

## Трофазно електрично коло: Y-Δ веза

Figure 1:

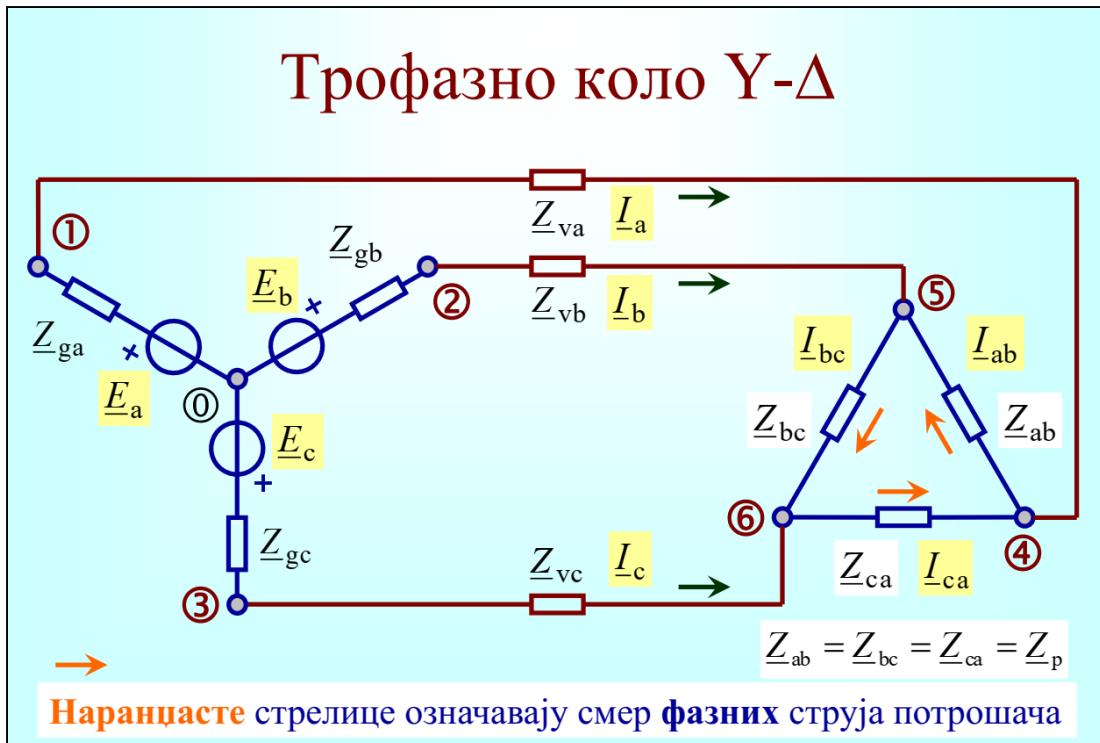
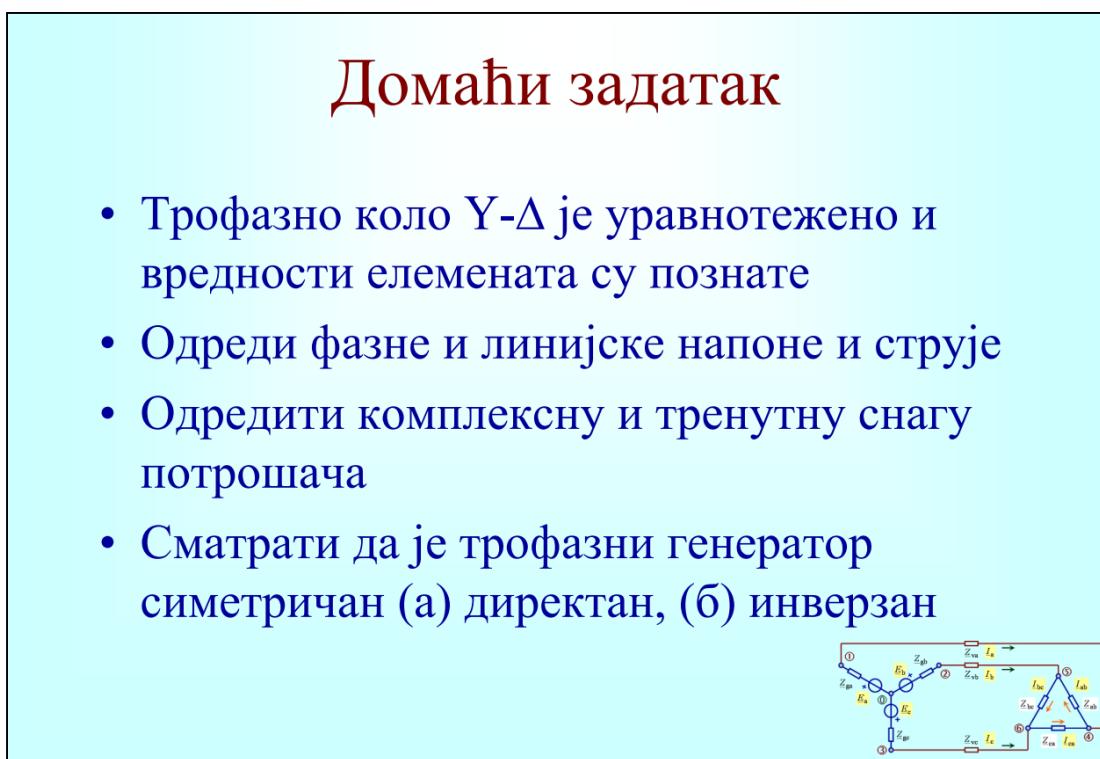


