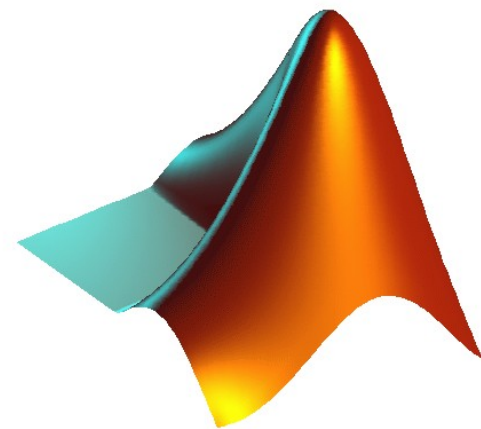
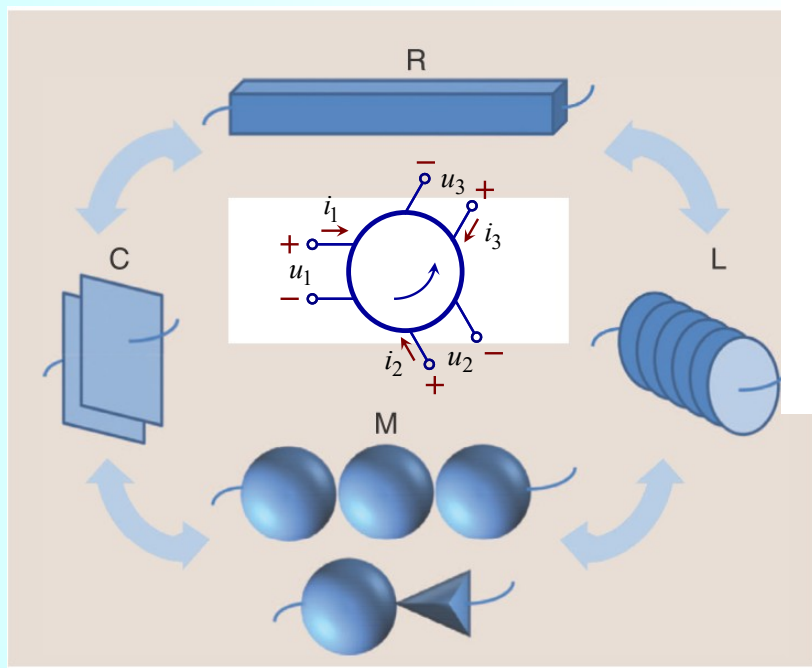


Практикум из рачунарске анализе кола



Милка Потребих



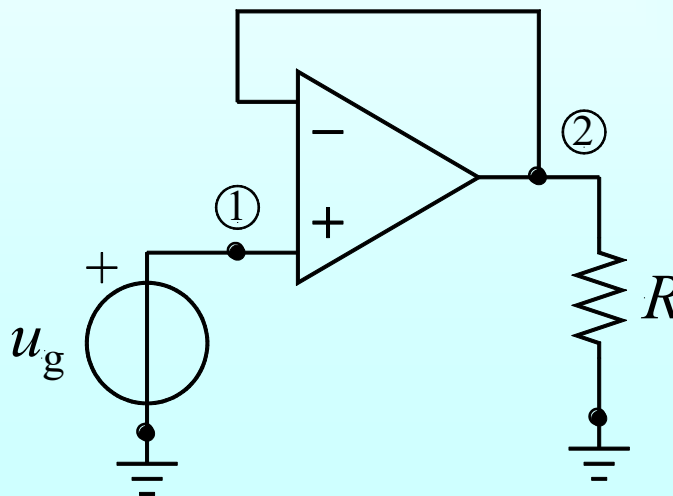
Рачунарски (софтверски) алати

- *Mathematica*, WolframAlpha, MuPAD, Maxima, SymPy, SymPy Live, SymPy Gamma
- MATLAB, Scilab, Octave, FreeMat, Julia
- LTspice, QucsStudio, ngspice, Xyce, XCircuit
- Python, MathCAD, MAPLE, GeoGebra
- Symbolab, SpeQ Mathematics, meta-calculator, Desmos, ...

Free/Libre Open Source Software (FLOSS)

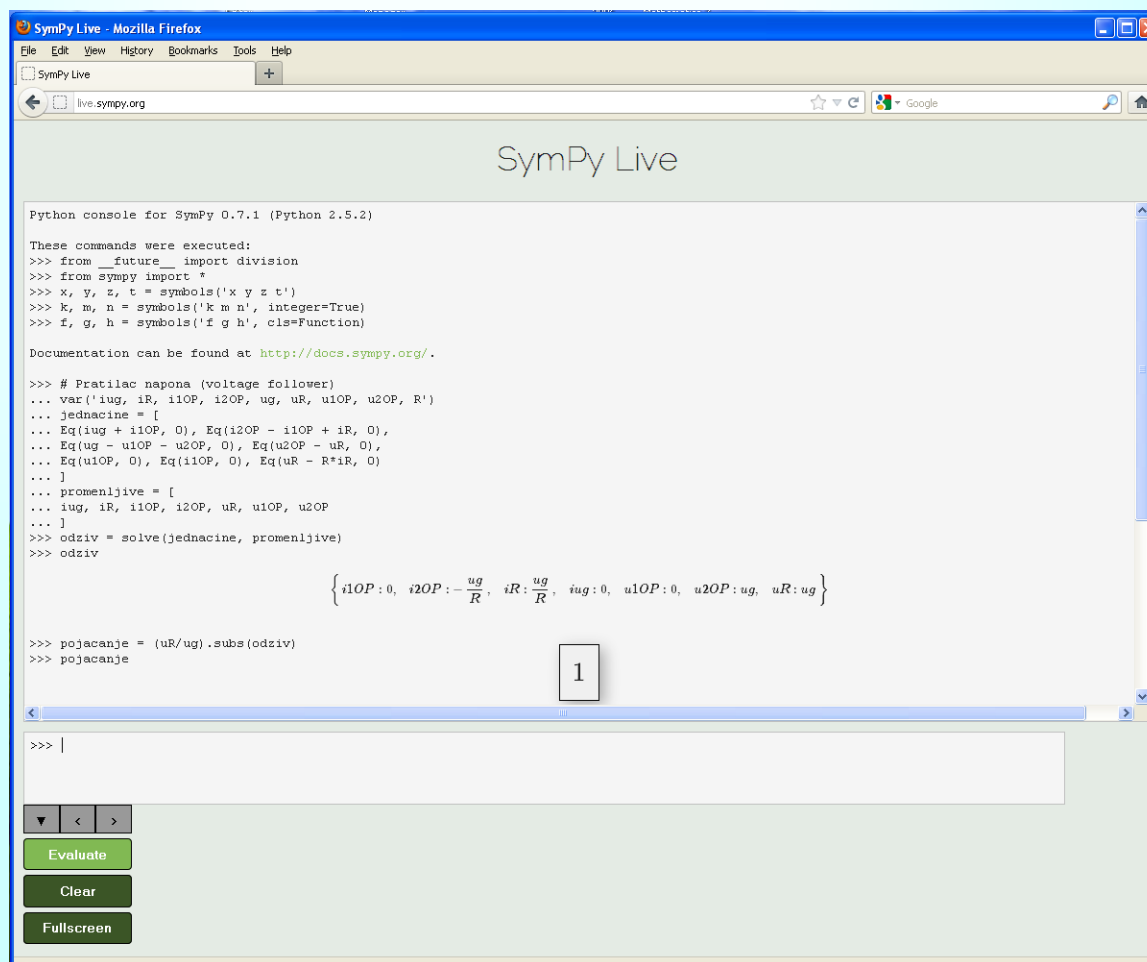
Пратилац напона

Одредити напонско појачање кола са слике, које се назива пратилац напона (voltage follower).



SymPy Live

Symbolic Python Live, <http://live.sympy.org/>



SymPy Live

Python console for SymPy 0.7.1 (Python 2.5.2)

These commands were executed:

```
>>> from __future__ import division
>>> from sympy import *
>>> x, y, z, t = symbols('x y z t')
>>> k, m, n = symbols('k m n', integer=True)
>>> f, g, h = symbols('f g h', cls=Function)
```

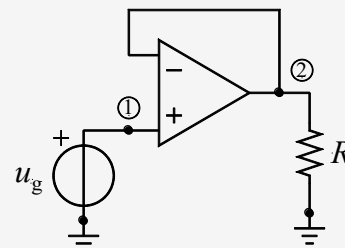
Documentation can be found at <http://docs.sympy.org/>.

```
>>> # Pratilac napona (voltage follower)
... var('iug, iR, i1OP, i2OP, ug, uR, u1OP, u2OP, R')
... jednacine = [
... Eq(iug + i1OP, 0), Eq(i2OP - i1OP + iR, 0),
... Eq(ug - u1OP - u2OP, 0), Eq(u2OP - uR, 0),
... Eq(u1OP, 0), Eq(i1OP, 0), Eq(uR - R*iR, 0)
... ]
... promenljive = [
... iug, iR, i1OP, i2OP, uR, u1OP, u2OP
... ]
>>> odz = solve(jednacine, promenljive)
>>> odz
```

$$\left\{ i1OP : 0, \quad i2OP : -\frac{ug}{R}, \quad iR : \frac{ug}{R}, \quad iug : 0, \quad u1OP : 0, \quad u2OP : ug, \quad uR : ug \right\}$$

```
>>> pojacanje = (uR/ug).subs(odz)
>>> pojacanje
```

