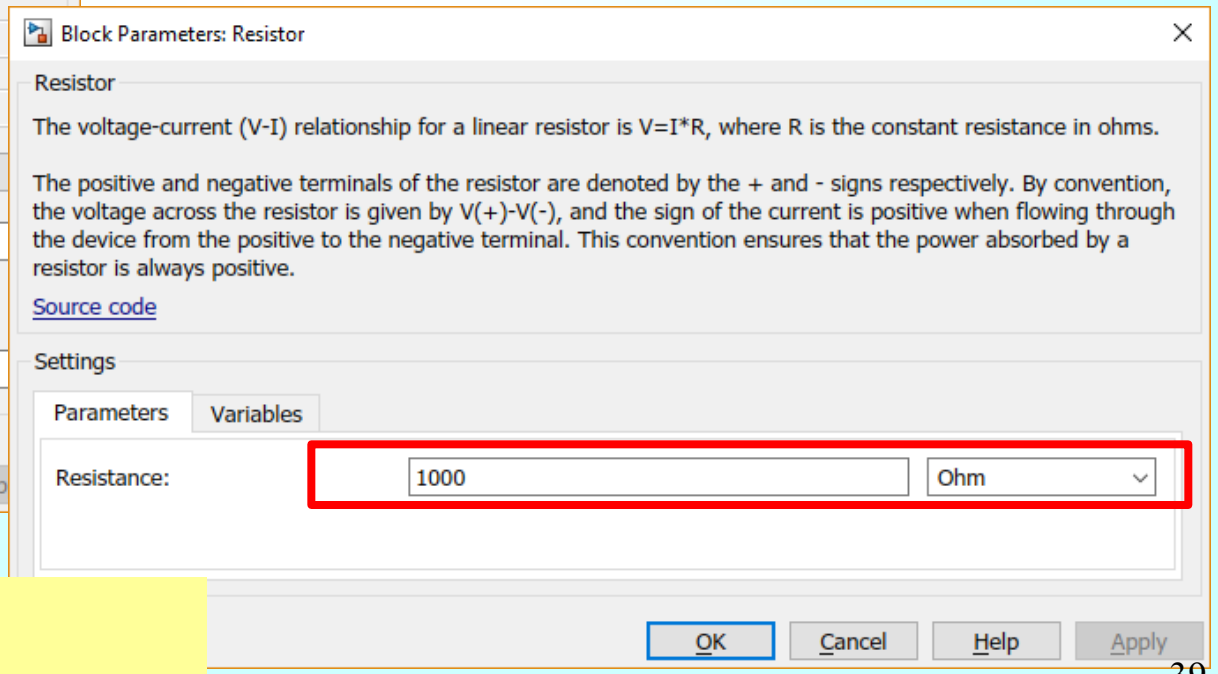
Симулација електричног кола коришћењем **Simscape > Foundation Library** библиотеке



MATLAB > Simscape > Foundation Library, Utilities







MATLAB > Simscape > Foundation Library, Utilities

Почетни услов...

Block Parameters: Capacitor

Capacitor

Models a linear capacitor. The relationship between voltage V and and current I is I=C*dV/dt where C is the capacitance in farads.

The Series resistance and Parallel conductance represent small parasitic effects. The parallel conductance can be used to model dielectric losses and the series resistance used to represent the effective series resistance (ESR) of the capacitor. Simulation of some circuits may require the presence of the small series resistance. Consult the documentation for further details.

Source code

Settings

Parameters | Variables

Capacitance:	1e-6	F
Series resistance:	0	Ohm
Parallel conductance:	0	1/Ohm

OK | Cancel | Help | Apply

*dV/dt where C is the capacitance in farads.

arallel conductance can be used to model
nce (ESR) of the capacitor. Simulation of
mentation for further details.

Settings

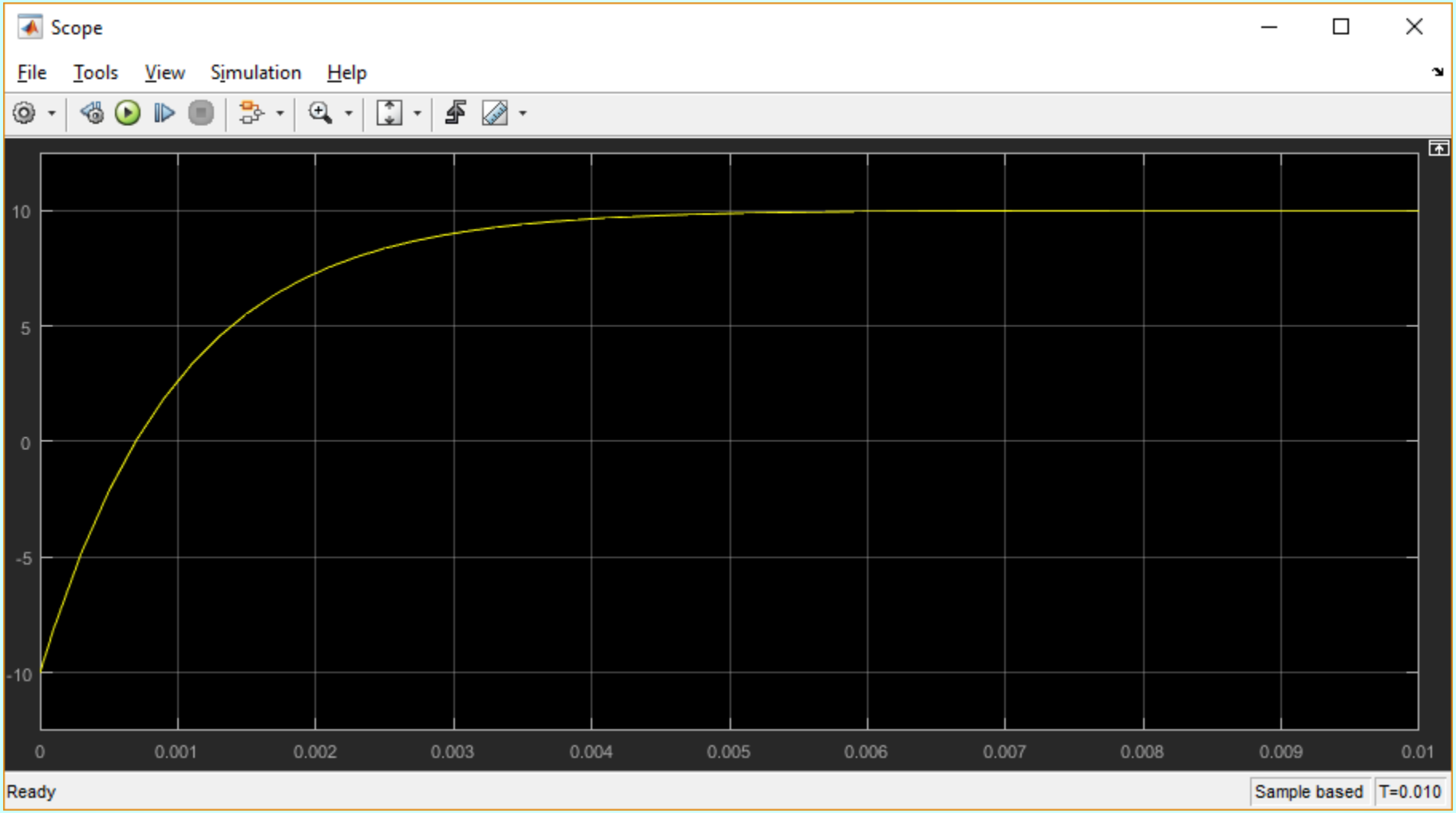
Parameters | Variables

Override	Variable	Priority	Beginning Value	Unit
☐	Current	None	0	A
☐	Voltage	None	0	V
☑	Capacitor voltage	High	-10	V

OK | Cancel | Help | Apply

MATLAB > Simscape > Foundation Library, Utilities

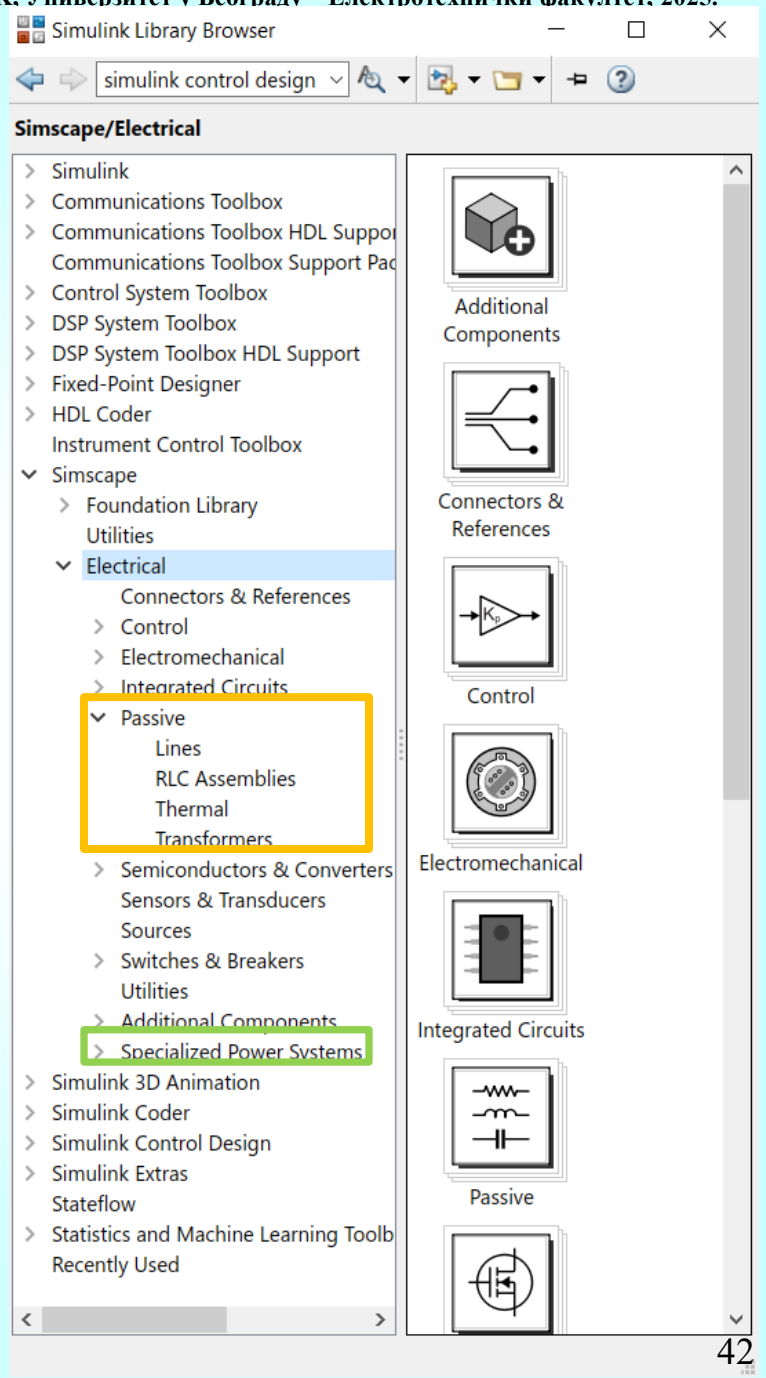
Налажење одзива...



MATLAB > Simscape > Foundation Library, Utilities

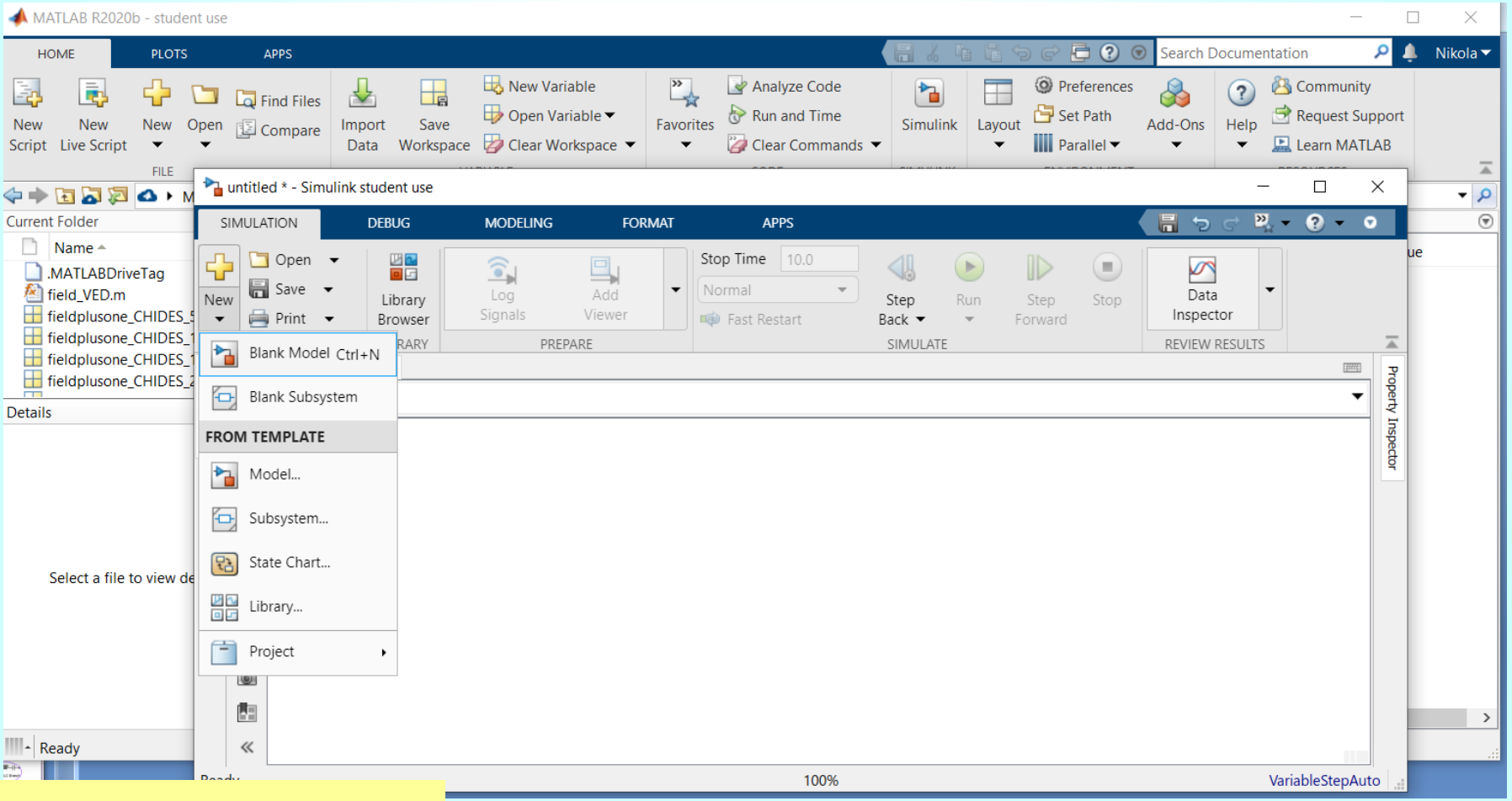
Simulink > Simscape > Electrical

- Библиотека за симулацију и нумеричку анализу електроенергетских система
- Трофазни извори, водови, трансформатори, потрошачи
- Термална анализа