Figure 2:



$$(\%i1) \quad a: \%e^{(\%i \cdot 2 \cdot \%pi / 3)};$$

$$(\%o1) \quad \frac{\sqrt{3} \%i}{2} - \frac{1}{2}$$

(%i2) zamene: [Zga=Zg, Zgb=Zg, Zgc=Zg, Zva=Zv, Zvb=Zv,  
Zvc=Zv, Zab=Zp, Zbc=Zp, Zca=Zp];

(%o2) [Zga=Zg, Zgb=Zg, Zgc=Zg, Zva=Zv, Zvb=Zv, Zvc=Zv,  
Zab=Zp, Zbc=Zp, Zca=Zp]

(%i3) pobude: [Eb=Ea·a<sup>-1</sup>, Ec=Ea·a<sup>-2</sup>];

(%o3) [Eb= $\frac{Ea}{\frac{\sqrt{3} \cdot \%i}{2} - \frac{1}{2}}$ , Ec= $\frac{Ea}{\left(\frac{\sqrt{3} \cdot \%i}{2} - \frac{1}{2}\right)^2}$ ]

(%i4) jednacine: [Ea-Ia·(Zga+Zva)+Zca·Ica+Ic·(Zvc+Zgc)-Ec=0,  
Eb-Ib·(Zgb+Zvb)+Zab·Iab+Ia·(Zva+Zga)-Ea=0,  
Ec-Ic·(Zgc+Zvc)+Zbc·Ibc+Ib·(Zvb+Zgb)-Eb=0,  
Ia=Iab-Ica, Ib=-Iab+Ibc, Ic=-Ibc+Ica];

(%o4) [Ic (Zvc + Zgc) - Ia (Zva + Zga) + Ica Zca - Ec + Ea = 0, - Ib  
(Zvb + Zgb) + Ia (Zva + Zga) + Iab Zab + Eb - Ea = 0, - Ic (Zvc + Zgc) +  
Ib (Zvb + Zgb) + Ibc Zbc + Ec - Eb = 0, Ia = Iab - Ica, Ib = Ibc - Iab,  
Ic = Ica - Ibc]

(%i5) promenljive: [Ia, Ib, Ic, Iab, Ibc, Ica];

(%o5) [Ia, Ib, Ic, Iab, Ibc, Ica]

(%i6) odziv: linsolve(jednacine, promenljive), zamene;

(%o6) [Ia= $-\frac{Ec + Eb - 2 \cdot Ea}{3 \cdot Zv + Zp + 3 \cdot Zg}$ , Ib= $-\frac{Ec - 2 \cdot Eb + Ea}{3 \cdot Zv + Zp + 3 \cdot Zg}$ , Ic=  
 $\frac{2 \cdot Ec - Eb - Ea}{3 \cdot Zv + Zp + 3 \cdot Zg}$ , Iab= $\frac{Ea - Eb}{3 \cdot Zv + Zp + 3 \cdot Zg}$ , Ibc= $-\frac{Ec - Eb}{3 \cdot Zv + Zp + 3 \cdot Zg}$ , Ica=  
 $\frac{Ec - Ea}{3 \cdot Zv + Zp + 3 \cdot Zg}$ ]

(%i7) ev(Ib/Ia, odziv), pobude, ratsimp;

(%o7)  $-\frac{\sqrt{3} \cdot \%i + 1}{2}$

(%i8) ratsimp(%-a<sup>-1</sup>);

(%o8) 0

(%i9) ev(Ic/Ia, odziv), pobude, ratsimp;

(%o9)  $\frac{\sqrt{3} \cdot \%i - 1}{2}$

(%i10) ratsimp(%-a<sup>-2</sup>);