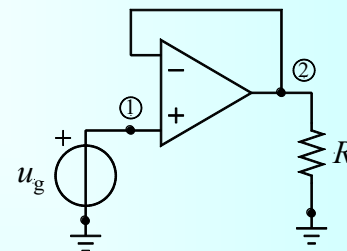
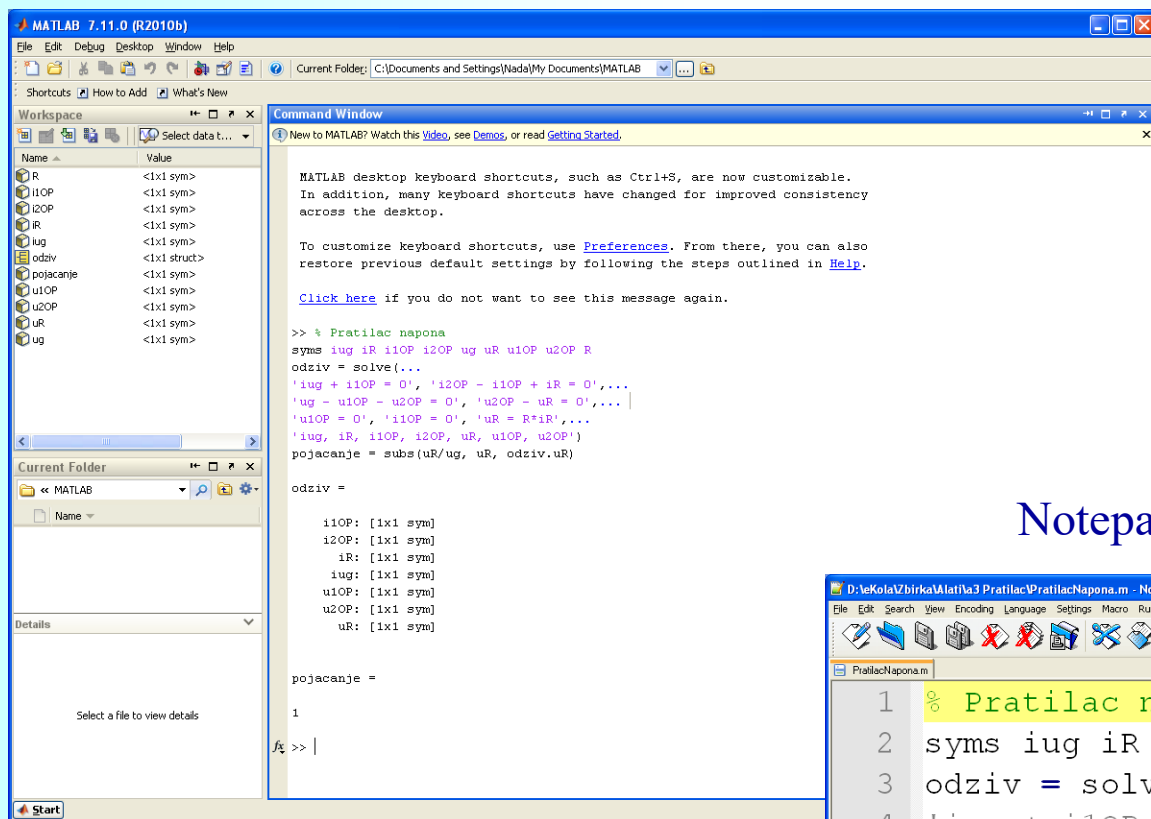
1



```
D:\eKola\Zbirka\Latila3 Pratilac\PratilacNapona.py - Notepad++
File Edit Search View Encoding Language Settings Macro Run Plugins Window ?
PratilacNapona.py
1 # Pratilac napona (voltage follower)
2 var('iug, iR, i1OP, i2OP, ug, uR, u1OP, u2OP, R')
3 jednacine = [
4 Eq(iug + i1OP, 0), Eq(i2OP - i1OP + iR, 0),
5 Eq(ug - u1OP - u2OP, 0), Eq(u2OP - uR, 0),
6 Eq(u1OP, 0), Eq(i1OP, 0), Eq(uR - R*iR, 0)
7 ]
8 promenljive = [
9 iug, iR, i1OP, i2OP, uR, u1OP, u2OP
10 ]
11 odziv = solve(jednacine, promenljive)
12 pojacanje = (uR/ug).subs(odziv)
```

Notepad++, <http://notepad-plus-plus.org/>

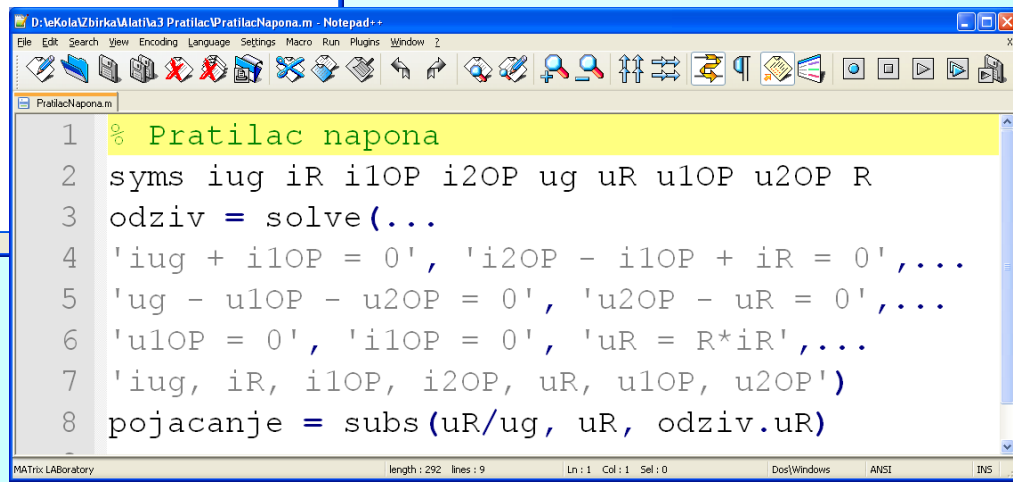
MATLAB: Symbolic Math Toolbox



Notepad++, <http://notepad-plus-plus.org/>

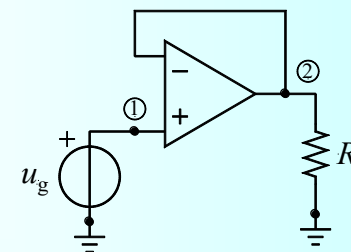
```
pojacanje =

1
```



MATLAB: MuPAD

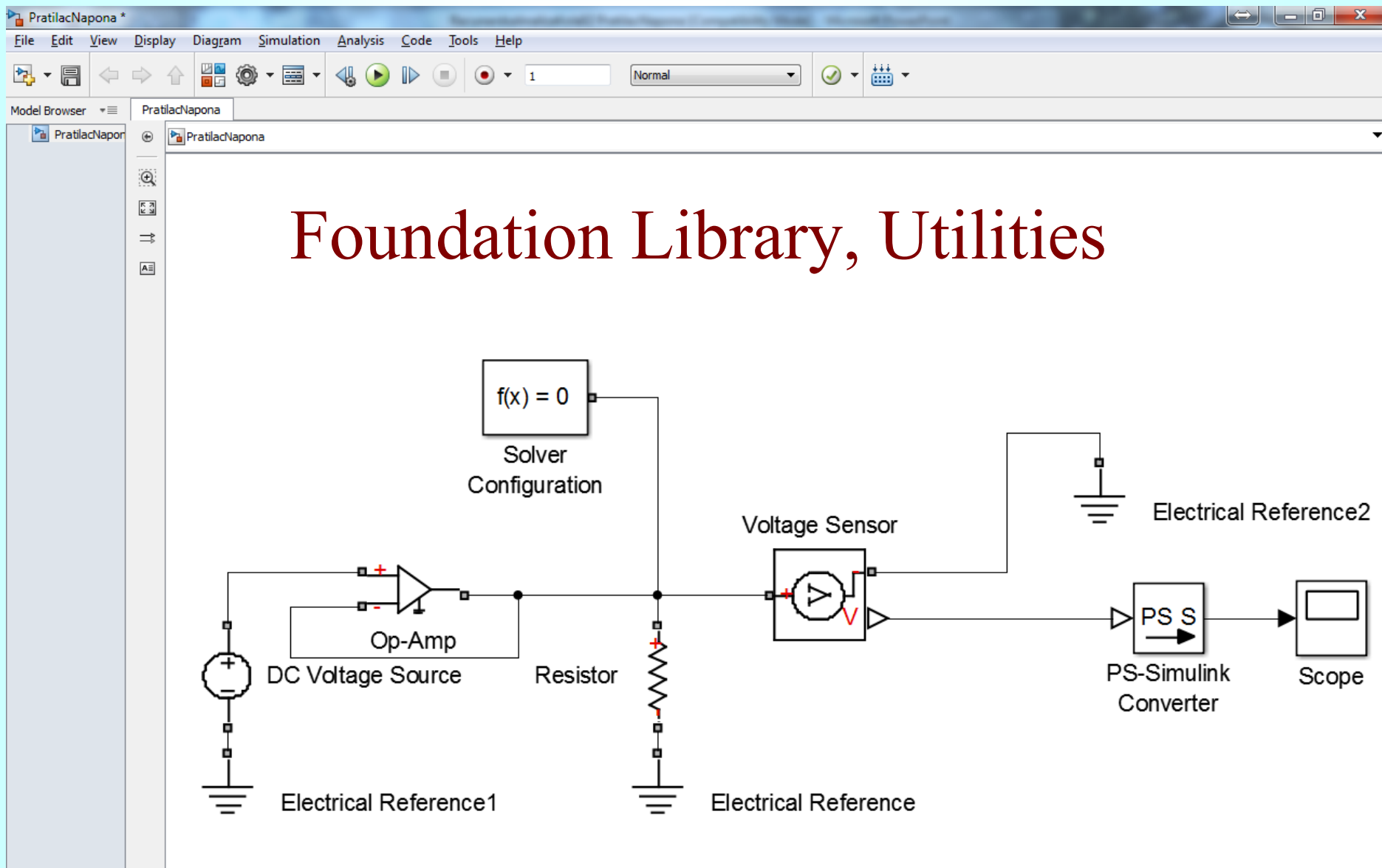
```
Pratilač napona - MuPAD
File Edit View Navigation Insert Format Notebook Window Help
Generic Sans Serif 11 B I U X2 »
Pratilač napona (voltage follower)
[ // Pratilač napona
jednacine := {iug + i1OP = 0, i2OP - i1OP + iR = 0,
ug - u1OP - u2OP = 0, u2OP - uR = 0, u1OP = 0, i1OP = 0, uR = R*iR}
{u2OP - uR = 0, i2OP - i1OP + iR = 0, i1OP + iug = 0, i1OP = 0, u1OP = 0, ug - u2OP - u1OP = 0, uR = R*iR}
assume(R>0)
promenljive := {iug, iR, i1OP, i2OP, uR, u1OP, u2OP}
{i1OP, i2OP, iR, iug, u1OP, u2OP, uR}
odziv := solve(jednacine, promenljive)
{ {i1OP = 0, i2OP = -ug/R, iR = ug/R, iug = 0, u1OP = 0, u2OP = ug, uR = ug} }
pojacanje := uR/ug | odziv
1
```



```
jednacine := {iug + i1OP = 0, i2OP - i1OP + iR = 0,
ug - u1OP - u2OP = 0, u2OP - uR = 0, u1OP = 0, i1OP = 0, uR = R*iR}
{u2OP - uR = 0, i2OP - i1OP + iR = 0, i1OP + iug = 0, i1OP = 0, u1OP = 0, ug - u2OP - u1OP = 0, uR = R*iR}
```