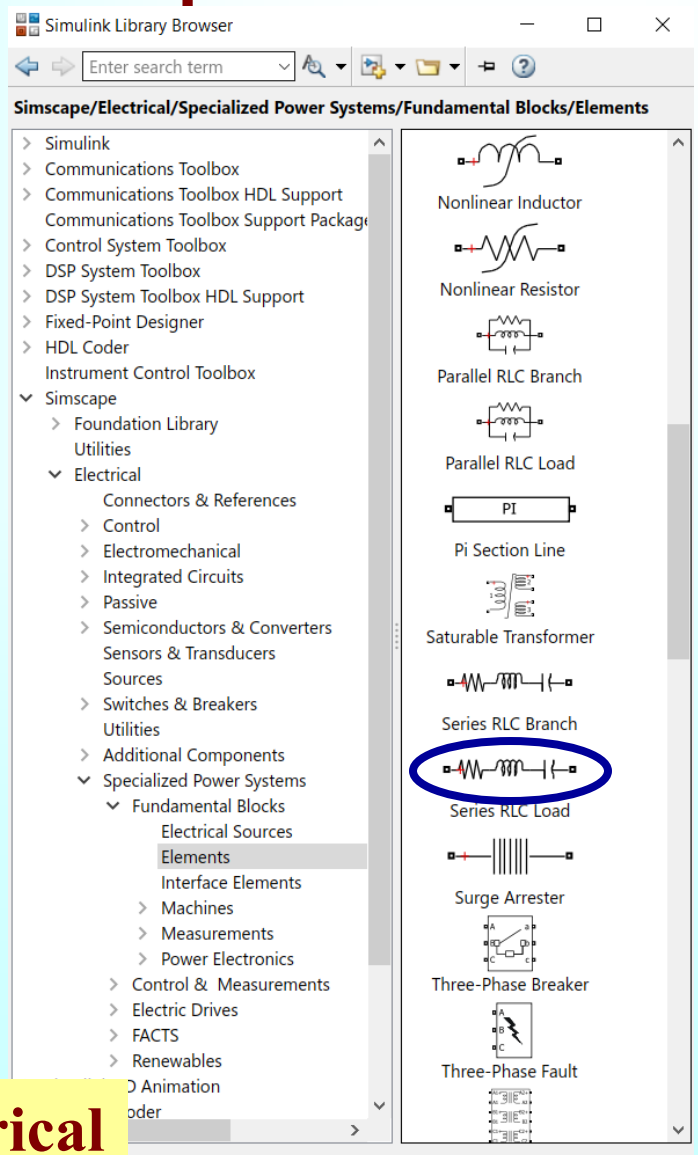
Simscape > Electrical

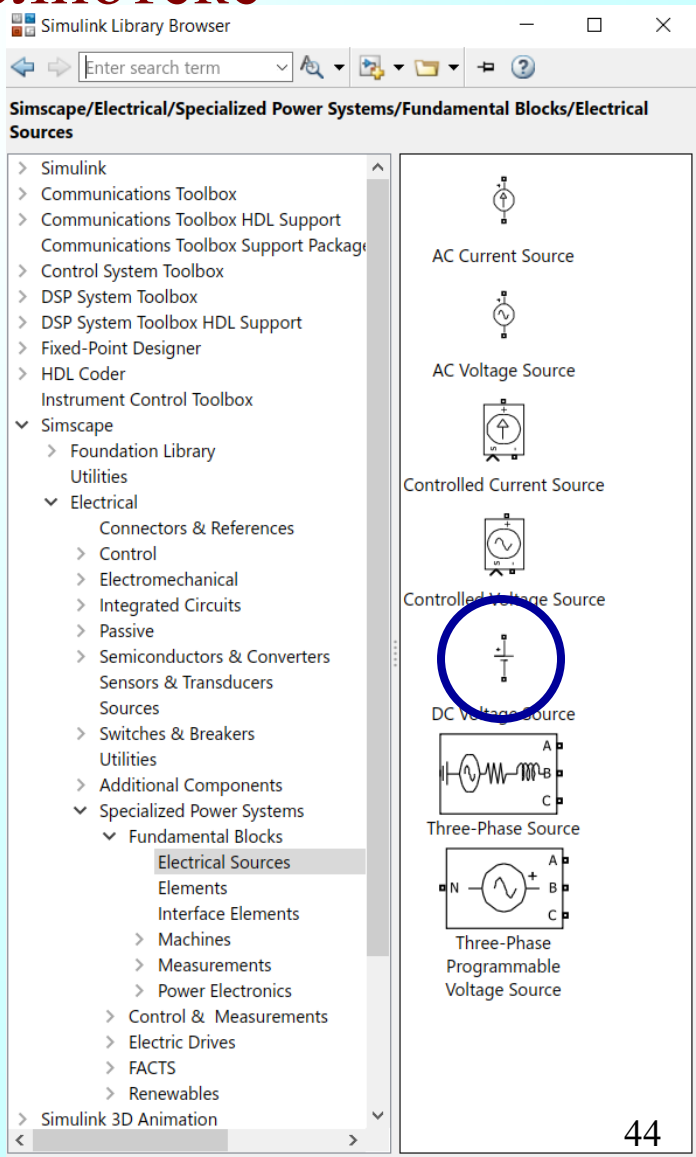
Симулација електричног кола коришћењем **Simscape>Electrical** библиотеке