(%o10) 0

(%i11) U45:  $\text{ev}(\text{Iab} \cdot \text{Zab}, \text{odziv}), \text{zamene};$

$$(\%o11) \frac{(Ea - Eb) Zp}{3 Zv + Zp + 3 Zg}$$

(%i12) U56:  $\text{ev}(\text{Ibc} \cdot \text{Zbc}, \text{odziv}), \text{zamene};$

$$(\%o12) - \frac{(Ec - Eb) Zp}{3 Zv + Zp + 3 Zg}$$

(%i13) U64:  $\text{ev}(\text{Ica} \cdot \text{Zca}, \text{odziv}), \text{zamene};$

$$(\%o13) \frac{(Ec - Ea) Zp}{3 Zv + Zp + 3 Zg}$$

(%i14) (U56/U45-a^(-1)), pobude, ratsimp;

$$(\%o14) 0$$

(%i15) (U64/U45-a^(-2)), pobude, ratsimp;

$$(\%o15) 0$$

### Комплексна снага трофазног пријемника Y-Δ кола

→ Sp:  $\text{ev}(\text{U45} \cdot \text{conjugate}(\text{Iab}) + \text{U56} \cdot \text{conjugate}(\text{Ibc}) + \text{U64} \cdot \text{conjugate}(\text{Ica}), \text{odziv}), \text{zamene}, \text{pobude}, \text{ratsimp};$

$$(\%o22) \frac{9 Ea^2 Zp}{9 Zv^2 + (6 Zp + 18 Zg) Zv + Zp^2 + 6 Zg Zp + 9 Zg^2}$$

(%i17)  $\text{ev}(\text{Sp} - 3 \cdot \text{ev}(\text{U45} \cdot \text{conjugate}(\text{Iab}), \text{odziv}), \text{pobude}), \text{ratsimp};$

$$(\%o17) 0$$

### Тренутна снага трофазног пријемника

→ Pp:  $\text{sqrt}(2) \cdot U \cdot \cos(\omega \cdot t + \theta 45) \cdot \text{sqrt}(2) \cdot I \cdot \cos(\omega \cdot t + \psi 45) + \text{sqrt}(2) \cdot U \cdot \cos(\omega \cdot t + \theta 45 - 2 \cdot \%pi/3) \cdot \text{sqrt}(2) \cdot I \cdot \cos(\omega \cdot t + \psi 45 - 2 \cdot \%pi/3) + \text{sqrt}(2) \cdot U \cdot \cos(\omega \cdot t + \theta 45 - 4 \cdot \%pi/3) \cdot \text{sqrt}(2) \cdot I \cdot \cos(\omega \cdot t + \psi 45 - 4 \cdot \%pi/3);$

$$(\%o18) 2 I U \cos\left(t \omega + \theta 45 - \frac{2 \%pi}{3}\right) \cos\left(t \omega + \psi 45 - \frac{2 \%pi}{3}\right) + 2 I U \cos\left(t \omega + \theta 45 - \frac{4 \%pi}{3}\right) \cos\left(t \omega + \psi 45 - \frac{4 \%pi}{3}\right) + 2 I U \cos(t \omega + \theta 45) \cos(t \omega + \psi 45)$$

(%i19) `trigexpand(Pp);`

$$\begin{aligned}
 & 2 I U \left( \frac{\sin(\theta45) \sin(t\omega)}{2} + \frac{\sqrt{3} \cos(\theta45) \sin(t\omega)}{2} + \right. \\
 & \left. \frac{\sqrt{3} \sin(\theta45) \cos(t\omega)}{2} - \frac{\cos(\theta45) \cos(t\omega)}{2} \right) \cdot \\
 & \left( \frac{\sin(\psi45) \sin(t\omega)}{2} + \frac{\sqrt{3} \cos(\psi45) \sin(t\omega)}{2} + \right. \\
 & \left. \frac{\sqrt{3} \sin(\psi45) \cos(t\omega)}{2} - \frac{\cos(\psi45) \cos(t\omega)}{2} \right) + 2 I U \left( \right. \\
 & \left. \frac{\sin(\theta45) \sin(t\omega)}{2} - \frac{\sqrt{3} \cos(\theta45) \sin(t\omega)}{2} - \right. \\
 & \left. \frac{\sqrt{3} \sin(\theta45) \cos(t\omega)}{2} - \frac{\cos(\theta45) \cos(t\omega)}{2} \right) \cdot \\
 & \left( \frac{\sin(\psi45) \sin(t\omega)}{2} - \frac{\sqrt{3} \cos(\psi45) \sin(t\omega)}{2} - \right. \\
 & \left. \frac{\sqrt{3} \sin(\psi45) \cos(t\omega)}{2} - \frac{\cos(\psi45) \cos(t\omega)}{2} \right) + 2 I U \\
 & (\cos(\theta45) \cos(t\omega) - \sin(\theta45) \sin(t\omega)) \\
 & (\cos(\psi45) \cos(t\omega) - \sin(\psi45) \sin(t\omega))
 \end{aligned}$$

(%i20) `trigsimp(%);`

$$3 I U \sin(\theta45) \sin(\psi45) + 3 I U \cos(\theta45) \cos(\psi45)$$

Тренутна снага не зависи од времена – константна је!

(%i21) `trigrat(%);`

$$3 I U \cos(\psi45 - \theta45)$$