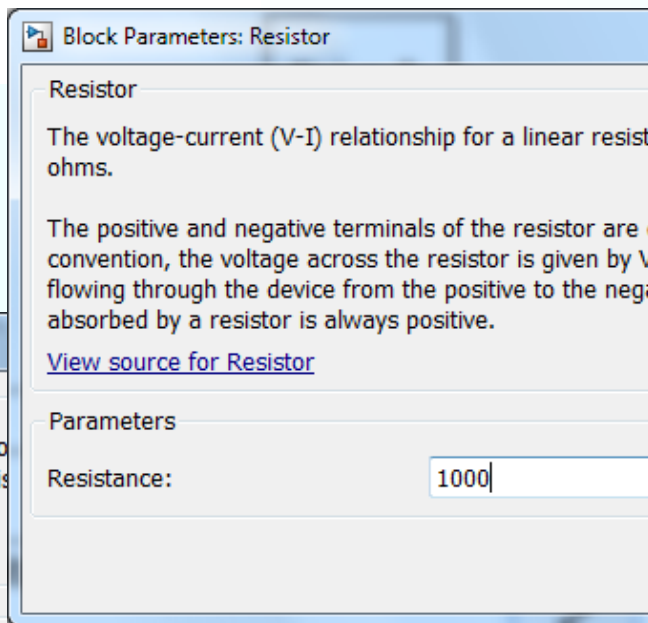
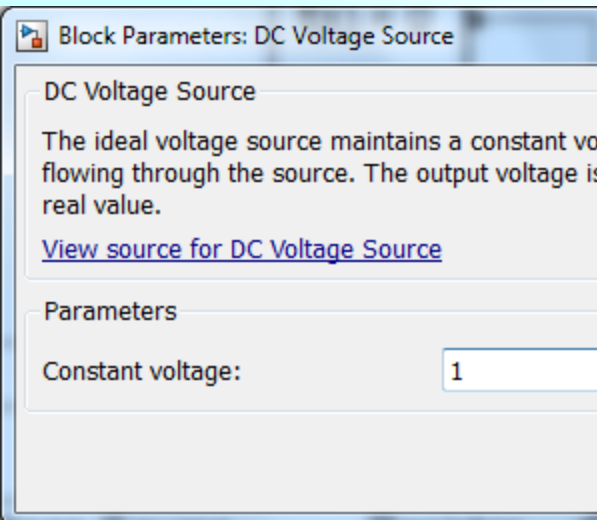
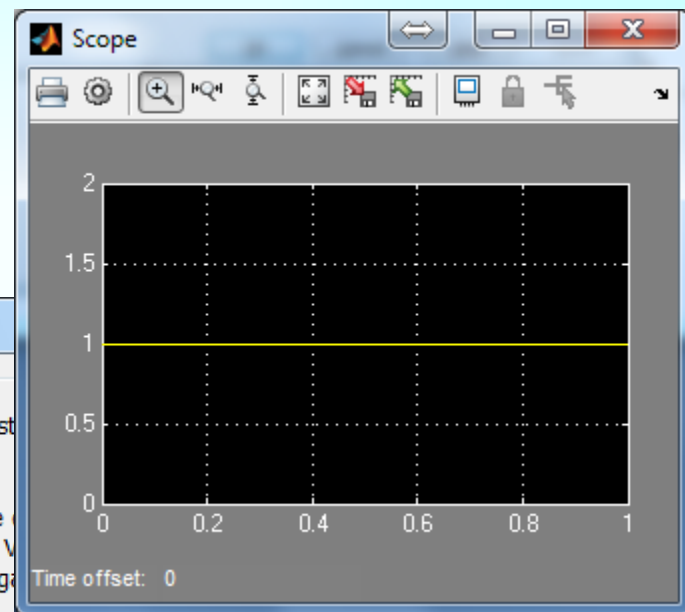
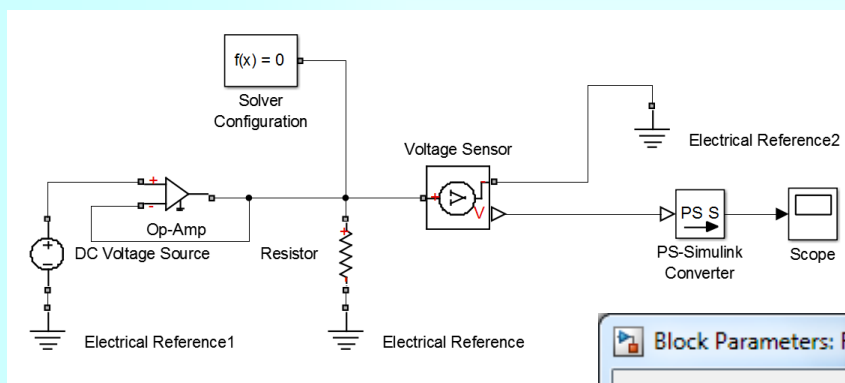
```
assume(R>0)
promenljive := {iug, iR, i1OP, i2OP, uR, u1OP, u2OP}
{i1OP, i2OP, iR, iug, u1OP, u2OP, uR}
odziv := solve(jednacine, promenljive)
{ {i1OP = 0, i2OP = -ug/R, iR = ug/R, iug = 0, u1OP = 0, u2OP = ug, uR = ug} }
pojacanje := uR/ug | odziv
1
```

MATLAB: Simscape



MATLAB: Simscape

Foundation Library, Utilities



Maxima

Maxima, <http://maxima.sourceforge.net/>

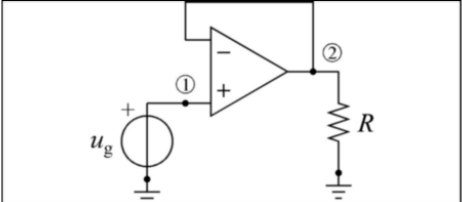
wxMaxima, <http://andrejv.github.com/wxmaxima/index.html>

wxMaxima 12.01.0 [PratilacNapona.wmx]

File Edit Cell Maxima Equations Algebra Calculus Simplify Plot Numeric Help

Pratilac napona (voltage follower)

Figure 1:



```
(%i1) jednacine: [iug + i1OP = 0, i2OP - i1OP + iR = 0,
ug - u1OP - u2OP = 0, u2OP - uR = 0,
u1OP = 0, i1OP = 0, uR = R*iR];

(%o1) [i1OP+iug=0,iR+i2OP-i1OP=0,-u2OP-u1OP+ug=0,u2OP-uR
=0,u1OP=0,i1OP=0,uR=iRR]

(%i2) odziv: linsolve(jednacine,
[iug, iR, i1OP, i2OP, uR, u1OP, u2OP]);

(%o2) [iug=0,iR=ug/R,i1OP=0,i2OP=-ug/R,uR=ug,u1OP=0,u2OP=ug]

(%i3) pojacanje: uR/ug, odziv;

(%o3) 1
```

Ready for user input

Инструментациони појачавач

- Одредити напон v_3 отпорника R_8 ?
- Нацртати график напона v_3 отпорника R_8 у функцији времена?
- Колика је снага извора?

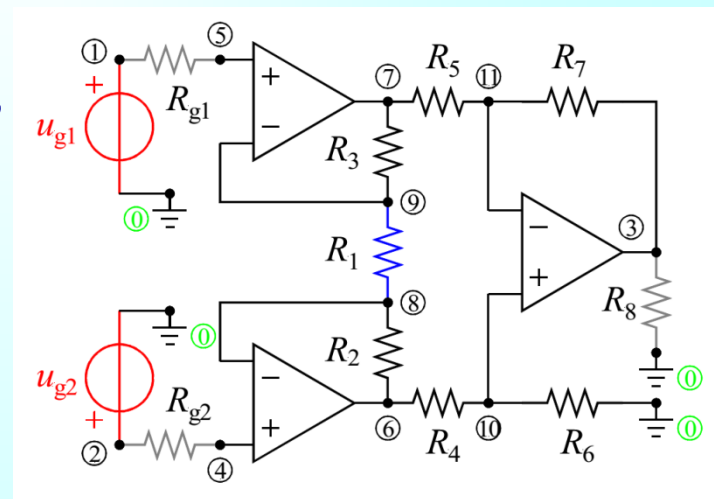
Вредности елемената су познате:

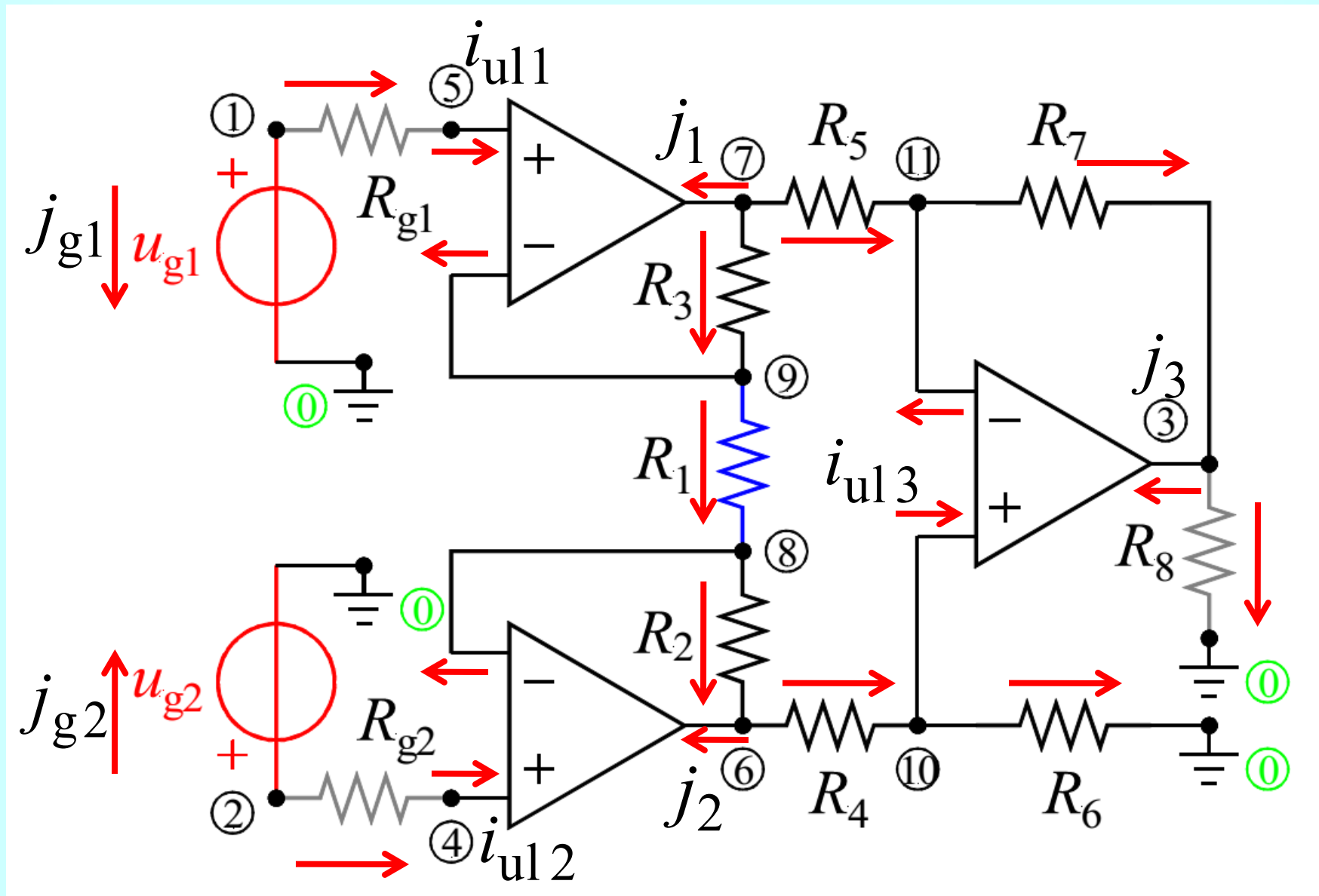
$$R_2 = R_3 = R_4 = R_5 = R_6 = R_7 = R_8 = R$$