Simscape > Electrical

Симулација електричног кола коришћењем **Simscape>Electrical** библиотеке

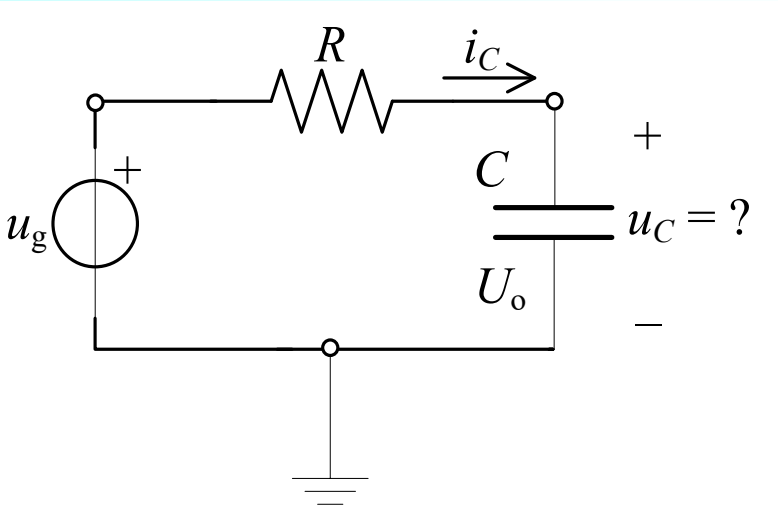




Simscape > Electrical

Симулација електричног кола коришћењем **Simscape>Electrical** библиотеке

коло је образовано у тренутку $t_0 = 0$

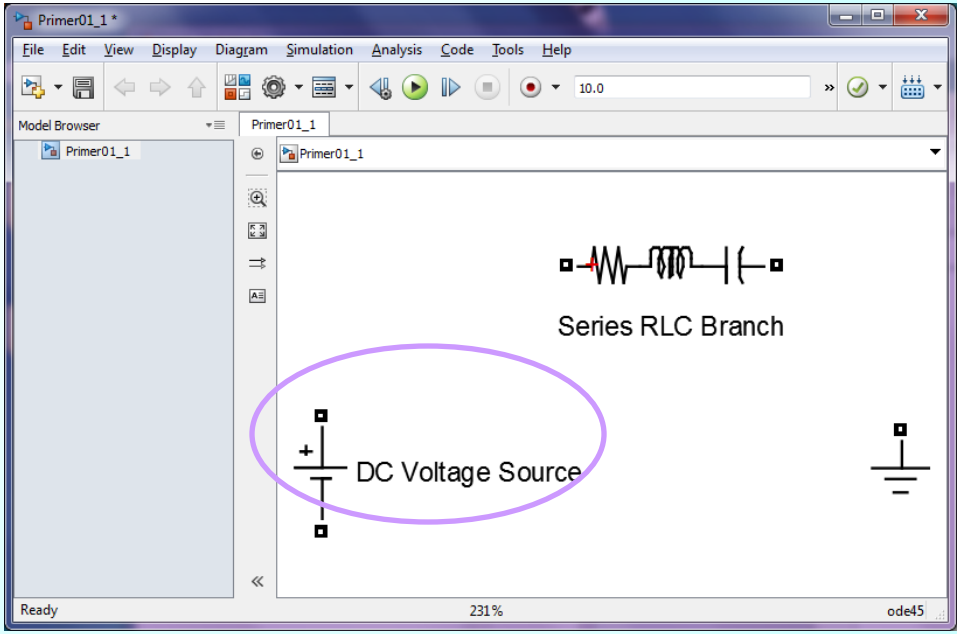


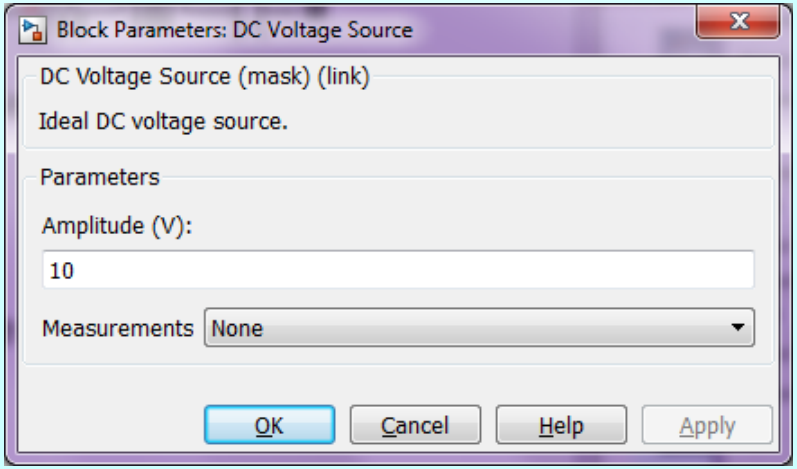
$R = 1\,\mathrm{k\Omega}$

$C = 1\,\mu\mathrm{F}$

$u_g(t) = U\,h(t), U = 10\,\mathrm{V}$

$u(t_0^-) = U_0 = -10\,\mathrm{V}$

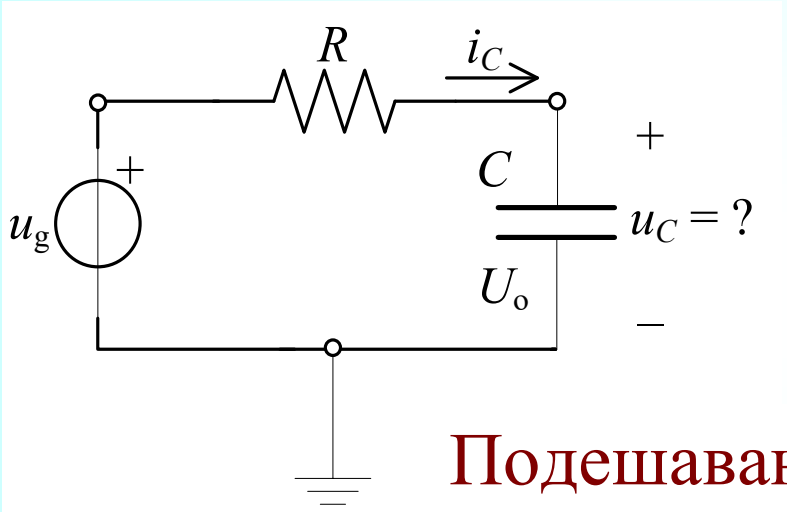




Simscape > Electrical

Симулација електричног кола коришћењем **Simscape>Electrical** библиотеке

коло је образовано у тренутку $t_0 = 0$



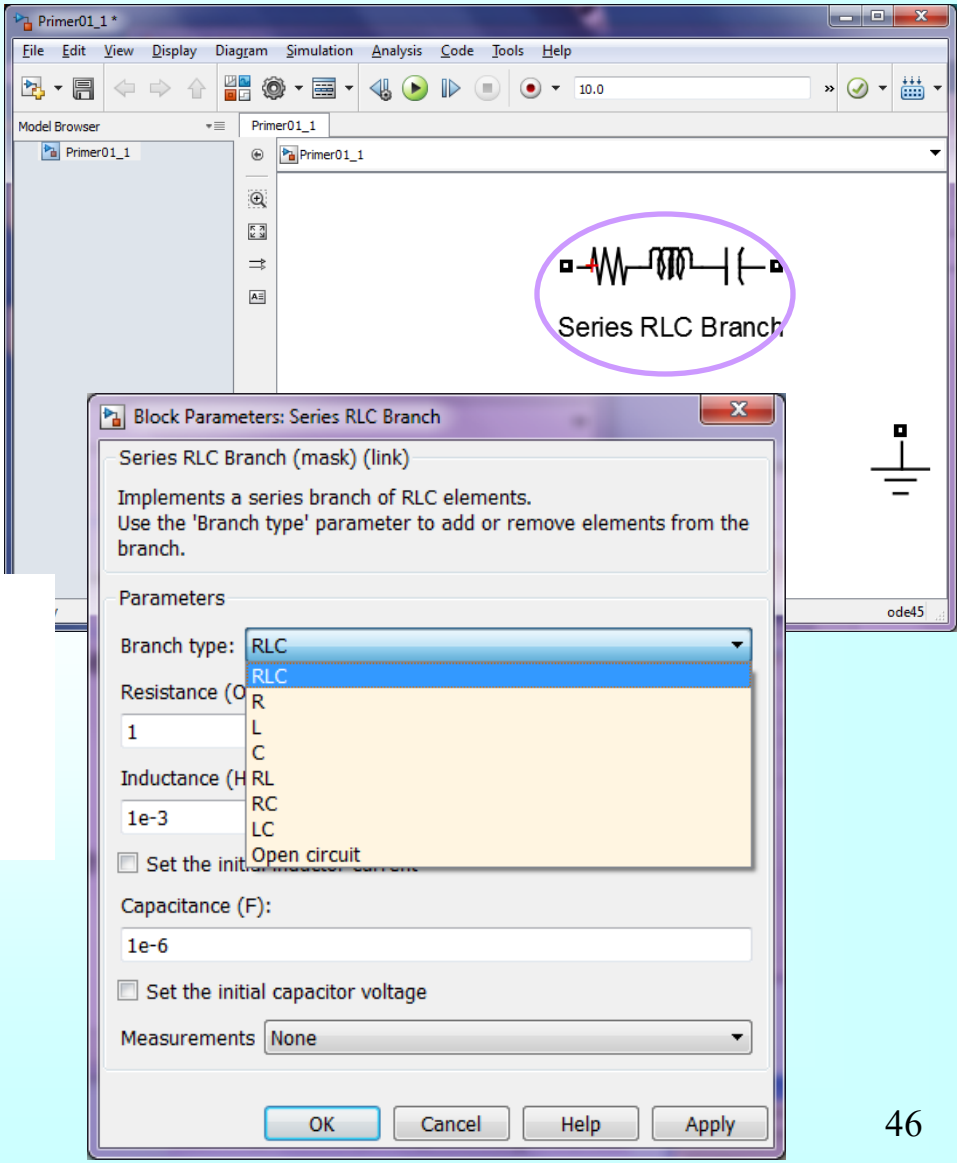
$R = 1\,\mathrm{k\Omega}$

$C = 1\,\mu\mathrm{F}$

$u_g(t) = U\,h(t), U = 10\,\mathrm{V}$

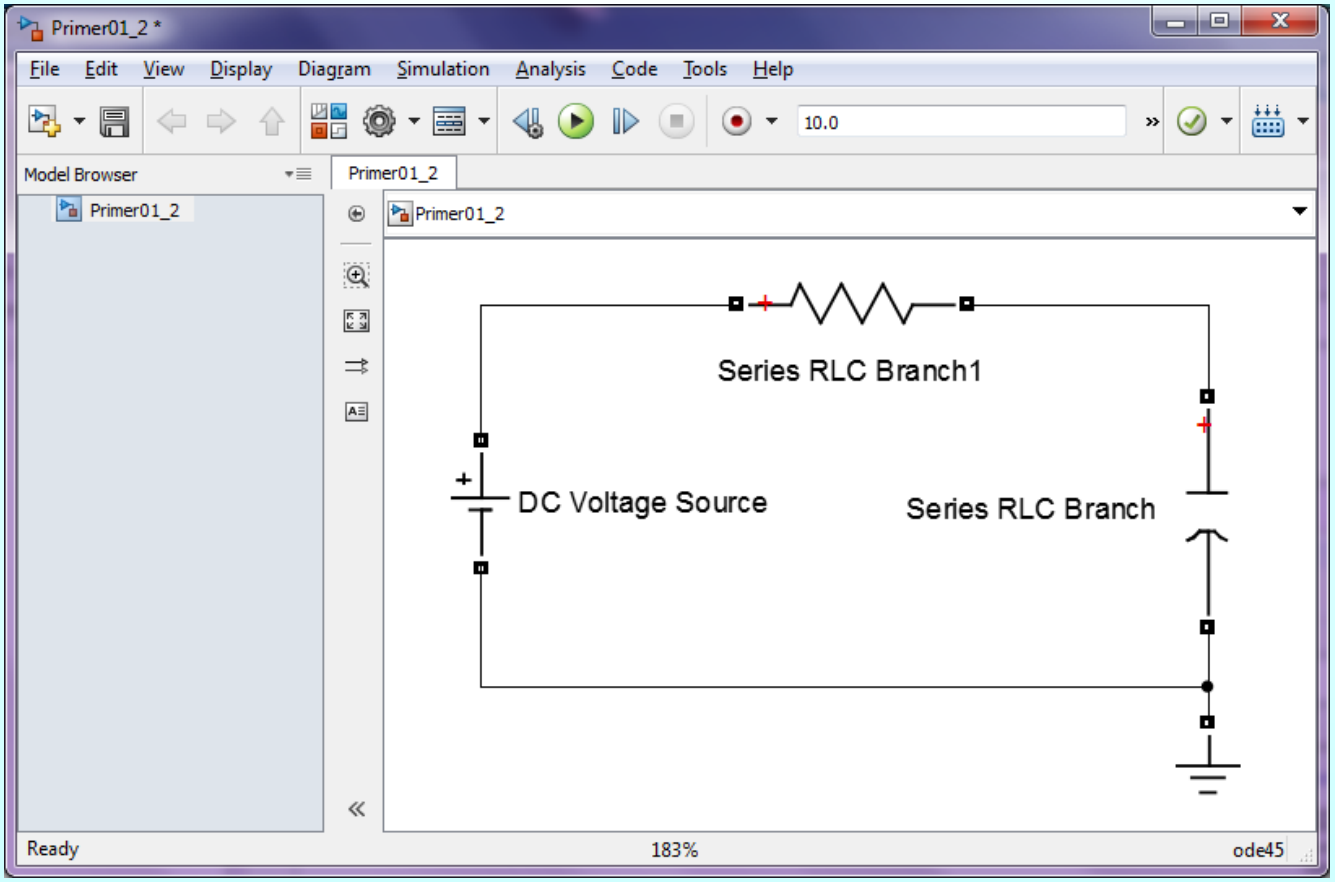
$u(t_0^-) = U_0 = -10\,\mathrm{V}$

Подешавање параметара симулације



Simscape > Electrical

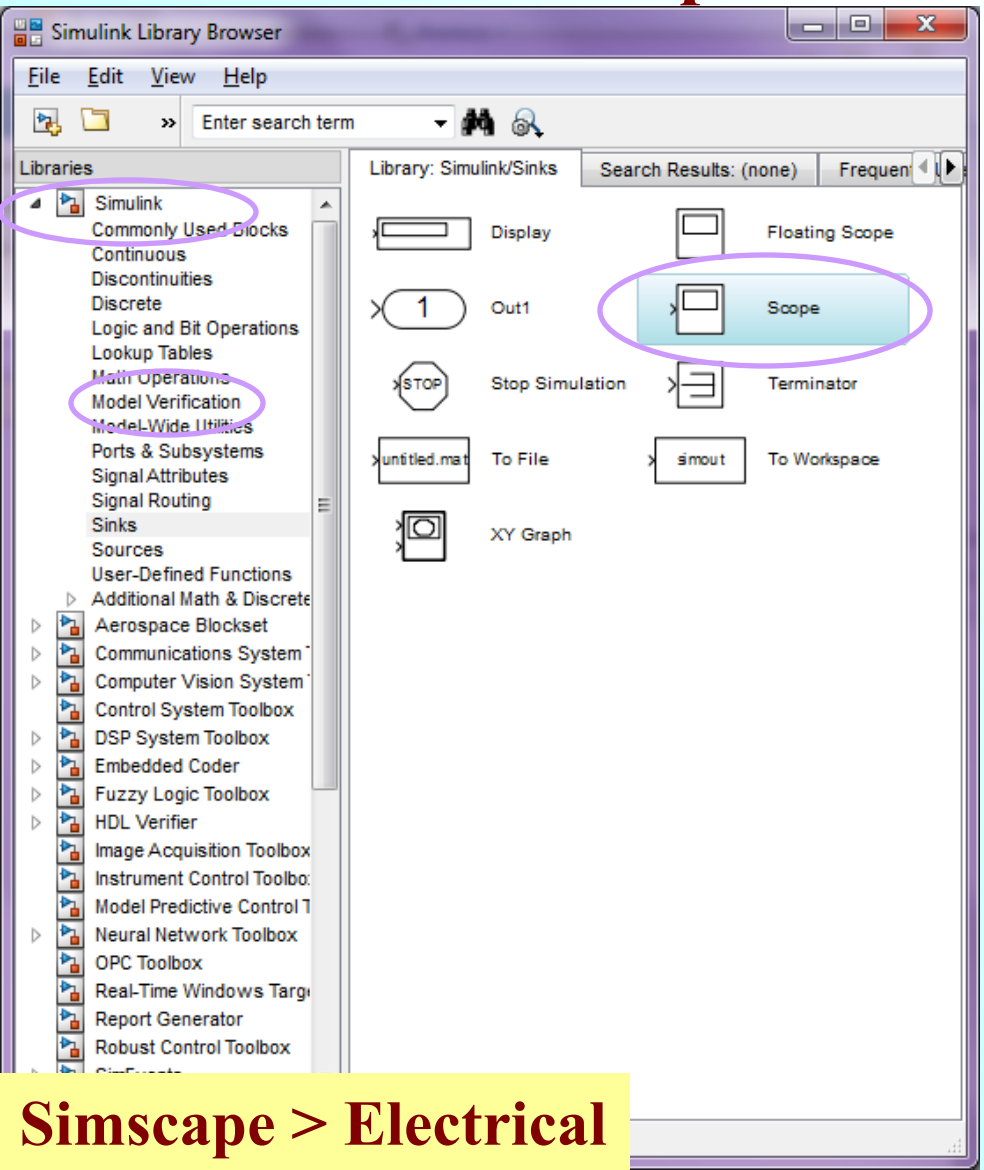
Симулација електричног кола коришћењем **Simscape>Electrical** библиотеке

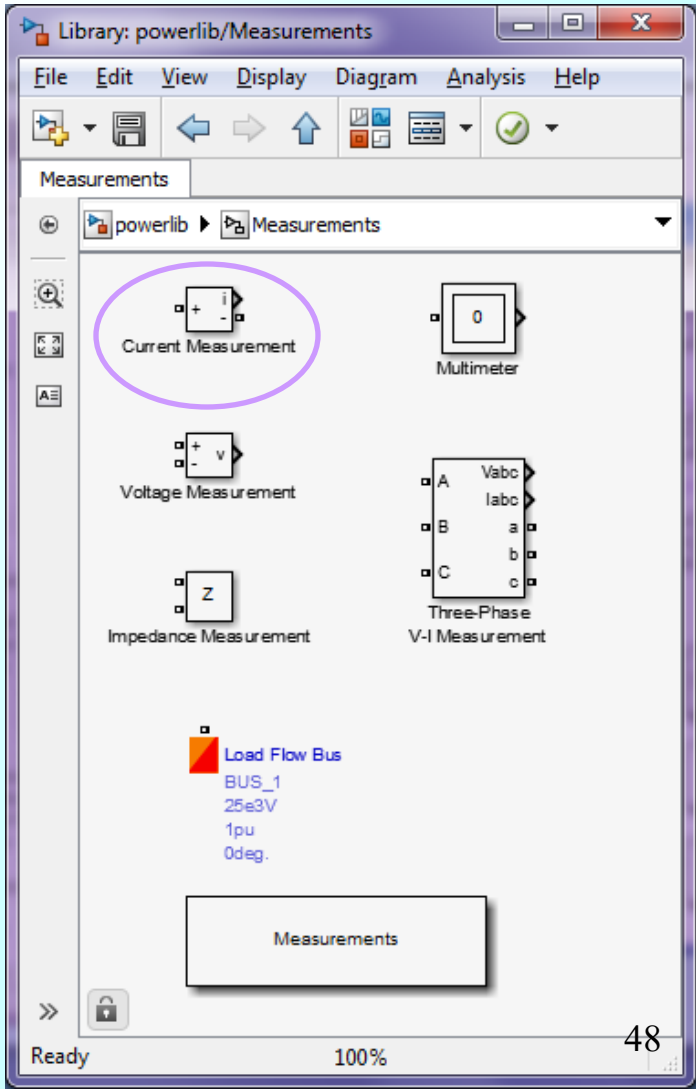


Како се мери напон?