$$R = 1 \text{ k}\Omega, R_1 = 2 \text{ k}\Omega, R_{g1} = R_{g2} = 1 \text{ k}\Omega$$

$$u_{g1}(t) = U h(t), u_{g2}(t) = U h(t-T)$$

$$U = 10 \text{ V}, T = 1 \text{ s}$$





Модификована метода потенцијала чворова

$$(1) j_{g1} + (V_1 - V_5)/R_{g1} = 0$$

$$(2) j_{g2} + (V_2 - V_4)/R_{g2} = 0$$

$$(3) j_3 - (V_{11} - V_3)/R_7 + V_3/R_8 = 0$$

$$(4) -(V_2 - V_4)/R_{g2} + i_{ul2} = 0$$

$$(5) -(V_1 - V_5)/R_{g1} + i_{ul1} = 0$$

$$(6) j_2 + (V_6 - V_{10})/R_4 - (V_8 - V_6)/R_2 = 0$$

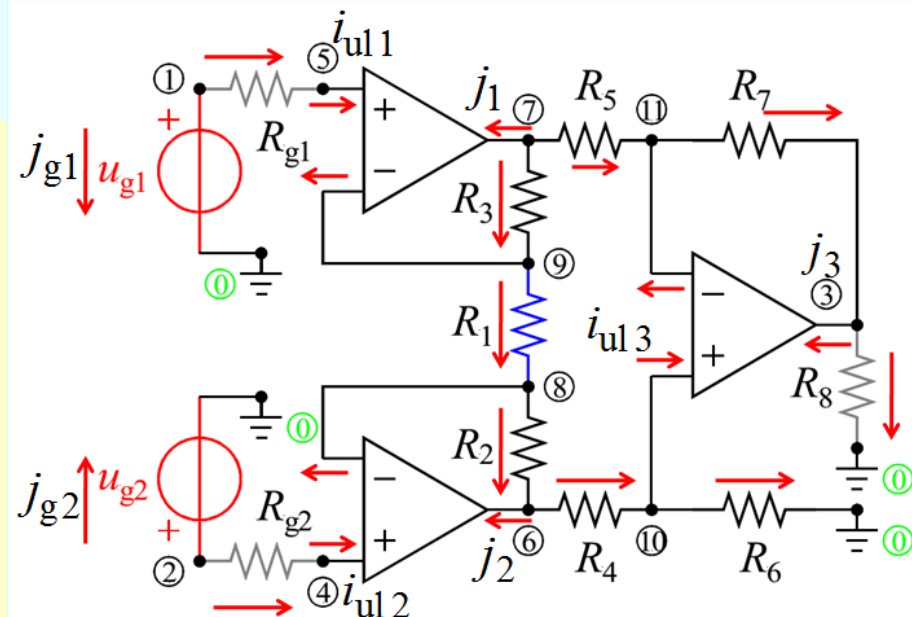
$$(7) j_1 + (V_7 - V_9)/R_3 + (V_7 - V_{11})/R_5 = 0$$

$$(8) -i_{ul2} - (V_9 - V_8)/R_1 + (V_8 - V_6)/R_2 = 0$$

$$(9) -i_{ul1} - (V_7 - V_9)/R_3 + (V_9 - V_8)/R_1 = 0$$

$$(10) -(V_6 - V_{10})/R_4 + i_{ul3} + V_{10}/R_6 = 0$$

$$(11) -(V_7 - V_{11})/R_5 - i_{ul3} + (V_{11} - V_3)/R_7 = 0$$



$$i_{ul1} = 0$$

$$i_{ul2} = 0$$

$$i_{ul3} = 0$$

$$V_5 = V_9$$

$$V_4 = V_8$$

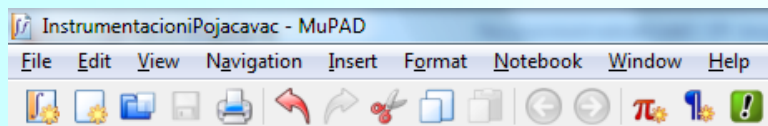
$$V_{10} = V_{11}$$

$$V_1 = u_{g1}$$

$$V_2 = u_{g2}$$

MATLAB: MuPAD

Једначине



```
jednacine:= {jg1 + (v1-v5)/Rg1 = 0,
v1 = ug1,
jg2 + (v2-v4)/Rg2 = 0,
v2 = ug2,
j3 + v3/R8 - (v11-v3)/R7 = 0,
-(v2-v4)/Rg2 + iul2 = 0, -(v1-v5)/Rg1 + iul1 = 0,
j2 + (v6-v10)/R4 - (v8-v6)/R2 = 0,
j1 + (v7-v11)/R5 + (v7-v9)/R3 = 0,
-iul2 - (v9-v8)/R1 + (v8-v6)/R2 = 0,
-iul1 - (v7-v9)/R3 + (v9-v8)/R1 = 0,
-(v6-v10)/R4 + v10/R6 + iul3 = 0,
-(v7-v11)/R5 + (v11-v3)/R7 -iul3 = 0,
iul1 = 0, iul2 = 0, iul3 = 0, v5=v9, v4=v8, v10 = v11}
```

$$\left\{ v1 = ug1, v2 = ug2, v4 = v8, v5 = v9, v10 = v11, iul3 + \frac{v10}{R6} - \frac{v6-v10}{R4} = 0, j3 + \frac{v3}{R8} + \frac{v3-v11}{R7} = 0, j2 + \frac{v6-v8}{R2} + \frac{v6-v10}{R4} = 0, j1 + \frac{v7-v9}{R3} + \frac{v7-v11}{R5} = 0, iul1 = 0, iul2 = 0, \right.$$

$$\left. iul3 = 0, iul1 - \frac{v1-v5}{Rg1} = 0, iul2 - \frac{v2-v4}{Rg2} = 0, jg1 + \frac{v1-v5}{Rg1} = 0, jg2 + \frac{v2-v4}{Rg2} = 0, \frac{v8-v9}{R1} - \frac{v6-v8}{R2} - iul2 = 0, -iul1 - \frac{v8-v9}{R1} - \frac{v7-v9}{R3} = 0, -iul3 - \frac{v3-v11}{R7} - \frac{v7-v11}{R5} = 0 \right\}$$

```
jednacine:= {jg1 + (v1-v5)/Rg1 = 0,
v1 = ug1,
jg2 + (v2-v4)/Rg2 = 0,
v2 = ug2,
j3 + v3/R8 - (v11-v3)/R7 = 0,
-(v2-v4)/Rg2 + iul2 = 0, -(v1-v5)/Rg1 + iul1 = 0,
j2 + (v6-v10)/R4 - (v8-v6)/R2 = 0,
j1 + (v7-v11)/R5 + (v7-v9)/R3 = 0,
-iul2 - (v9-v8)/R1 + (v8-v6)/R2 = 0,
-iul1 - (v7-v9)/R3 + (v9-v8)/R1 = 0,
-(v6-v10)/R4 + v10/R6 + iul3 = 0,
-(v7-v11)/R5 + (v11-v3)/R7 -iul3 = 0,
iul1 = 0, iul2 = 0, iul3 = 0, v5=v9, v4=v8, v10 = v11}
```

MATLAB: MuPAD

assume, променљиве, solve...

```
[assume(Rg1>0 and Rg2>0 and R1>0 and R2>0 and R3>0 and R4>0 and R5>0 and R6>0 and R7>0 and R8>0 and R>0)
promenljive:={v1, v2, v3, v4, v5, v6, v7, v8, v9, v10, v11, j1, j2, j3, iul1, iul2, iul3, jg1, jg2}
{iul1, iul2, iul3, j1, j2, j3, jg1, jg2, v1, v2, v3, v4, v5, v6, v7, v8, v9, v10, v11}
odziv:=solve(jednacine, promenljive)
```

```
[assume(Rg1>0 and Rg2>0 and R1>0 and R2>0 and R3>0 and R4>0 and R5>0 and R6>0 and R7>0 and R8>0 and R>0)
promenljive:={v1, v2, v3, v4, v5, v6, v7, v8, v9, v10, v11, j1, j2, j3, iul1, iul2, iul3, jg1, jg2}
{iul1, iul2, iul3, j1, j2, j3, jg1, jg2, v1, v2, v3, v4, v5, v6, v7, v8, v9, v10, v11}
odziv:=solve(jednacine, promenljive)
{
  iul1 = 0, iul2 = 0, iul3 = 0, j1 =  $-\frac{R1 R4 ug1 + R1 R6 ug1 + R3 R4 ug1 - R1 R6 ug2 + R2 R6 ug1 - R3 R4 ug2 - R2 R6 ug2 + R3 R6 ug1 + R4 R5 ug1 - R3 R6 ug2 - R4 R5 ug2 + R5 R6 ug1 - R5 R6 ug2}{\sigma_1}$ ,
  j2 =  $-\frac{R1 ug2 - R2 ug1 + R2 ug2 - R4 ug1 + R4 ug2 - R6 ug1 + R6 ug2}{R1 (R4 + R6)}$ , j3 =  $(R1 R4 R7 ug1 + R1 R4 R8 ug1 - R1 R5 R6 ug2 + R2 R5 R6 ug1 + R1 R6 R7 ug1 - R2 R5 R6 ug2 + R3 R4 R7 ug1 - R1 R6 R7 ug2$ 
  +  $R1 R6 R8 ug1 + R2 R6 R7 ug1 - R3 R4 R7 ug2 + R3 R4 R8 ug1 - R1 R6 R8 ug2 - R2 R6 R7 ug2 + R2 R6 R8 ug1 - R3 R4 R8 ug2 + R3 R6 R7 ug1 - R2 R6 R8 ug2 - R3 R6 R7 ug2 + R3 R6 R8 ug1$ 
  -  $R3 R6 R8 ug2)/(R1 R5 R8 (R4 + R6))$ , jg1 = 0, jg2 = 0, v1 = ug1, v2 = ug2,
  v3 =  $-\frac{R1 R4 R7 ug1 - R1 R5 R6 ug2 + R2 R5 R6 ug1 + R1 R6 R7 ug1 - R2 R5 R6 ug2 + R3 R4 R7 ug1 - R1 R6 R7 ug2 + R2 R6 R7 ug1 - R3 R4 R7 ug2 - R2 R6 R7 ug2 + R3 R6 R7 ug1 - R3 R6 R7 ug2}{\sigma_1}$ , v4 = ug2, v5 = ug1, v6 =  $\frac{\sigma_3}{R1}$ ,
  v7 =  $\frac{R1 ug1 + R3 ug1 - R3 ug2}{R1}$ , v8 = ug2, v9 = ug1, v10 =  $\sigma_2$ , v11 =  $\sigma_2$ 
}
```