Simscape > Electrical

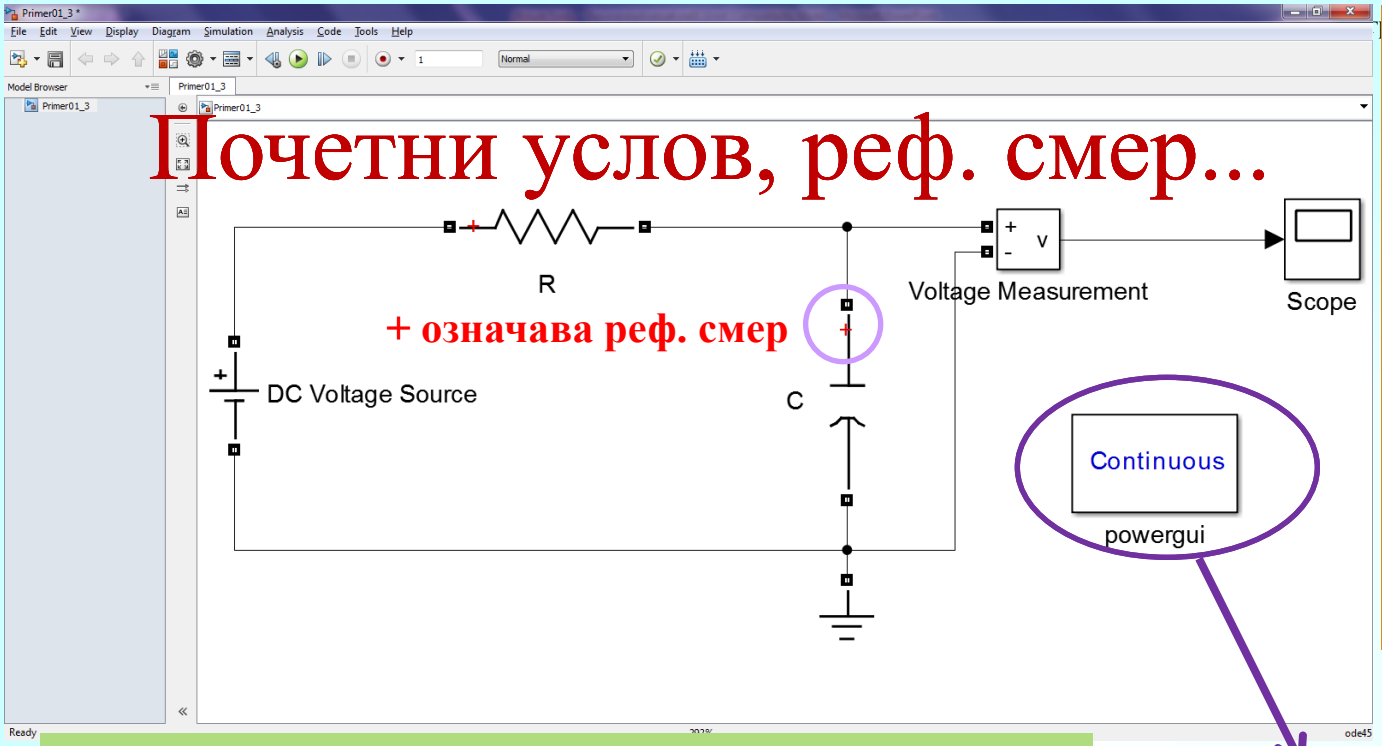
Симулација електричног кола коришћењем **Simscape>Electrical** библиотеке



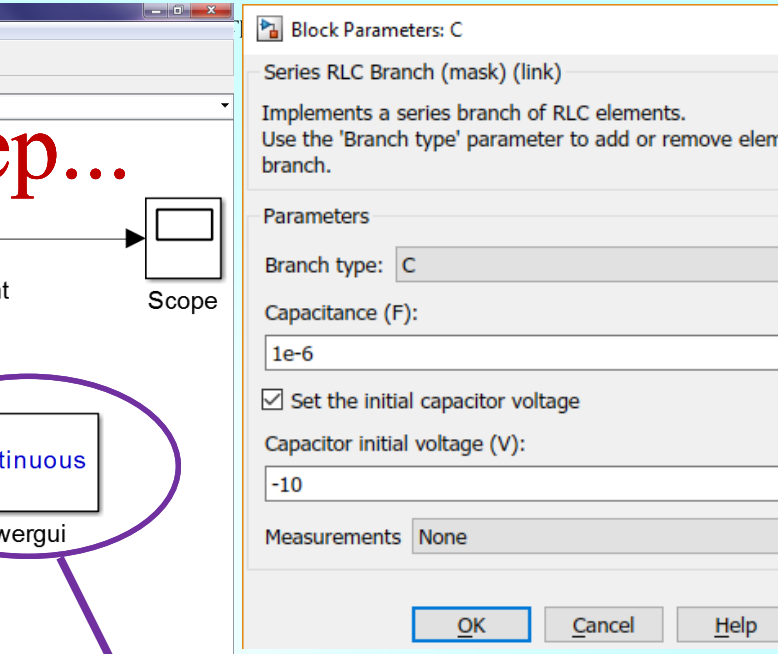


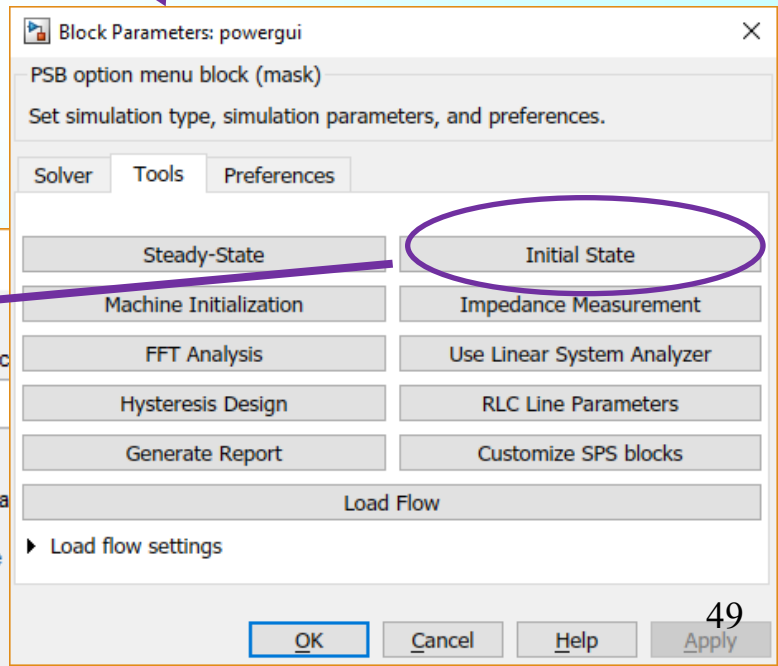
Simscape > Electrical

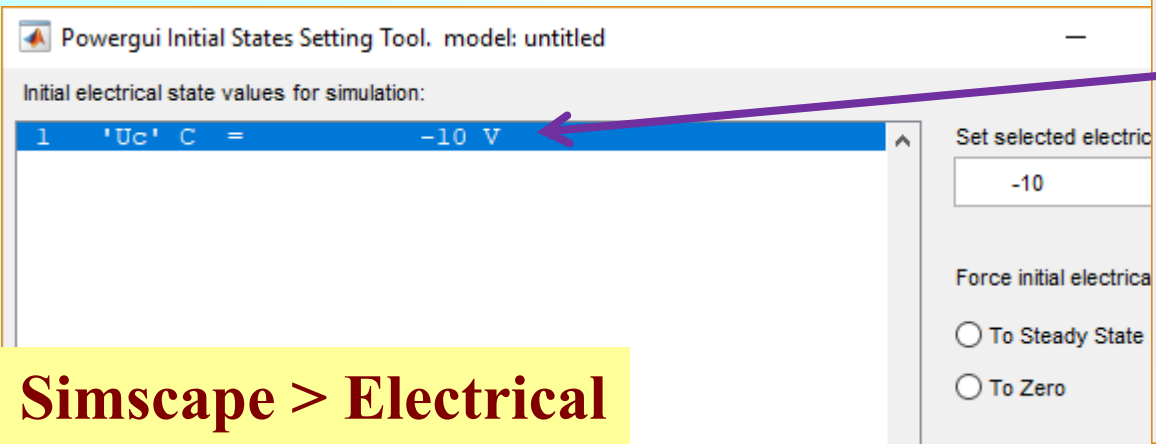
Почетни услов, реф. смер...



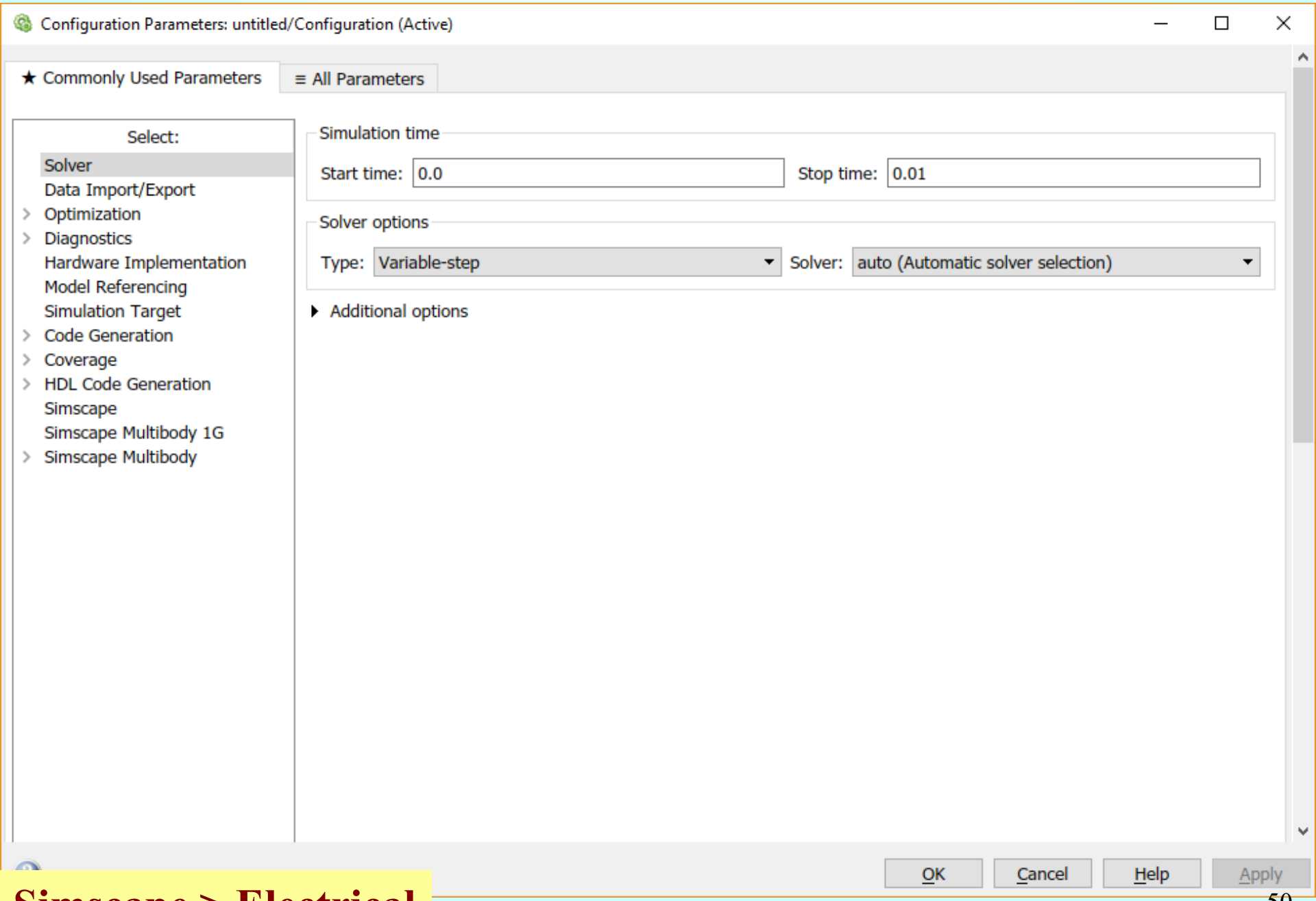
Плоче кондезатора се разликују!!!