where

$$\sigma_1 = R1 R5 (R4 + R6)$$

$$\sigma_2 = \frac{R6 \sigma_3}{R1 (R4 + R6)}$$

$$\sigma_3 = R1 ug2 - R2 ug1 + R2 ug2$$

MATLAB: MuPAD

Замена..., simplify, heaviside, plot...

```
zamena:={R2=R, R3=R, R4=R, R5=R, R6=R, R7=R}
{R2 = R, R3 = R, R4 = R, R5 = R, R6 = R, R7 = R}

v3|odziv
- 
$$\frac{R_1 R_4 R_7 u_{g1} - R_1 R_5 R_6 u_{g2} + R_2 R_5 R_6 u_{g1} + R_1 R_6 R_7 u_{g1} - R_2 R_5 R_6 u_{g2} + R_3 R_4 R_7 u_{g1} - R_1 R_6 R_7 u_{g2} + R_2 R_6 R_7 u_{g1} - R_3 R_4 R_7 u_{g2} - R_2 R_6 R_7 u_{g2} + R_3 R_6 R_7 u_{g1} - R_3 R_6 R_7 u_{g2}}{R_1 R_5 (R_4 + R_6)}$$


v3|odziv|zamena
- 
$$\frac{4 R^3 u_{g1} - 4 R^3 u_{g2} + 2 R^2 R_1 u_{g1} - 2 R^2 R_1 u_{g2}}{2 R^2 R_1}$$

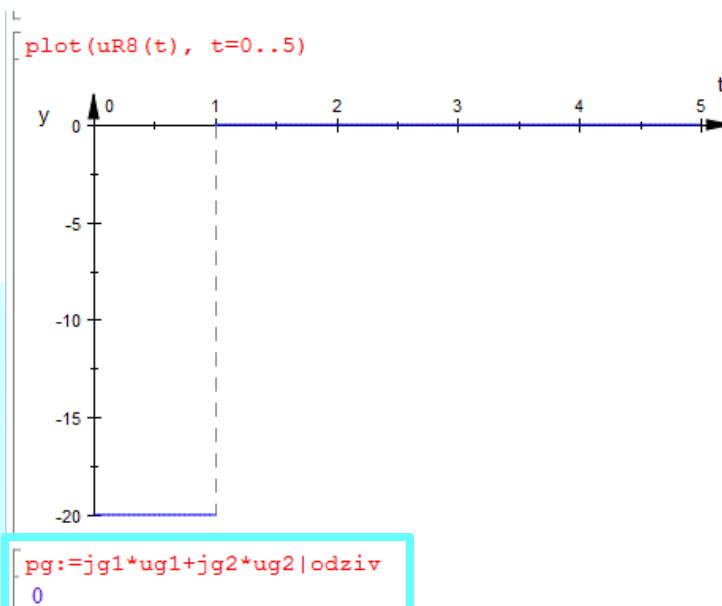

simplify(v3|odziv|zamena)
- 
$$\frac{(2 R + R_1) (u_{g1} - u_{g2})}{R_1}$$


assume(U>0 and T>0 and t>0): pobude:={ug1=U*heaviside(t), ug2=U*heaviside(t-T)}
{ug1 = U, ug2 = U heaviside(t-T)}

simplify(v3|odziv|zamena)|pobude
- 
$$\frac{(U - U \text{heaviside}(t-T)) (2 R + R_1)}{R_1}$$

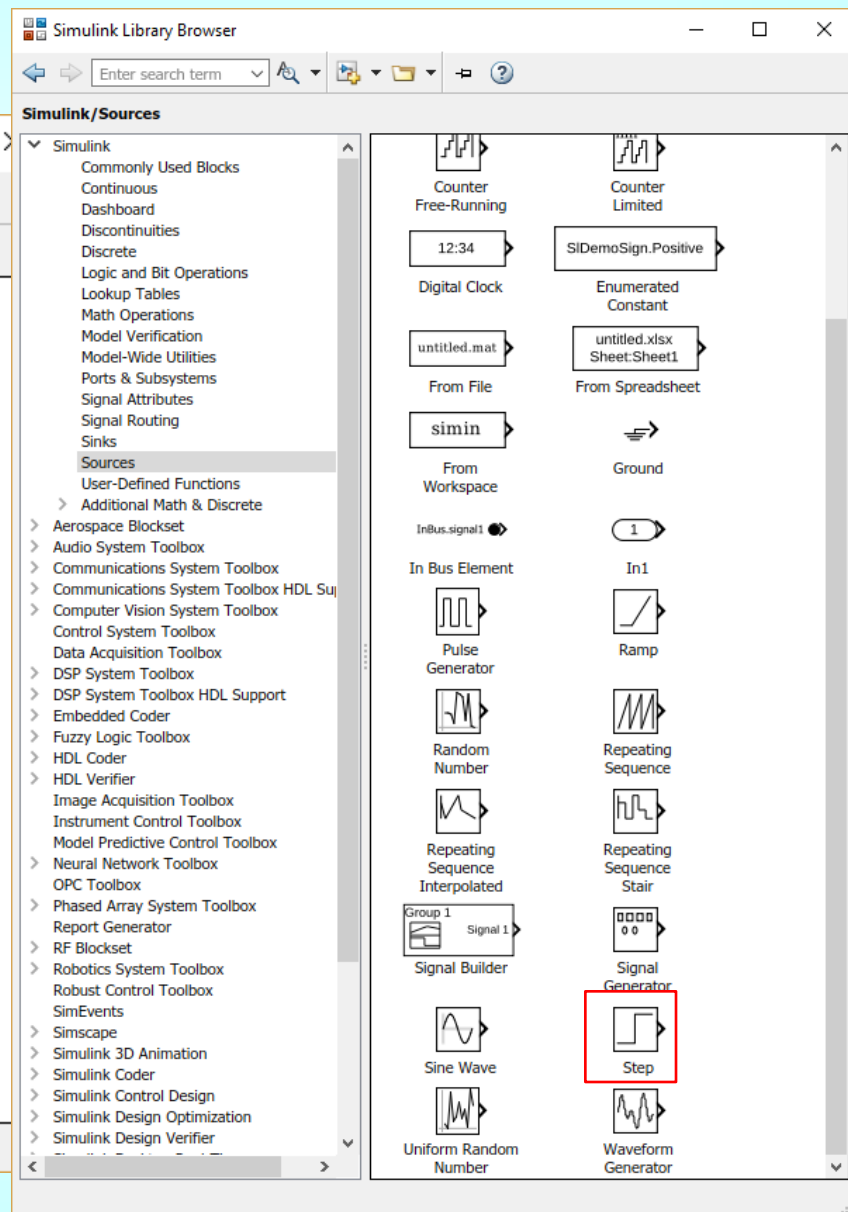
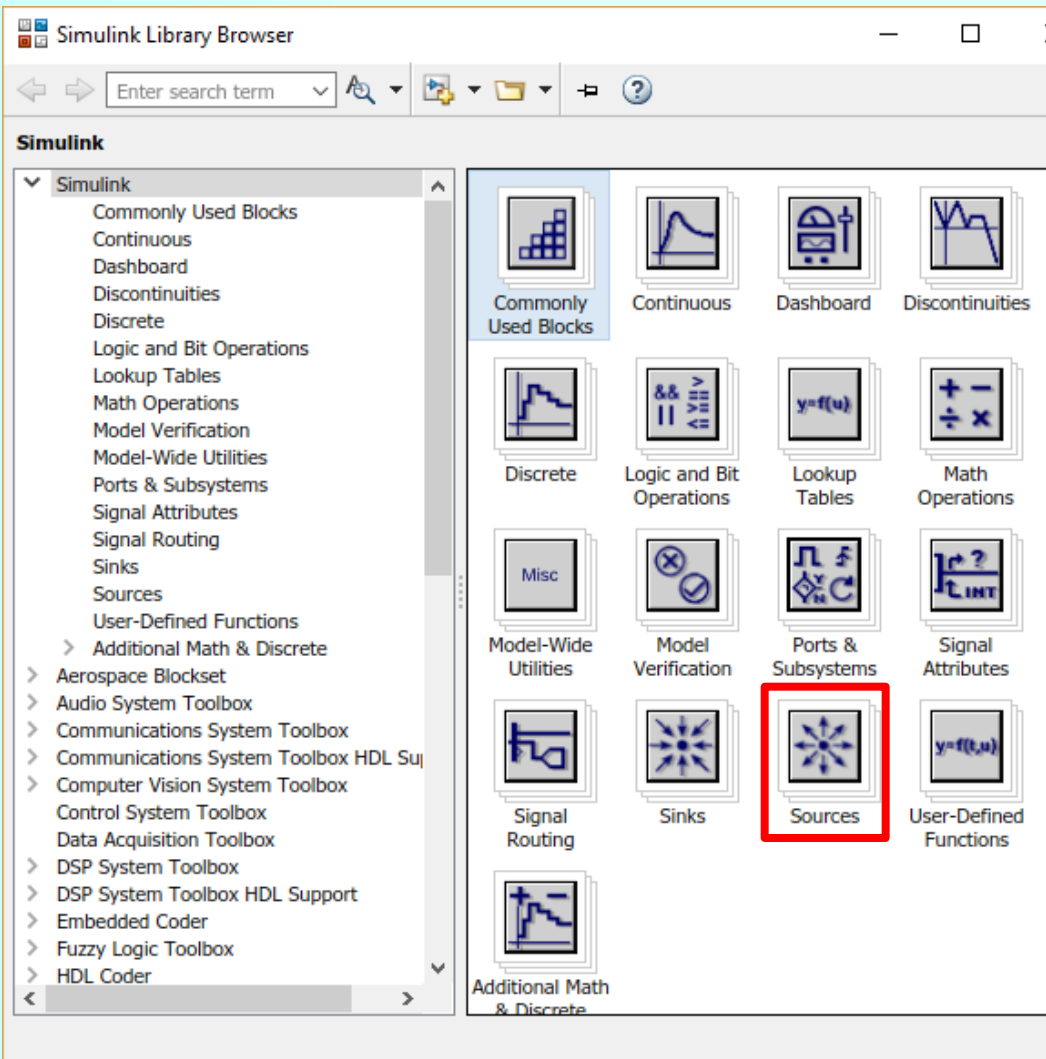

vrednosti:={U=10, T=1, R=1000, R1=2000}
{R = 1000, R1 = 2000, T = 1, U = 10}

ur8(t):=simplify(v3|odziv|zamena)|pobude|vrednosti
20 heaviside(t-1) - 20
```