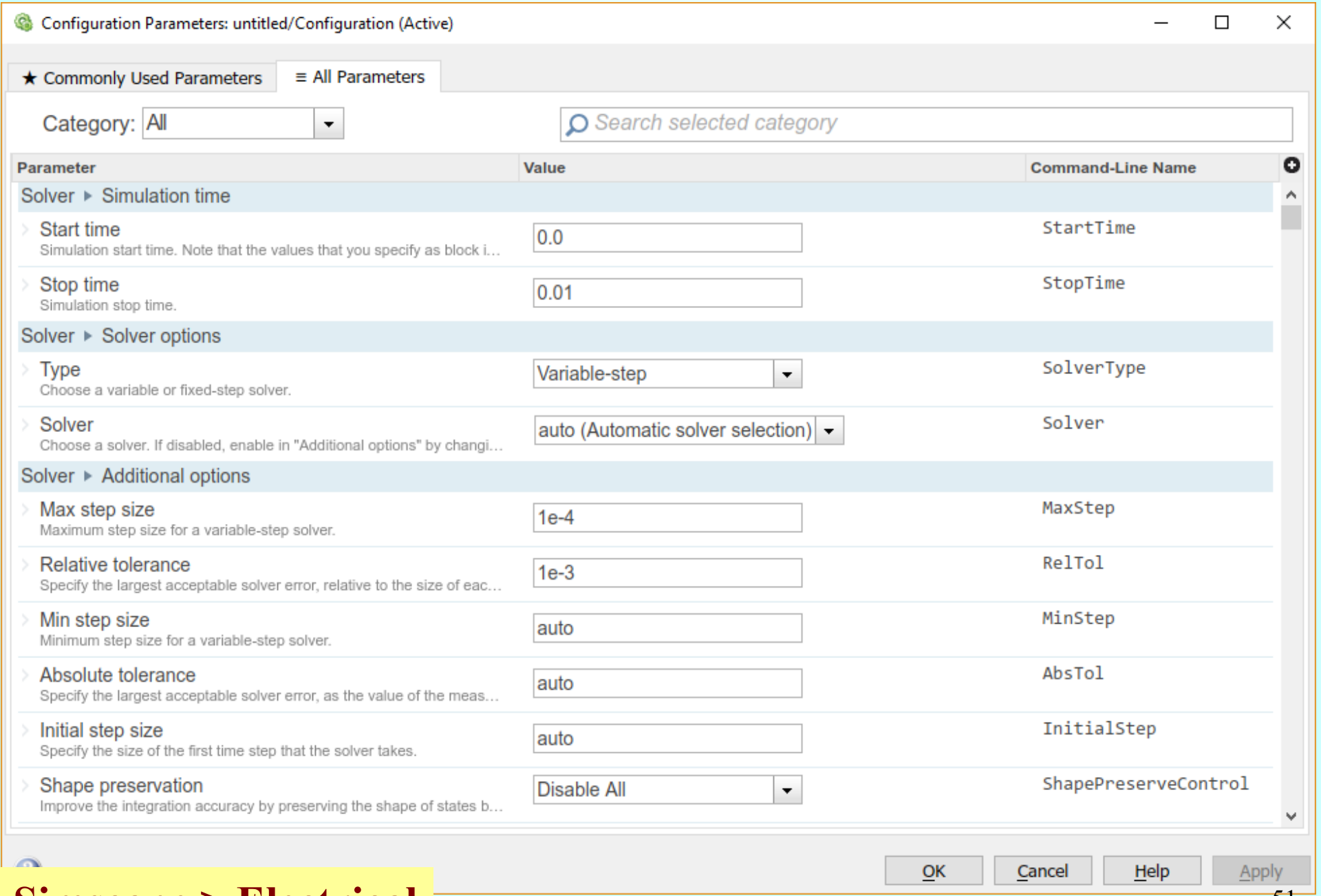




Simscape > Electrical

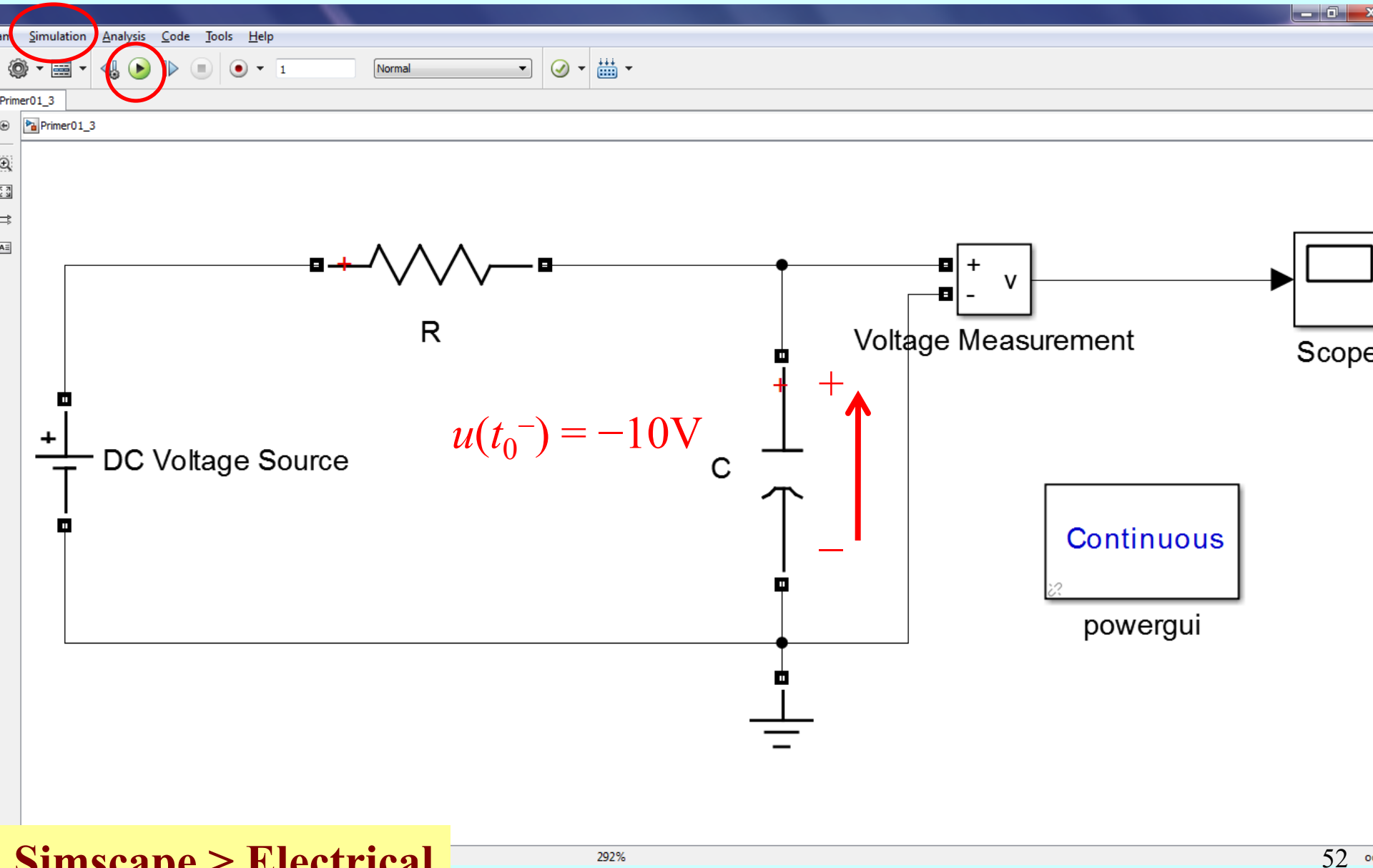


Simscape > Electrical



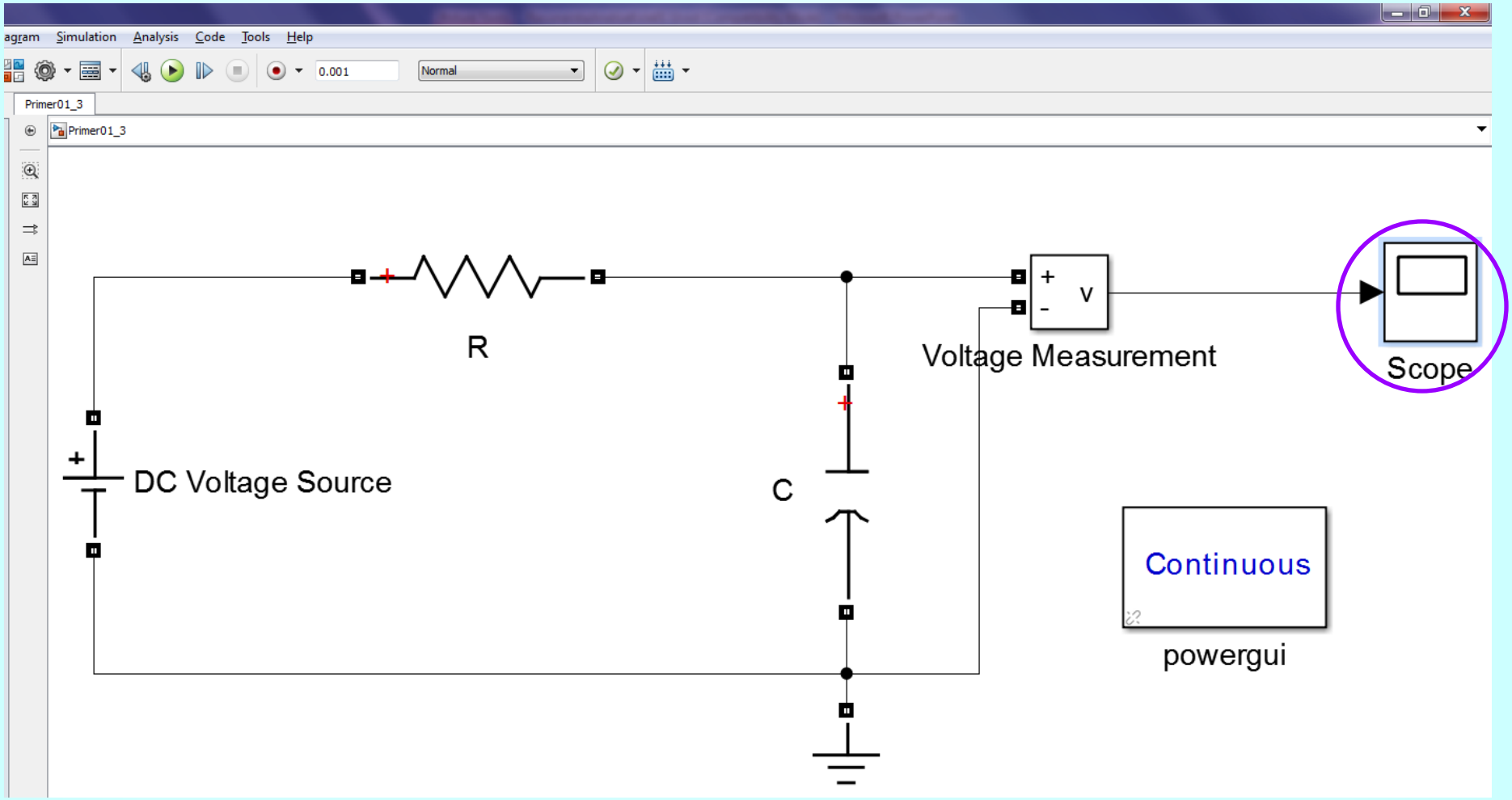
Simscape > Electrical

Покретање симулације...