MATLAB: Simulink

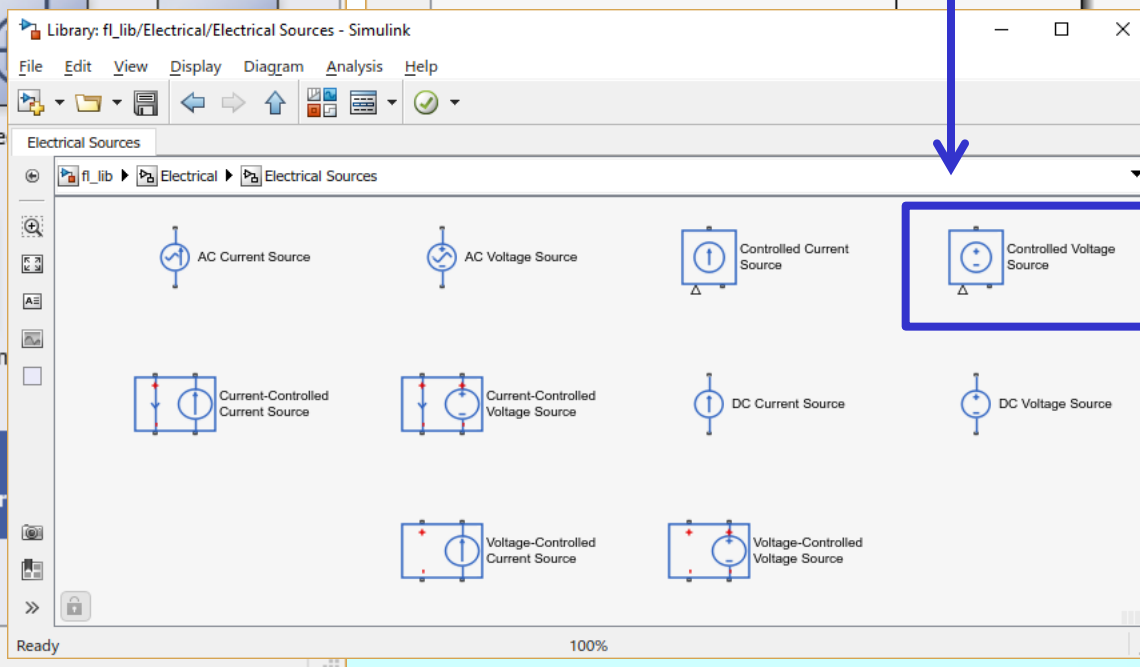
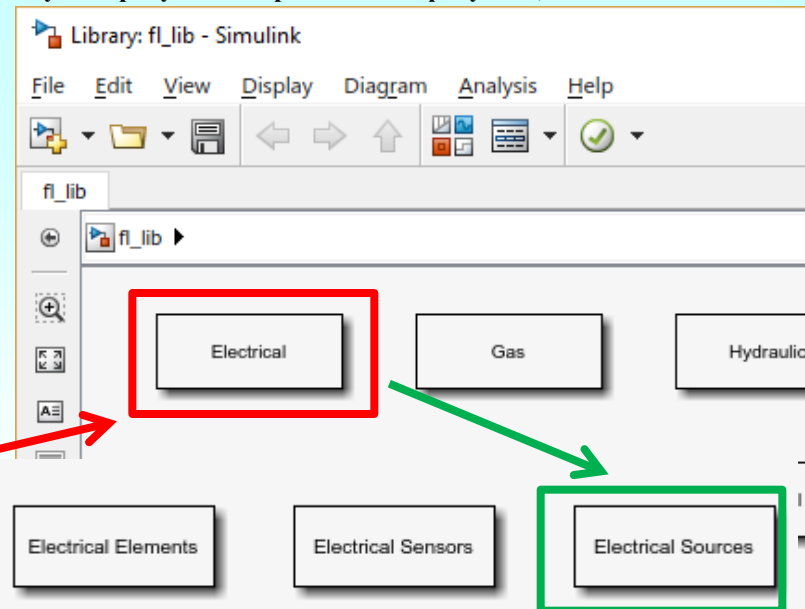
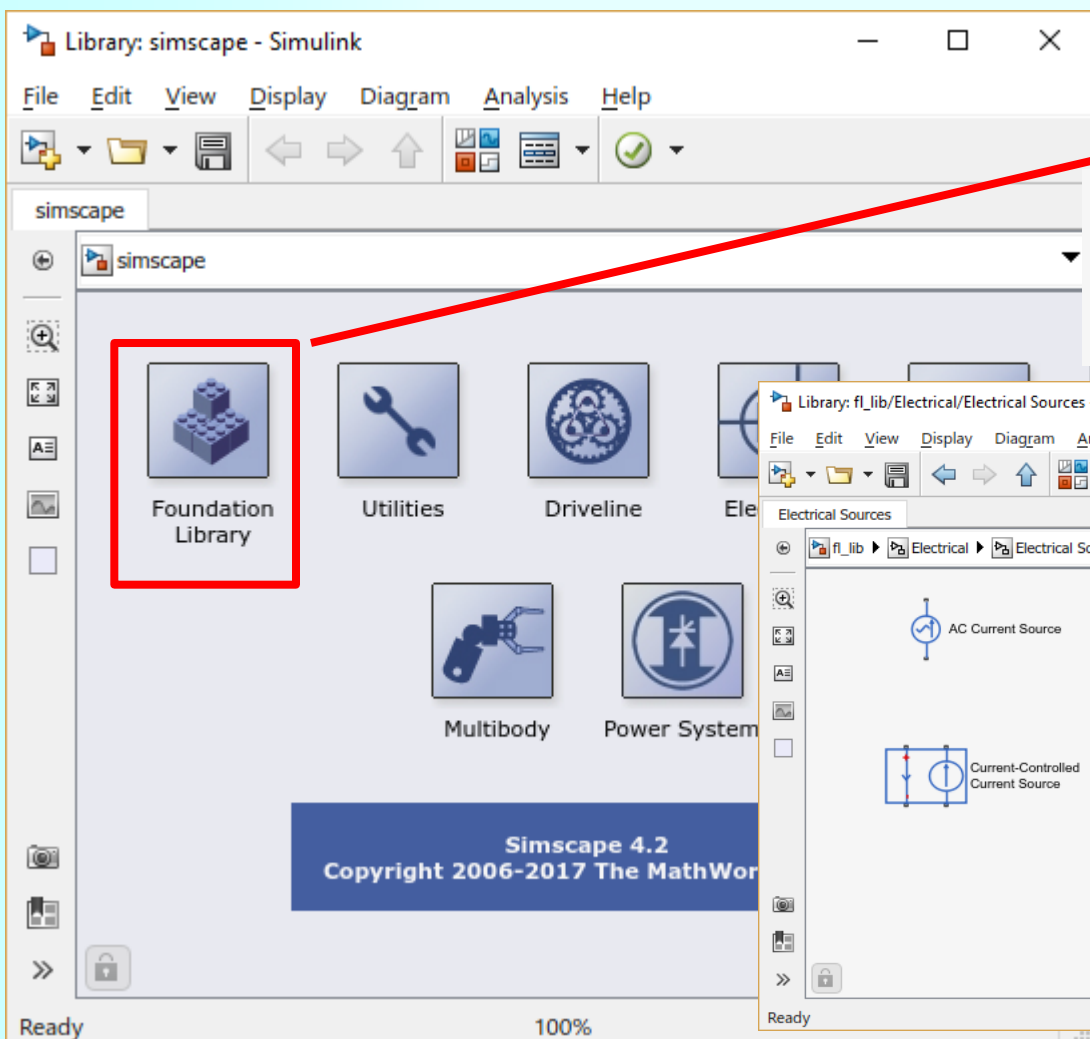
Sources