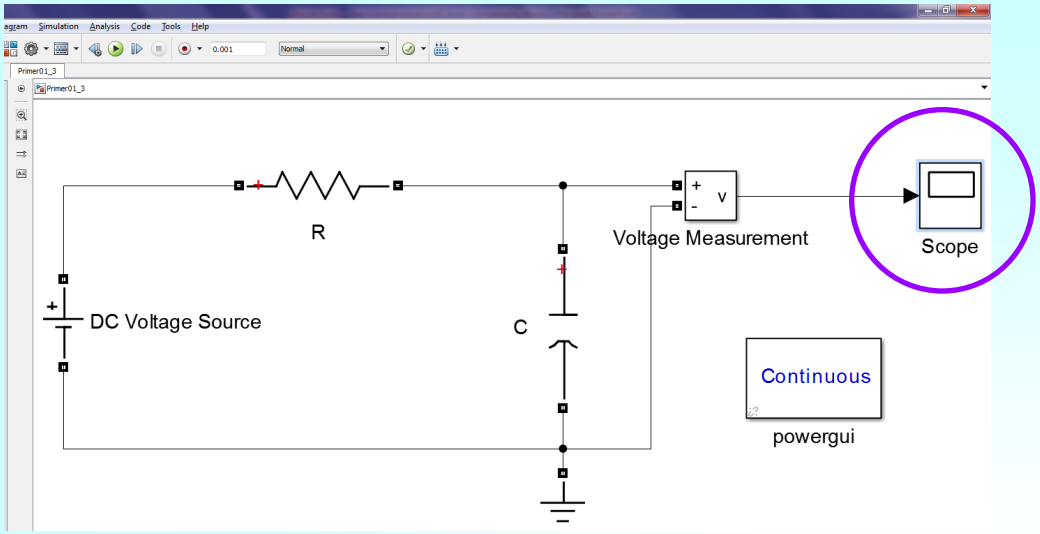


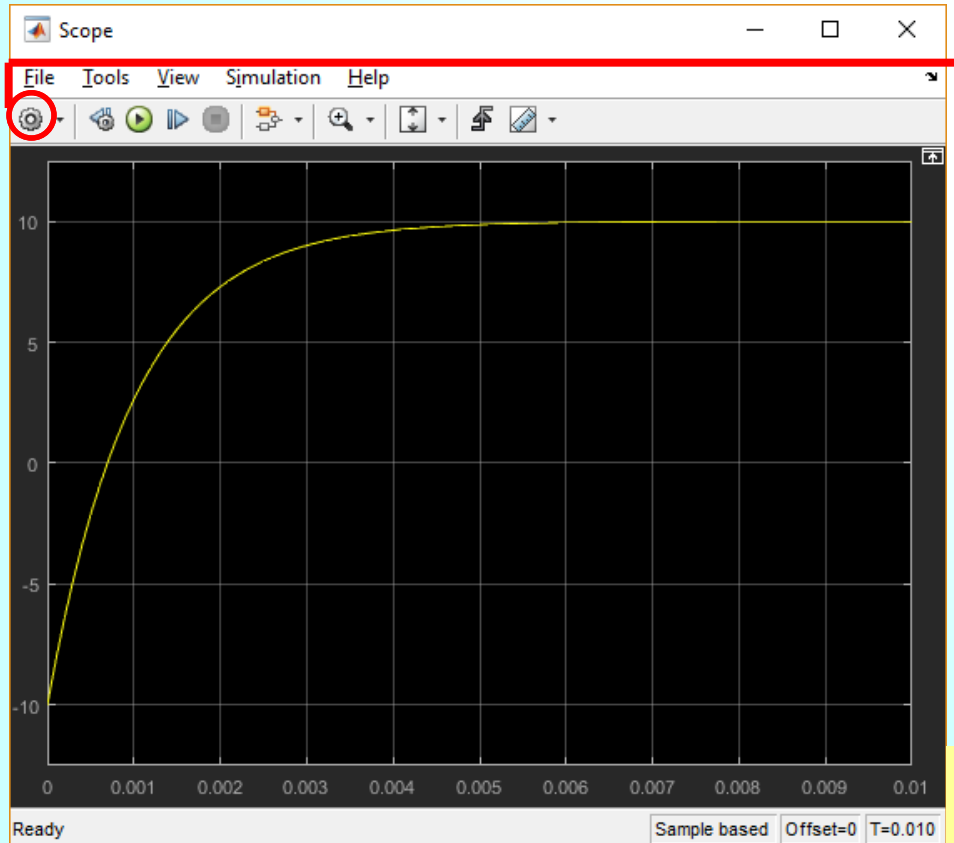
Simscape > Electrical

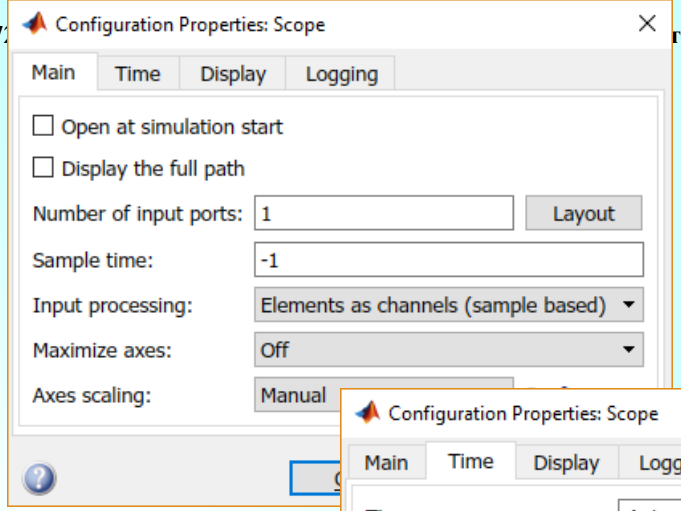
Налажење одзива... Осцилоскоп

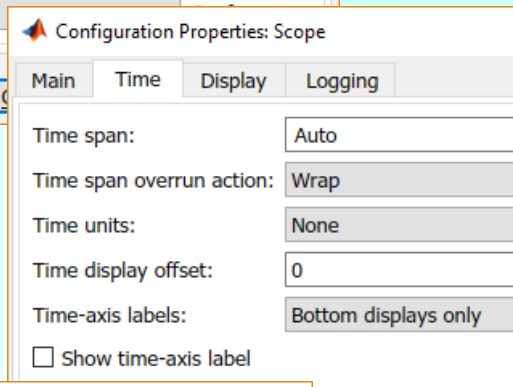


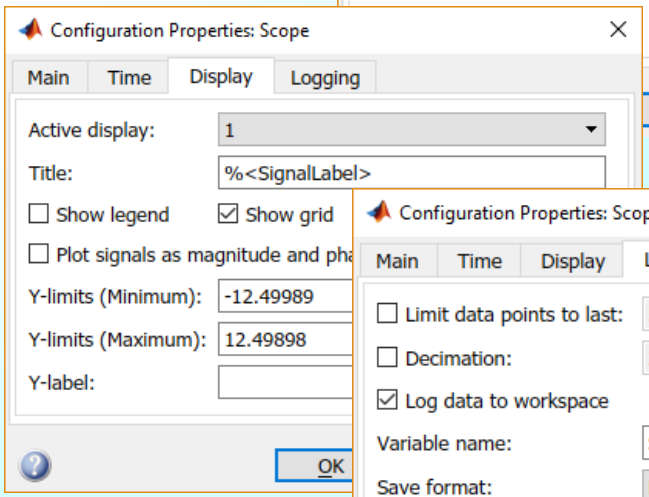
Simscape > Electrical

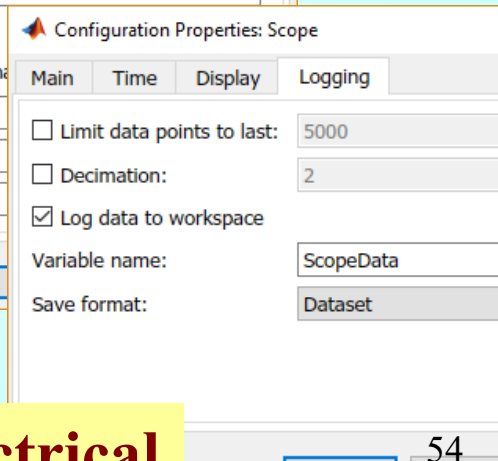












Simscape > Electrical

Configuration Parameters: untitled/Configuration (Active)

★ Commonly Used Parameters | ≡ All Parameters

Category: All

Search selected category

Parameter	Value	Command-Line Name
Solver ▸ Simulation time		
Start time Simulation start time. Note that the values that you specify as block i…	0.0	StartTime
Stop time Simulation stop time.	0.01	StopTime
Solver ▸ Solver options		
Type Choose a variable or fixed-step solver.	Variable-step	SolverType
Solver Choose a solver. If disabled, enable in "Additional options" by changi…	auto (Automatic solver selection)	Solver
Solver ▸ Additional options		
Max step size Maximum step size for a variable-step solver.	1e-4	MaxStep
Relative tolerance Specify the largest acceptable solver error, relative to the size of eac…	1e-3	RelTol
Min step size Minimum step size for a variable-step solver.	auto	MinStep
Absolute tolerance Specify the largest acceptable solver error, as the value of the meas…	auto	AbsTol
Initial step size Specify the size of the first time step that the solver takes.	auto	InitialStep
Shape preservation Improve the integration accuracy by preserving the shape of states b…	Disable All	ShapePreserveControl

$\tau = RC = 1\text{ms}$

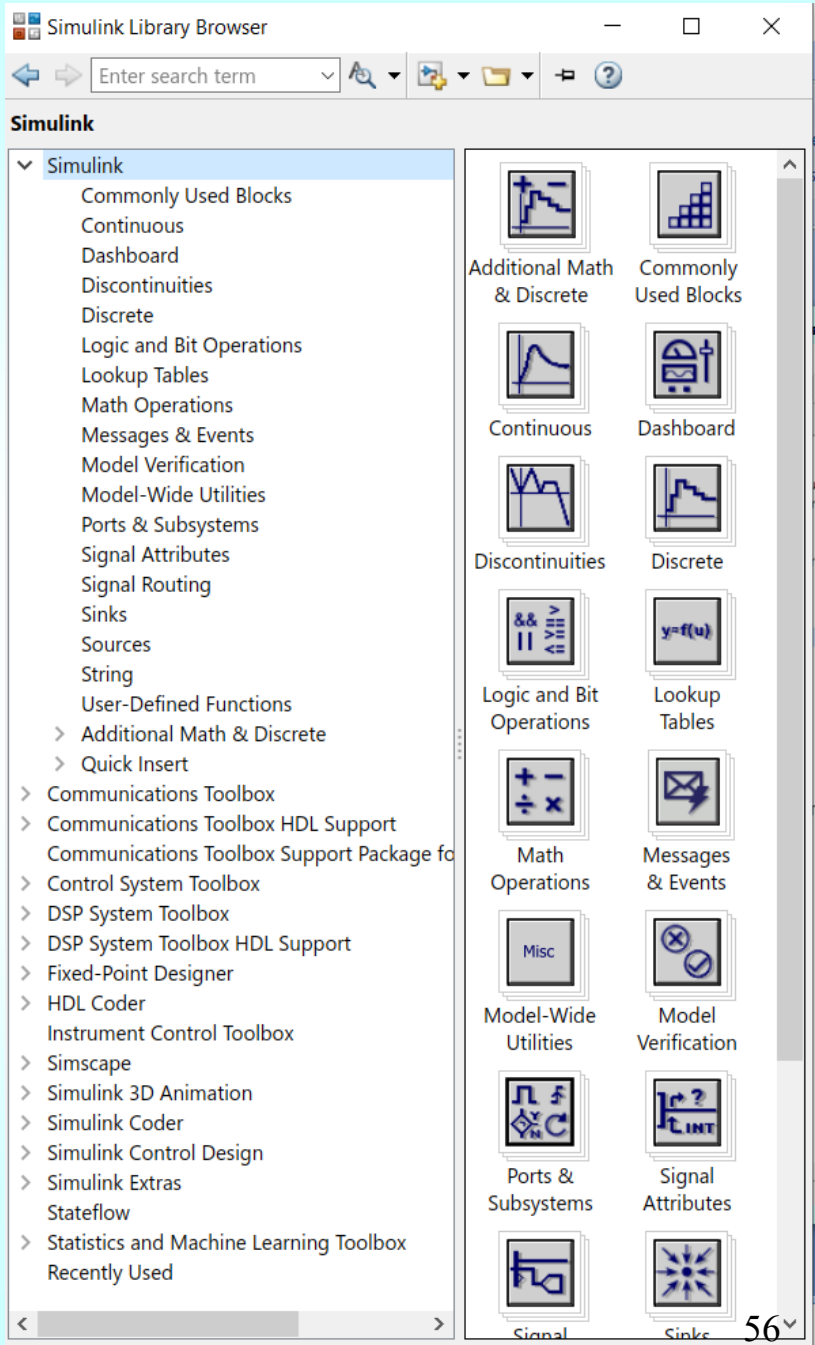
$\tau/10 = 0.1\text{ms}$

OK | Cancel | Help | Apply

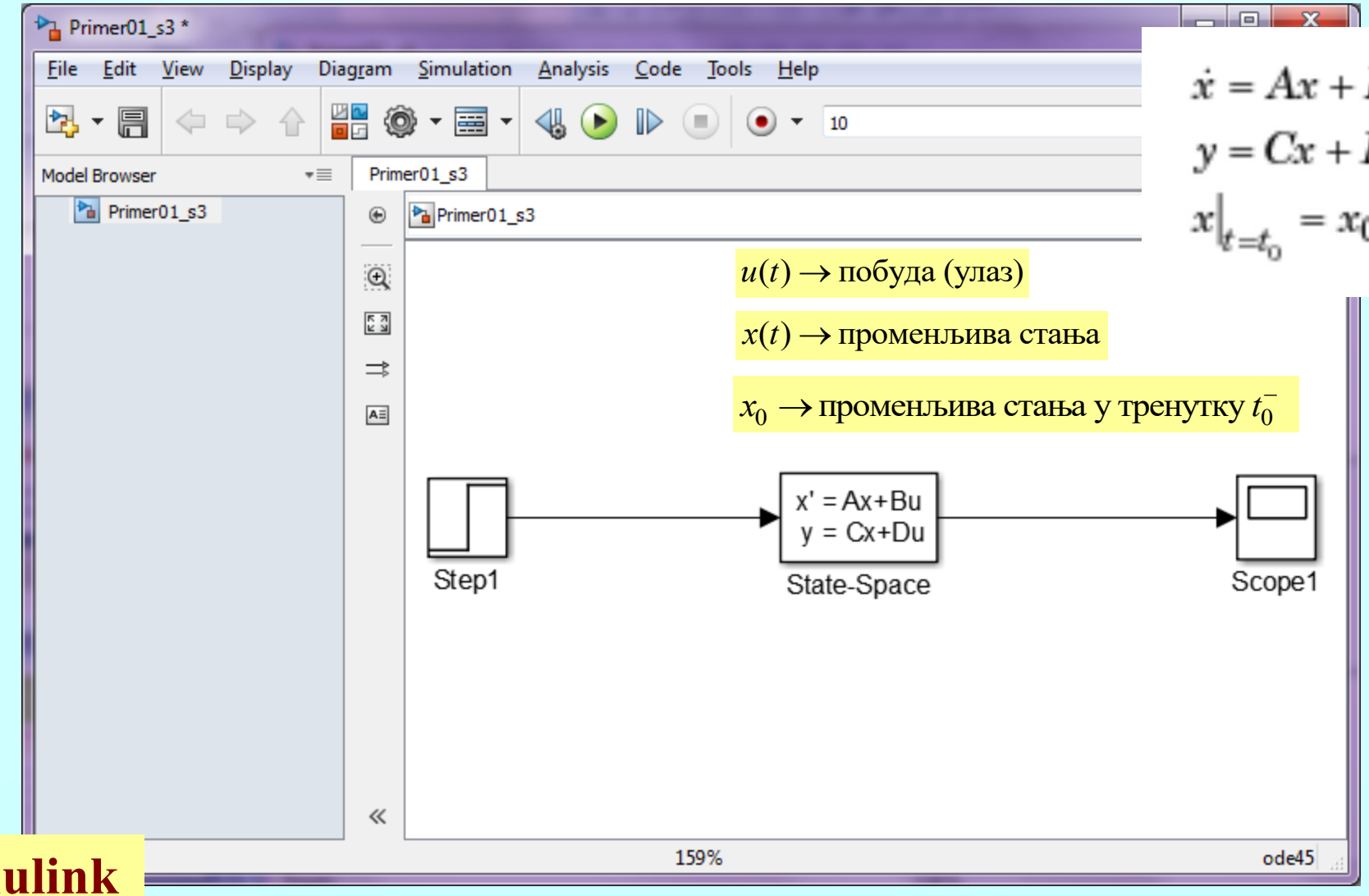
Simscape > Electrical

Simulink

- Решавање кола може се извршити његовим свођењем на систем линеарних диференцијалних једначина
- Дати систем може се представити једноставним блоком са више улаза и излаза

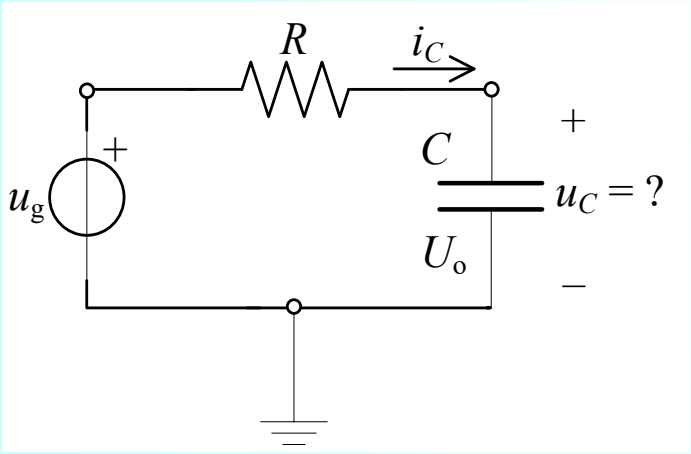


Симулација електричног кола као уопштеног система коришћењем **Simulink**-а



Simulink

Симулација електричног кола као уопштеног система коришћењем **Simulink**-а



$u = u_g(t) \rightarrow$ побуда (улаз)

$x = u_C(t) \rightarrow$ напон кондензатора (промењљива стања и одзив - излаз)

$x(t_0^-) = U_0 \rightarrow$ напон кондензатора у тренутку t_0^-

коло је образовано у тренутку $t_0 = 0$

$$u_g(t) = Ri_C + u_C(t)$$

$$i_C = C\frac{du_C}{dt}$$

$$u_C(t_0^-) = U_0 = -10\,\text{V}$$

$$\dot{x} = Ax + Bu$$
$$y = Cx + Du$$
$$x\big|_{t=t_0} = x_0\,,$$

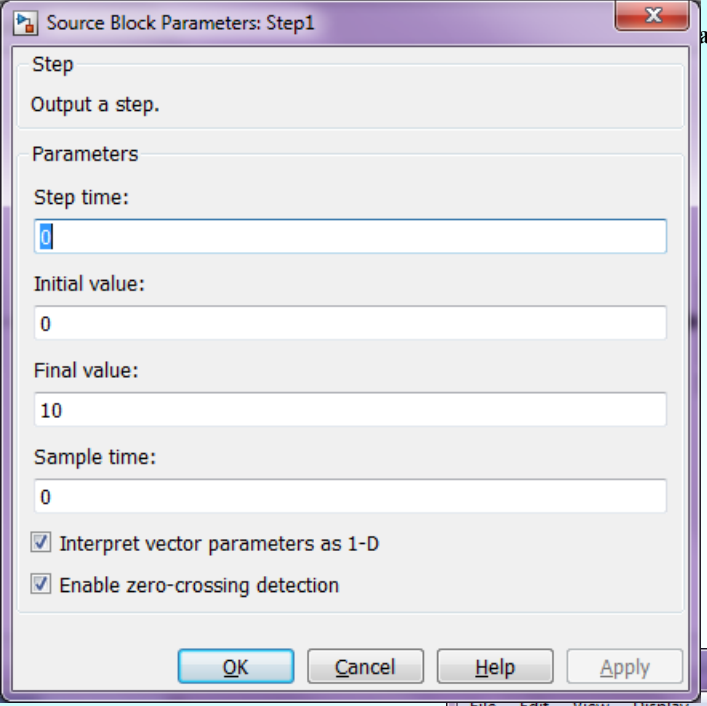
$$R = 1\,\text{k}\Omega$$
$$C = 1\,\mu\text{F}$$
$$u_g(t) = U\,h(t), U = 10\,\text{V}$$
$$u_C(t_0^-) = U_0 = -10\,\text{V}$$

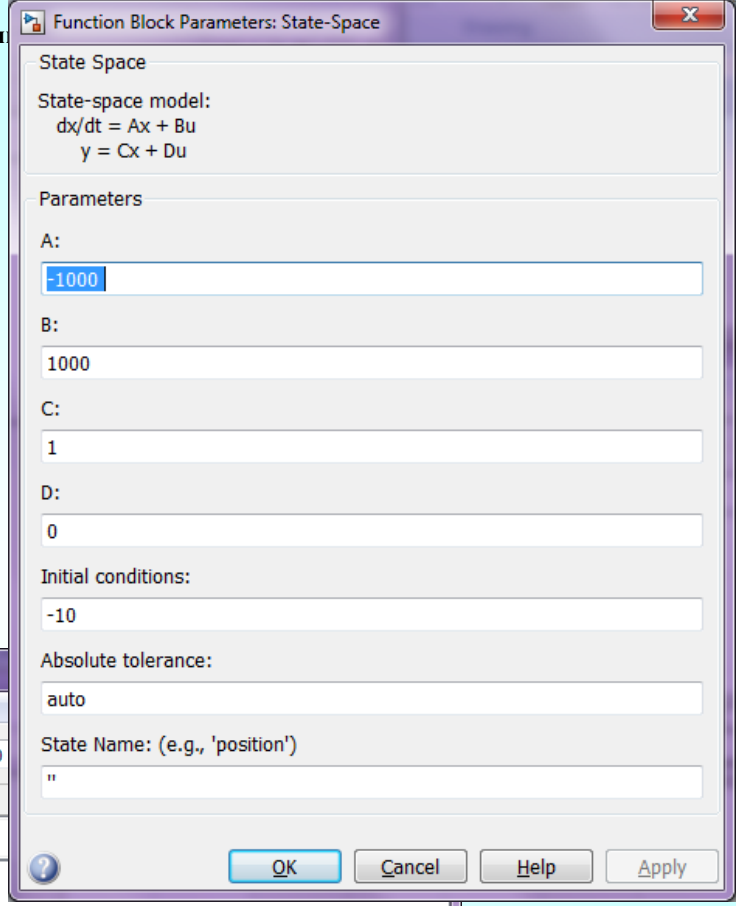
$$\frac{du_C(t)}{dt} = \frac{i_C}{C} = \frac{u_g(t) - u_C(t)}{RC} = -\frac{u_C(t)}{RC} + \frac{u_g(t)}{RC}$$
$$u_C(t_0^-) = U_0 = -10\,\text{V}$$

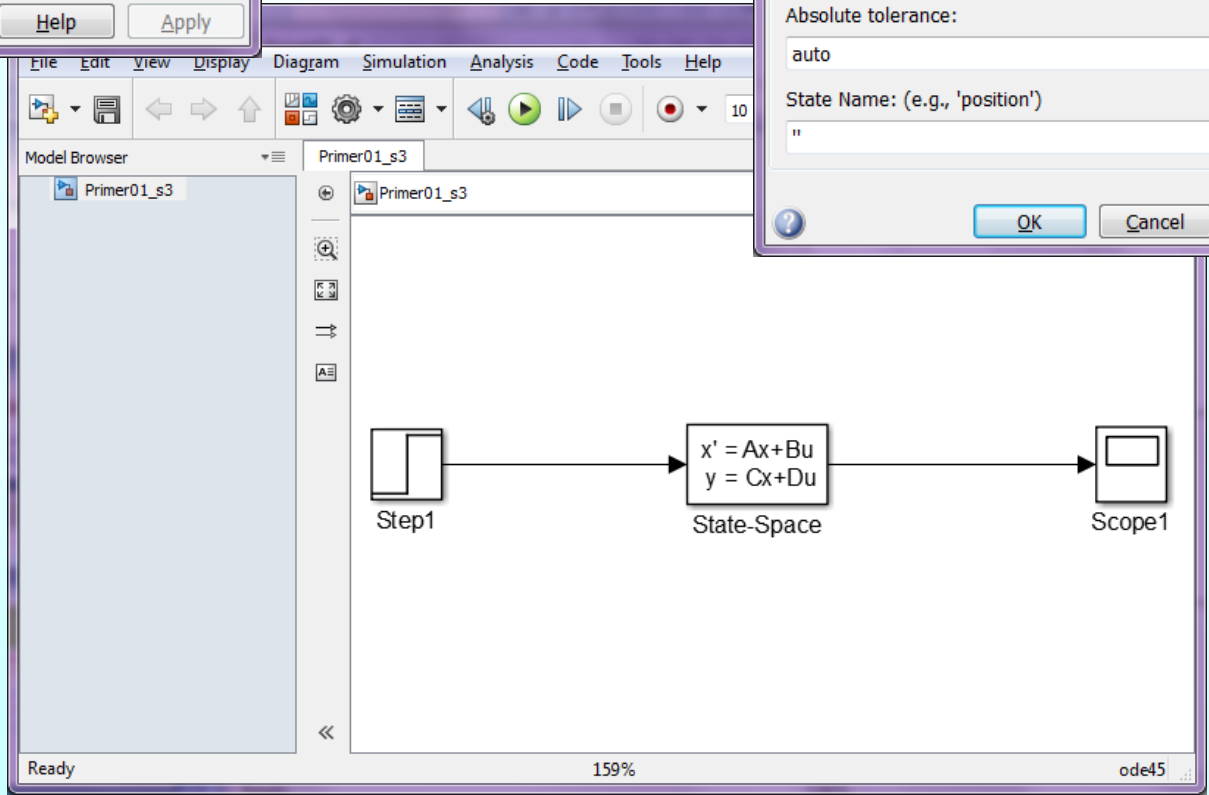
$$\frac{du_C(t)}{dt} = Au_C(t) + Bu_g(t) \Rightarrow A = -\frac{1}{RC} = -1000\,, B = 1000$$
$$u_C(t) = Cu_C(t) \Rightarrow C = 1, D = 0$$
$$u_C(t_0^-) = U_0 = -10\,\text{V}$$