MATLAB: Simscape

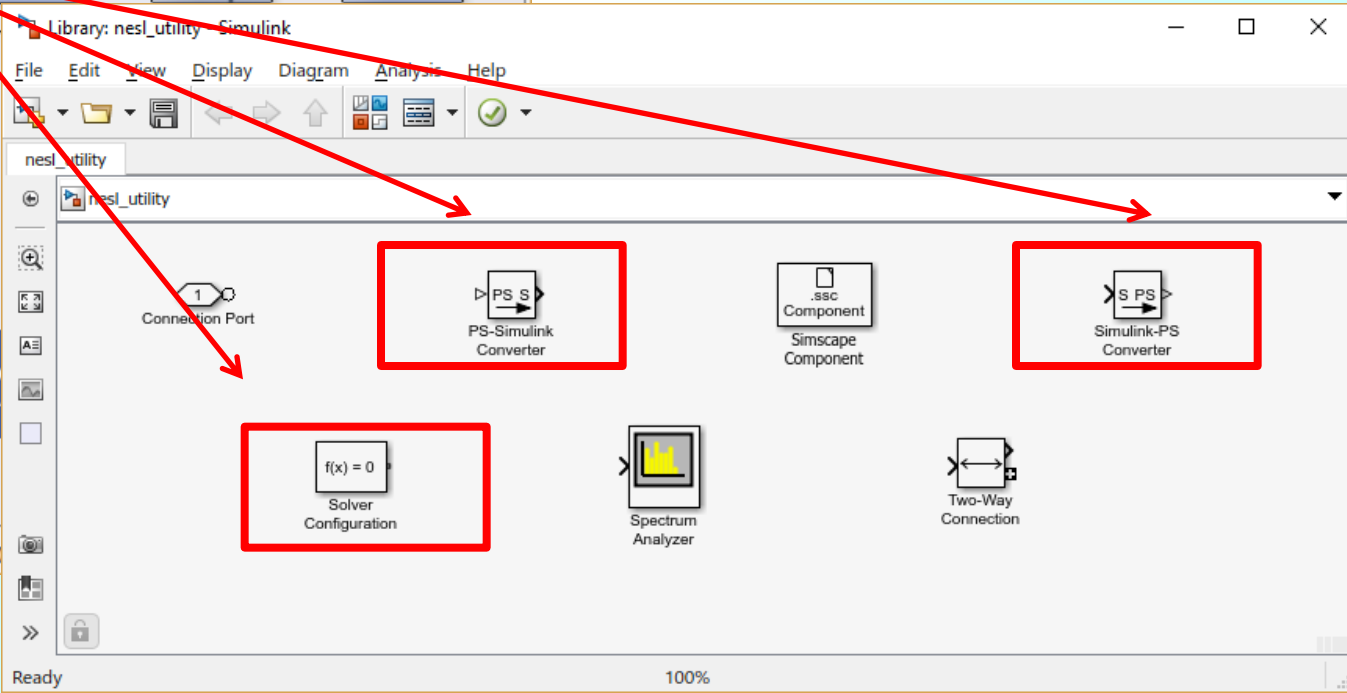
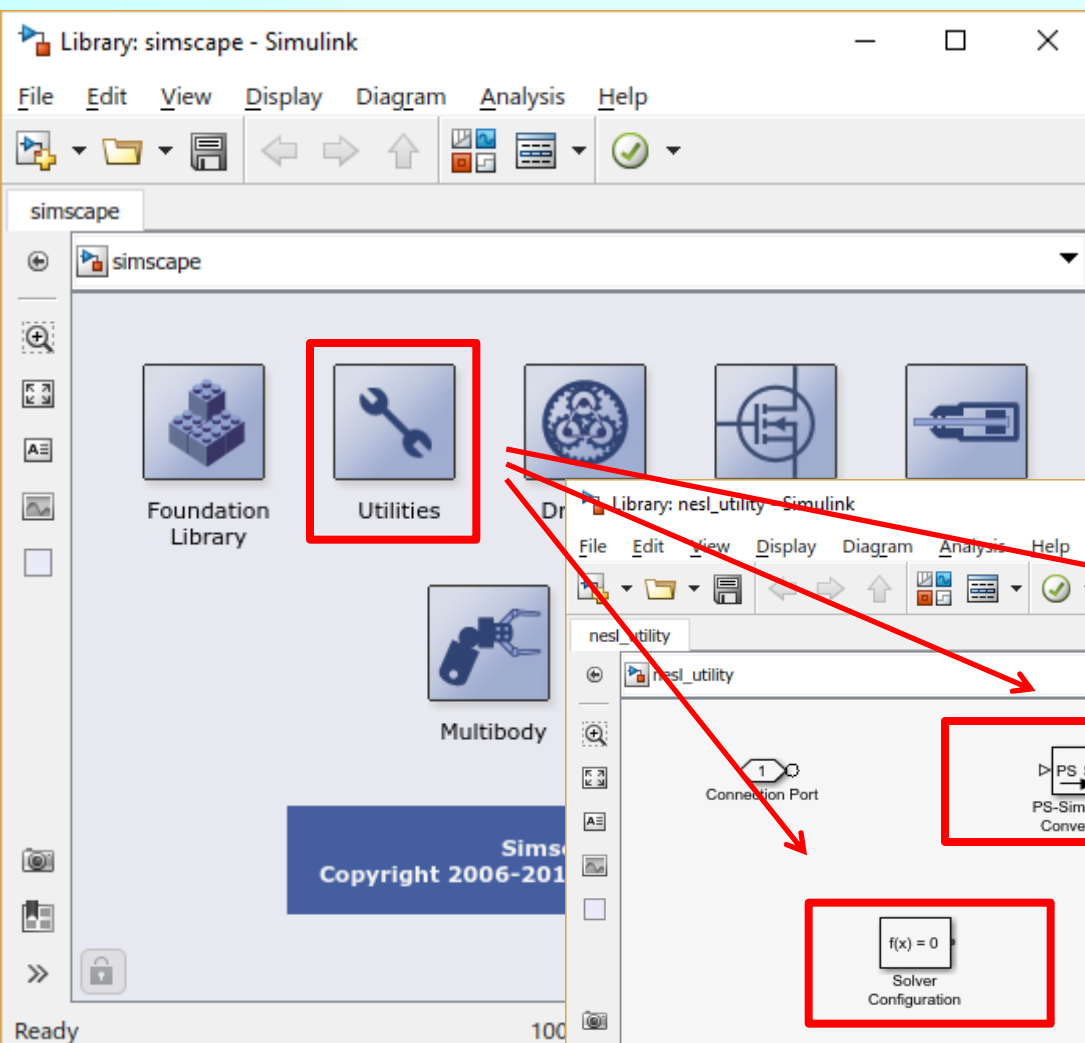
Foundation Library