Simulink

Симулација електричног кола као уопштеног система коришћењем **Simulink**-а

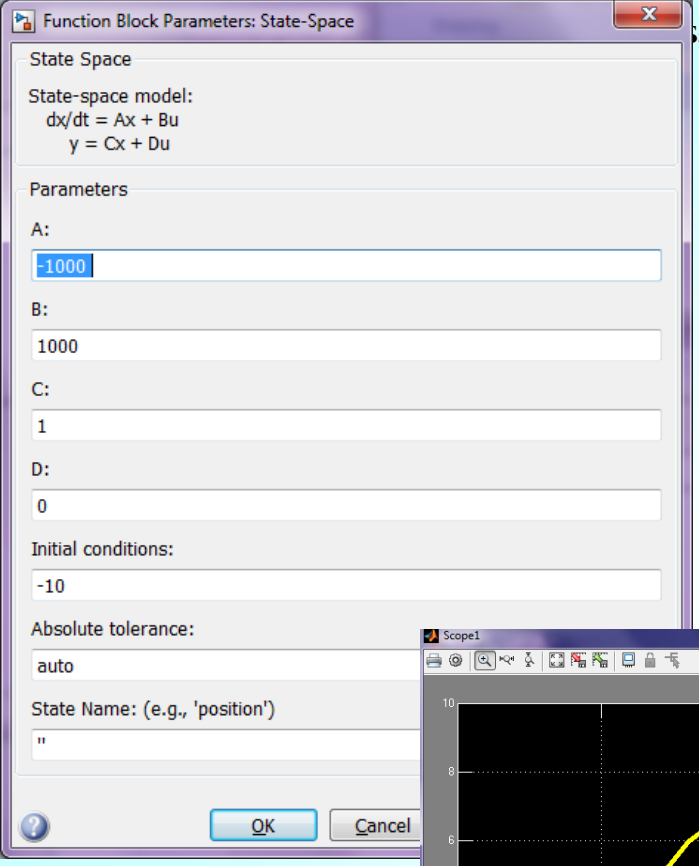


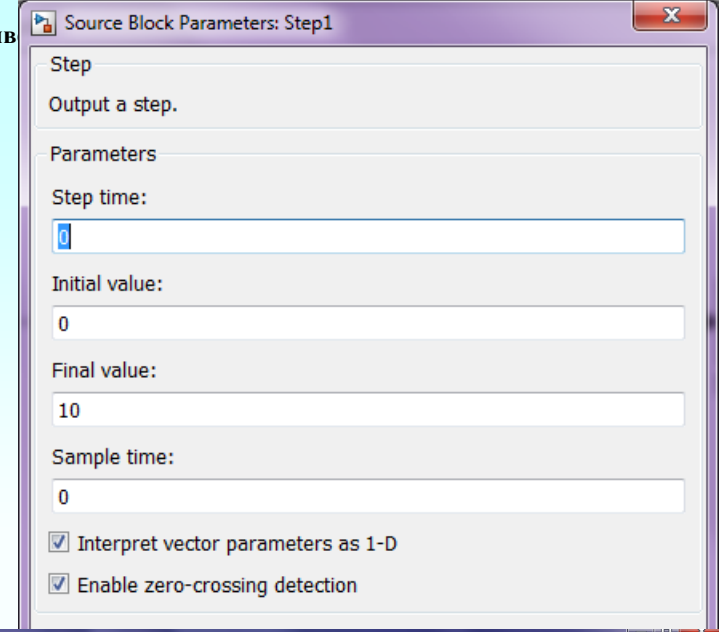


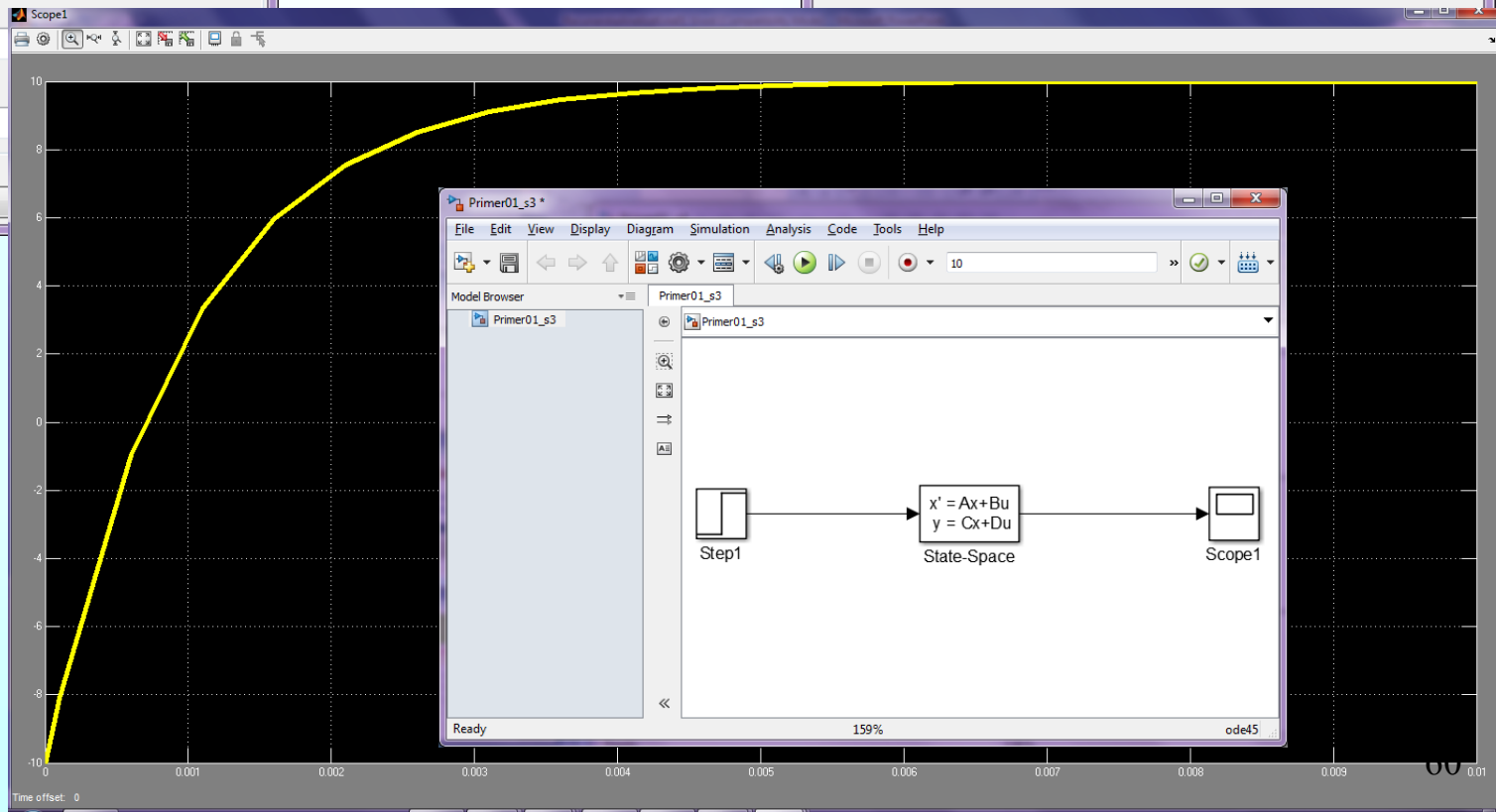


Simulink

Симулација електричног кола као уопштеног система коришћењем **Simulink**-а

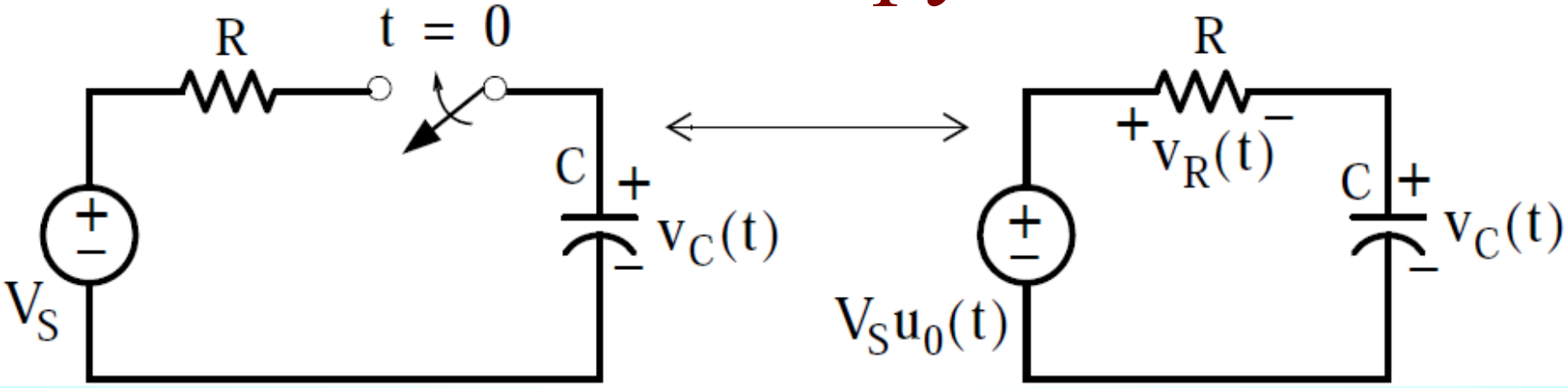






Simulink

Решимо “ручно”



$$v_R + v_C = V_S u_0(t)$$

$$RC\frac{dv_C}{dt} + v_C = V_S u_0(t)$$

$$i = i_C = C\frac{dv_C}{dt}$$

$t > 0$

$$RC\frac{dv_C}{dt} + v_C = V_S$$

$$v_R = Ri = RC\frac{dv_C}{dt}$$

$$v_C(t) = (V_S - V_S e^{-(1/RC)t})u_0(t)$$

MATLAB
Symbolic Math Toolbox (Live Script)

- Библиотека за симболички рачун (алгебарске и диференцијалне једначине)
- Омогућава нумеричку анализу резултата прорачуна

Live Editor - D:\Engineering_Work\ETF\Teaching\PRK\RCKolo_v0.mlx

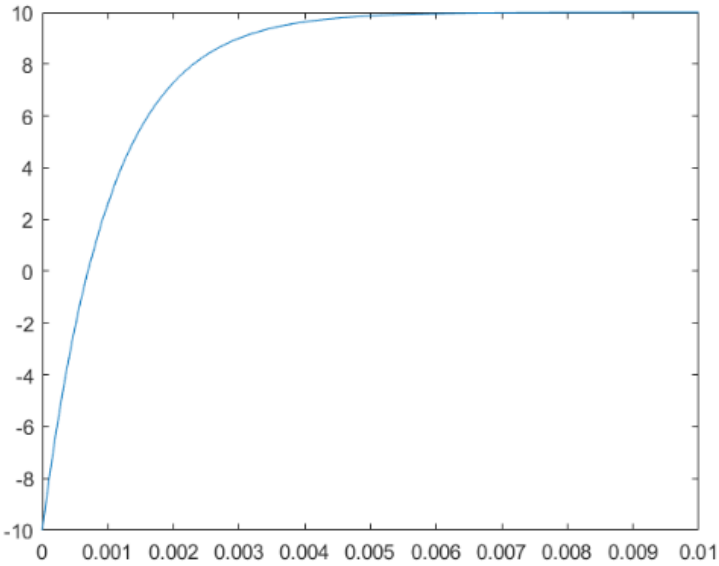
pluto1.m | template1.m | RCKolo_v0.mlx

```
1  syms v_c(t)
2  ode = diff(v_c,t) ==-1000*v_c+10000
3  cond = v_c(0)==-10
4  v_csol = dsolve(ode,cond)
5
6  fplot(v_csol, [0 0.01])
```

$$\frac{\partial}{\partial t} v_c(t) = 10000 - 1000\, v_c(t)$$

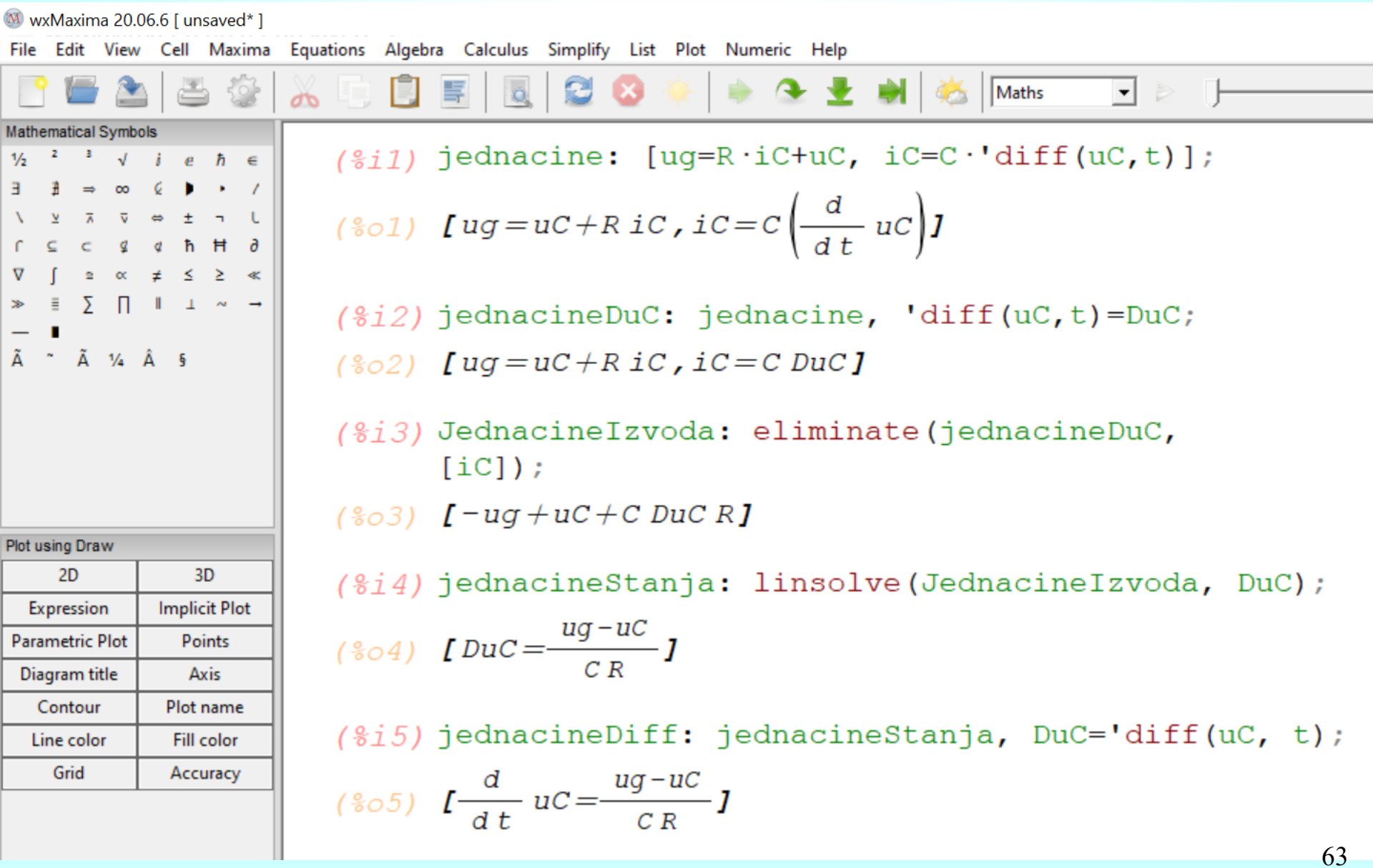
cond = $v_c(0) = -10$

v_csol = $10 - 20\, e^{-1000\, t}$



Simbolic Math Toolbox

wxMaxima


wxMaxima 20.06.6 [unsaved*]
File Edit View Cell Maxima Equations Algebra Calculus Simplify List Plot Numeric Help
Maths
Mathematical Symbols
Plot using Draw
2D
3D
Expression
Implicit Plot
Parametric Plot
Points
Diagram title
Axis
Contour
Plot name
Line color
Fill color
Grid
Accuracy
(%i1) jednacine: [ug=R·iC+uC, iC=C·'diff(uC,t)];
(%o1) [ug = uC + R iC, iC = C (d/dt uC)]
(%i2) jednacineDuC: jednacine, 'diff(uC,t)=DuC;
(%o2) [ug = uC + R iC, iC = C DuC]
(%i3) JednacineIzvoda: eliminate(jednacineDuC, [iC]);
(%o3) [−ug + uC + C DuC R]
(%i4) jednacineStanja: linsolve(JednacineIzvoda, DuC);
(%o4) [DuC = (ug − uC)/(C R)]
(%i5) jednacineDiff: jednacineStanja, DuC='diff(uC, t);
(%o5) [d/dt uC = (ug − uC)/(C R)]

```
(%i6) zamene: [ug=U];
```

(%o6) $[ug=U]$

```
(%i7) vrednosti: [R=1000, C=10^(-6), U=10, U0=-10];
```

(%o7) $[R=1000, C=\frac{1}{1000000}, U=10, U0=-10]$

```
(%i8) JednacineDiffZamena: jednacineDiff, zamene;
```

(%o8) $[\frac{d}{dt}\,uC=\frac{U-uC}{CR}]$

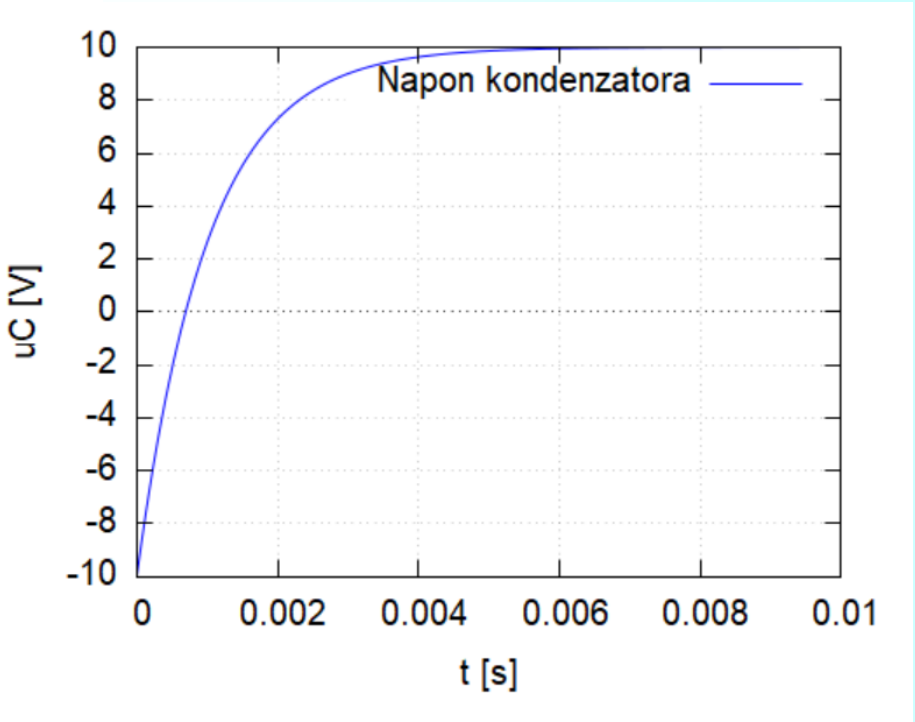
```
(%i9) ode2(JednacineDiffZamena, uC, t);
```

(%o9) $uC=\%e^{-\frac{t}{CR}}\left(U\,\%e^{\frac{t}{CR}}+\%c\right)$

```
(%i10) ic1(%, t=0, uC=U0);
```

(%o10) $uC=\%e^{-\frac{t}{CR}}\left(U\,\%e^{\frac{t}{CR}}+U0-U\right)$

```
(%i11) uCt: ev(%, vrednosti);
```

(%o11) $uC=\%e^{-1000\,t}\left(10\,\%e^{1000\,t}-20\right)$

```
(%i17) wxplot2d(rhs(uCt), [t,0,0.01],
       [xlabel," t [s] "],
       [ylabel," uC [V] "],
       [xtics, 0, 0.002, 0.01],
       [legend, "Napon kondenzatora"], grid2d)$
```