Electrical, Electrical Sources



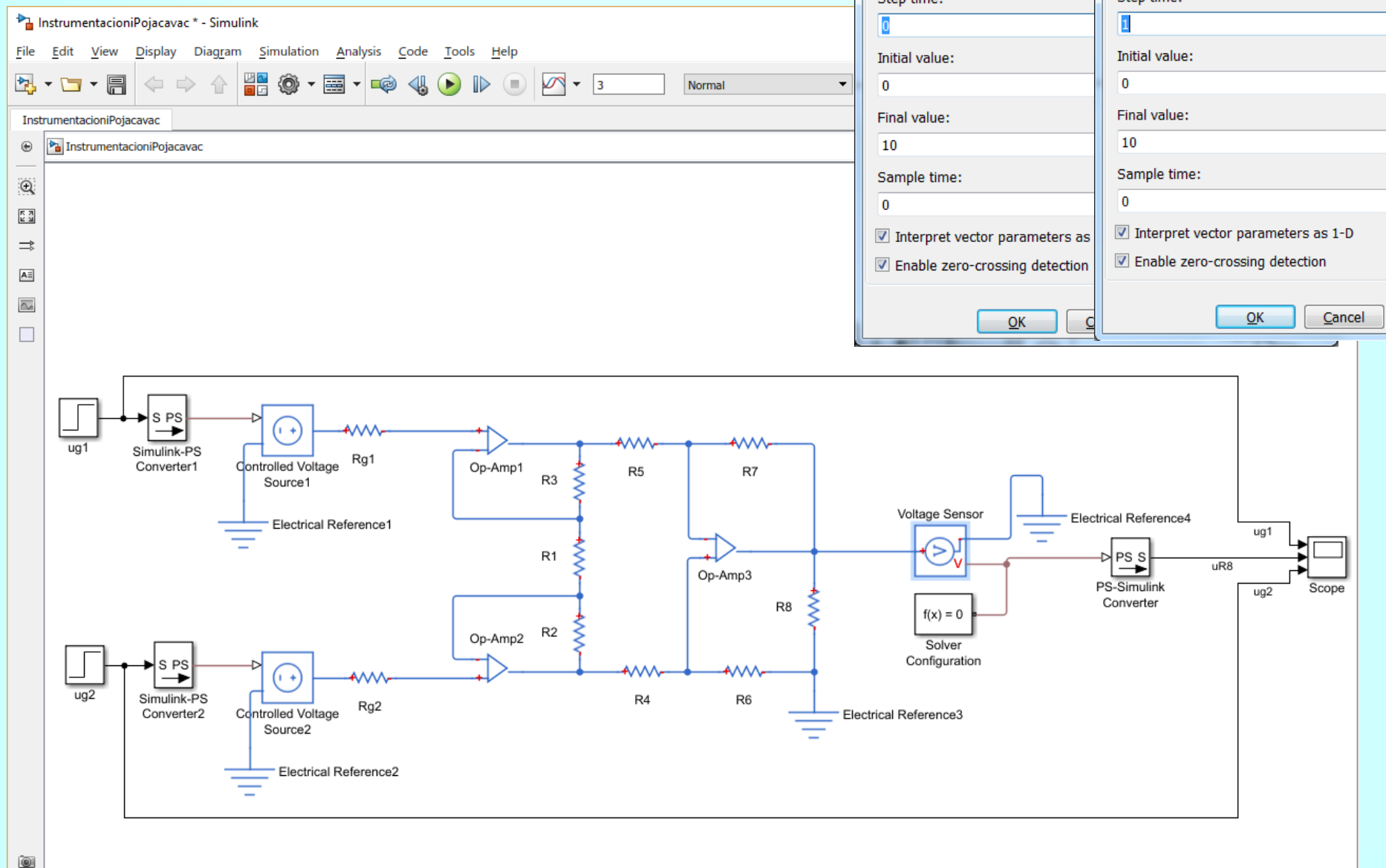
MATLAB: Simscape

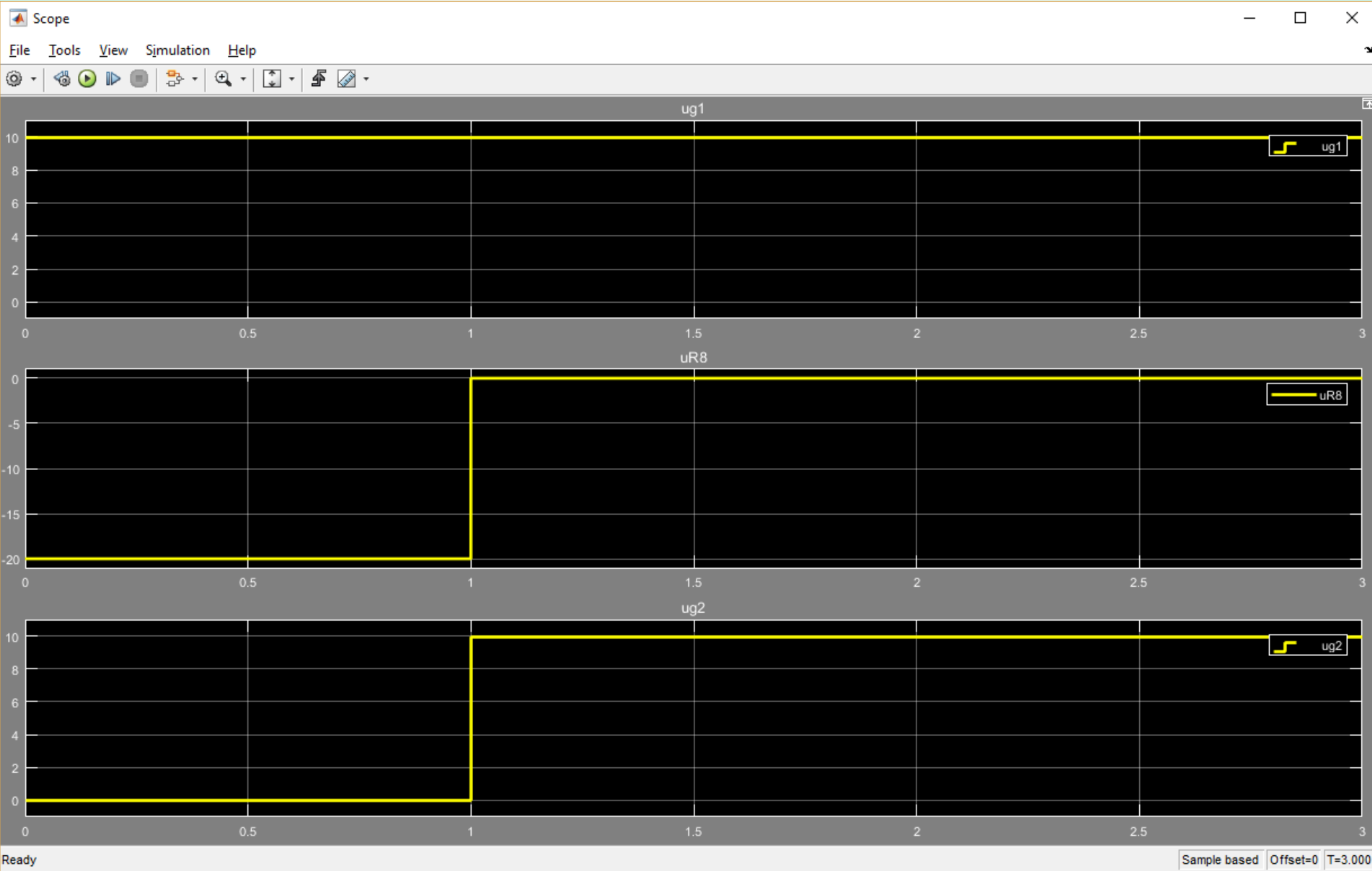
Utilities



MATLAB: Simscape

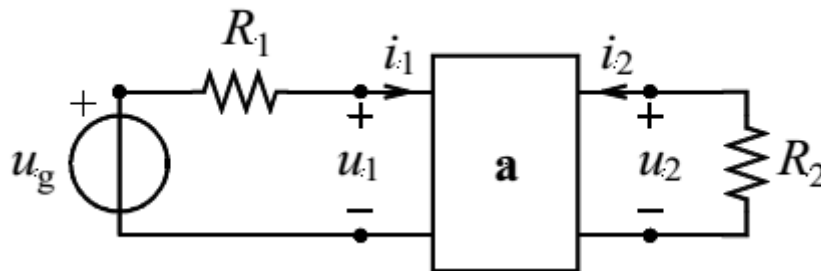
Foundation Library, Utilities, Simulink

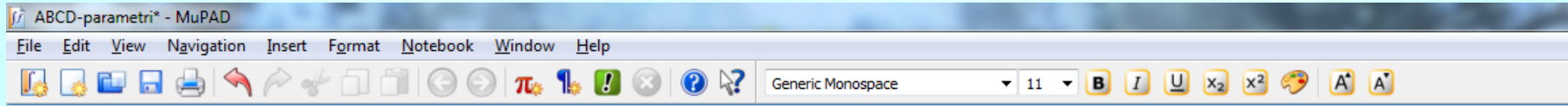




ABCD параметри мреже

- Отпорности отпорника и a -параметри ($ABCD$ -параметри) мреже са два приступа су познати.
- Одредити:
 - Количник напона $A = u_2 / u_g$
 - Улазну отпорност мреже $R_u = u_1 / i_1$
 - Струју i_1
- Сматрати да су a -параметри мреже реални и различити од нуле.





MATLAB: MuPAD

```
jednacine:= {ug = R1*i1 + u1,
u2 + R2*i2 = 0,
u1 = a11*u2 + a12*(-i2),
i1 = a21*u2 + a22*(-i2)}

{u2+R2 i2 = 0, i1 = a21 u2 - a22 i2, u1 = a11 u2 - a12 i2, ug = u1 + R1 i1}

promenljive:={u1, u2, i1, i2}
{i1, i2, u1, u2}
```

```
odziv:=solve(jednacine, promenljive)
```

$\left\{ \begin{array}{l} [i1 = 0, i2 = z, u1 = 0, u2 = 0] \\ [i1 = \frac{ug}{R1}, i2 = -\frac{ug}{R1 a22}, u1 = 0, u2 = 0] \\ [i1 = -a22 z, i2 = z, u1 = 0, u2 = 0] \\ \left\{ \begin{array}{l} i1 = \frac{a22 ug}{a12 + R1 a22}, i2 = -\frac{ug}{a12 + R1 a22}, u1 = \frac{a12 ug}{a12 + R1 a22}, u2 = 0 \end{array} \right\} \\ \left\{ \begin{array}{l} i1 = \frac{a22 z}{a12}, i2 = -\frac{z}{a12}, u1 = z, u2 = 0 \end{array} \right\} \\ \left\{ \begin{array}{l} i1 = \frac{a22 ug + R2 a21 ug}{\sigma_1}, i2 = -\frac{ug}{\sigma_1}, u1 = \frac{ug (a12 + R2 a11)}{\sigma_1}, u2 = \frac{R2 ug}{\sigma_1} \end{array} \right\} \\ \left\{ \begin{array}{l} i1 = \frac{z (a22 + R2 a21)}{R2}, i2 = -\frac{z}{R2}, u1 = \frac{z (a12 + R2 a11)}{R2}, u2 = z \end{array} \right\} \\ \emptyset \end{array} \right.$	if $R2 = 0 \wedge a12 = 0 \wedge a22 = 0 \wedge ug = 0$ if $R2 = 0 \wedge R1 \neq 0 \wedge a12 = 0 \wedge a22 \neq 0$ if $R2 = 0 \wedge a12 = 0 \wedge (R1 = 0 \vee a22 = 0) \wedge ug = 0$ if $a12 + R1 a22 \neq 0 \wedge R2 = 0 \wedge a12 \neq 0$ if $a12 + R1 a22 = 0 \wedge R2 = 0 \wedge a12 \neq 0 \wedge ug = 0$ if $\sigma_1 \neq 0 \wedge R2 \neq 0$ if $\sigma_1 = 0 \wedge R2 \neq 0 \wedge ug = 0$ if $(\sigma_1 = 0 \wedge (R2 = 0 \vee ug \neq 0)) \vee ((a12 + R1 a22 = 0 \vee (a12 = 0 \wedge (R1 = 0 \vee a22 = 0))) \wedge R2 = 0 \wedge ug \neq 0)$
---	---

where

$$\sigma_1 = a12 + R2 a11 + R1 a22 + R1 R2 a21$$

```
odziv[6]
```

$$\left\{ \begin{array}{l} i1 = \frac{a22 ug + R2 a21 ug}{\sigma_1}, i2 = -\frac{ug}{\sigma_1}, u1 = \frac{ug (a12 + R2 a11)}{\sigma_1}, u2 = \frac{R2 ug}{\sigma_1} \end{array} \right\}$$

where

$$\sigma_1 = a12 + R2 a11 + R1 a22 + R1 R2 a21$$

```
odziv[6]
```

$$\left\{ \begin{array}{l} i1 = \frac{a22 ug + R2 a21 ug}{\sigma_1}, i2 = -\frac{ug}{\sigma_1}, u1 = \frac{ug (a12 + R2 a11)}{\sigma_1}, u2 = \frac{R2 ug}{\sigma_1} \end{array} \right\}$$

where

$$\sigma_1 = a12 + R2 a11 + R1 a22 + R1 R2 a21$$

```
A:=(u2|odziv[6])/ug
      R2
a12 + R2 a11 + R1 a22 + R1 R2 a21
```

```
Ru:=simplify((u1/i1)|odziv[6])
      a12 + R2 a11
a22 + R2 a21
```

```
simplify(i1|odziv[6])
      ug (a22 + R2 a21)
a12 + R2 a11 + R1 a22 + R1 R2 a21
```

MATLAB: MuPAD

assume...

```
ABCD-parametri_assumption* - MuPAD
File Edit View Navigation Insert Format Notebook Window Help

jednacine:= {ug = R1*i1 + u1,
u2 + R2*i2 = 0,
u1 = a11*u2 + a12*(-i2),
i1 = a21*u2 + a22*(-i2)}

{u2 + R2 i2 = 0, i1 = a21 u2 - a22 i2, u1 = a11 u2 - a12 i2, ug = u1 + R1 i1}

promenljive:={u1, u2, i1, i2}
{i1, i2, u1, u2}

assume(a11*a22-a12*a21<>0 and R1>0 and R2>0 and ug<>0):
odziv:=solve(jednacine, promenljive)

{
  if  $\sigma_1 = 0$ 
  { }
  if  $\sigma_1 \neq 0$ 
  {  $i1 = \frac{a22 ug + R2 a21 ug}{\sigma_1}$ ,  $i2 = -\frac{ug}{\sigma_1}$ ,  $u1 = \frac{ug (a12 + R2 a11)}{\sigma_1}$ ,  $u2 = \frac{R2 ug}{\sigma_1}$  }
}

where
 $\sigma_1 = a12 + R2 a11 + R1 a22 + R1 R2 a21$ 
```

```
odziv[2]
{  $i1 = \frac{a22 ug + R2 a21 ug}{\sigma_1}$ ,  $i2 = -\frac{ug}{\sigma_1}$ ,  $u1 = \frac{ug (a12 + R2 a11)}{\sigma_1}$ ,  $u2 = \frac{R2 ug}{\sigma_1}$  }

where
 $\sigma_1 = a12 + R2 a11 + R1 a22 + R1 R2 a21$ 
```

```
A:=(u2|odziv[2])/ug
 $\frac{R2}{a12 + R2 a11 + R1 a22 + R1 R2 a21}$ 
Ru:=simplify((u1/i1)|odziv[2])
 $\frac{a12 + R2 a11}{a22 + R2 a21}$ 
simplify(i1|odziv[2])
 $\frac{ug (a22 + R2 a21)}{a12 + R2 a11 + R1 a22 + R1 R2 a21}$ 
```